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An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty

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Abstract

In this work the design of a reverse distribution network is studied. Most of the proposed models on the subject are case based and, for that reason, they lack generality. In this paper we try to overcome this limitation and a generalized model is proposed. It contemplates the design of a generic reverse logistics network where capacity limits, multi-product management and uncertainty on product demands and returns are considered. A mixed integer formulation is developed which is solved using standard B&B techniques. The model is applied to an illustrative case.

Keywords: Reverse logistics; Optimization model; Uncertainty

1. Introduction

One of the first definitions of reverse logistics was provided by Stock (1992): "... the term often used to the role of logistics in recycling, waste disposal and management of hazardous materials; a broader perspective includes all issues relating to logistics activities carried out in source reduction, recycling, substitution, reuse of materials and disposal".

Later, Fleischmann (2001) proposes a new definition: "reverse logistics is the process of planning, implementing and controlling the efficient, effective inbound flow and storage of secondary goods and related information opposite to the traditional supply chain directions for the purpose of recovering value and proper disposal". In this definition products do not have to return to the origin. The author uses a broad concept of secondary products, which includes non-used products. When referring to *inbound flow* (...) *opposite to the traditional supply chain directions*, Fleischmann leaves out from this definition the reuse of organic waste.

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In the operations research perspective, the reverse logistics chain can be divided into three major areas: distribution, production planning and inventory.

In the distribution area, one of the first works published was by Gottinger (1988). Since then several models have been proposed which focus on such aspects as product recycling and planning/distribution. Some relevant and important works concerning product recycling, including closed loop chains, are Caruso et al. (1993), Kroon and Vrijens (1995), Barros et al. (1998), Listes and Dekker (2001), Giannikos (1998), Jayaraman et al. (1999) and Fleischmann et al. (2001). With the exception of this last work, all others are case dependent, which makes them difficult to apply to a variety of cases.

On the matter of planning, the works of Ammons et al. (1997), Spengler et al. (1997) and Krikke et al. (2001) simultaneously handle distribution and production planning. In addition Veerakamolmal and Gupta (2000), Sodhi and Reimer (2001) and Voigt (2001) explore aspects of production planning when product recovery is considered.

From the above mentioned works the most generic model for the design of reverse logistic networks is the one proposed by Fleischmann et al. (2001) (recovery network model – RNM). It considers the forward flow together with the reverse flow, allowing the simultaneous definition of the optimal distribution and recovery networks. A MILP formulation is proposed that constitutes an extension of the traditional warehouse location problem where two such models are integrated: one for the forward chain connecting factories to customers through warehouses, and the other for the reverse chain connecting customers to factories, through disassembly centres. The two chains are integrated by means of a balance constraint that assures, for each factory, that its total return is not greater than its total production. Two previously published case studies were used to test the model and a study on the benefit of integrating both chains was performed.

While the RNM appears quite general it still does not take into account three important characteristics of real problems, such as production/storage capacity limits, multi-product production and uncertainty in demand/return flows.

In this work, we try to overcome these drawbacks by presenting a new model which contemplates both distribution and planning aspects in the context of a reverse logistics network, and which allows for a multi-product environment with limited capacities and uncertainty in the demand and return flows. A MILP formulation model is proposed that extends the RNM model and allows easier application to real life problems, without a loss in generality. As each new model characteristic is added, we solve the model for an Iberian company case study and present the results.

This paper is structured as follows. In the next section, the problem is described in detail. Section 3 describes the Iberian case study. Section 4 presents the mathematical formulation of the model, with each choice of constraint being illustrated with the Iberian case study. The paper concludes with some final remarks and a discussion of future research.

2. Problem description

A reverse logistics network establishes a relationship between the market that releases used products and the market for "new" products. When these two markets coincide, we talk of a closed loop network, otherwise of an open loop.

The reverse logistics distribution problem is faced by industries that want to recover their end-of-life products or are in the recycling business, where used products are collected and partly incorporated into new products.

Fleischmann et al. (2001) define this problem as follows:

Given

- customers' demand and return,
- minimal disposal fraction,
- unit costs of demand and return,
- unit cost of disposal,
- penalty costs for non-satisfied demand and return,
- fixed costs for open/use factories, warehouses and distribution centres,

Determine
- the distribution and recovery networks,

So as to
- minimize total cost.

3. Case 1: Iberian company

To test the RNM model, a case of an office document company that operates in the Iberian market is solved. In addition to new products that are released and supplied to customers, the company is also considering remanufacturing some of its products collected after end use. The production and remanufacturing take place in factories, which supply customers through a set of warehouses. The collection of used products from customers is directed to a set of disassembly centres that in turn supply factories. This recover policy requires considerable foresight on the adequacy of the distribution/recovery network structure.

As a first case, a single product network will be created. The manager board requested a study on the best production location for this new product. Five different sites were proposed: Seville (factory 1), Salamanca (factory 2), Saragossa (factory 3), Viseu (factory 4) and Madrid (factory 5). In addition, eight possible locations were proposed for warehouses and five for disassembly centres.

Fifteen clusters of customers located in the same region were considered (for simplicity, these will be simply referred to as customers). Although this may seem to be a small number, we stress that this is a strategic model. It is not intended to create a detailed network, but rather to establish the major connections between customers' clusters and different potential locations for facilities, within a strategic environment.

All the necessary data were supplied by the company, but since this is treated as confidential, the data used in all cases were randomly generated, following different uniform distributions, as shown in Table 1. Note that, for the reverse chain, the "non-satisfied return" corresponds to the amount of the end use products that was not collected by disassembly centres. This amount is equivalent to the "non-satisfied demand" in the forward chain.

The solution of the model resulted in the network shown in Fig. 1.

The solution shows that the factories located in Viseu, Madrid and Seville were opened to serve five warehouses and four disassembly centres, so as to satisfy all customers' demand and return. Note that some customers are supplied by the factory that receives their returns, while others are supplied by a different one. Even

Table 1
Iberian case parameters, based in Fleischmann et al. (2001)

Description	Parameter	Values (m.u./u.)
Factory fixed cost	f^p	~Unif[32 000, 55 000]
Warehouse fixed cost	f^w	~Unif[8000, 12 000]
Disassembly centre fixed cost	f^r	~Unif[50 000, 72 000]
Transportation cost by distance and product		
Factory–warehouse	c^{pw}	2
Warehouse–costumer	c^{wc}	7
Costumer–disassembly centre	c^{cr}	5
Disassembly centre–factory	c^{rp}	4
Demand	d	~Unif[7000, 20 000]
Return	r	~Unif[5000, 13 000]
Not satisfied demand cost	c^u	~Unif[6000, 12 000]
Not satisfied return cost	c^w	~Unif[5000, 8000]
Model coefficients		
$c^f_{mijk} = c^{pw}t_{ij} + c^{wc}t_{jk}$, $c^r_{mkli} = c^{cr}t_{kl} + c^{rp}t_{li}$ where t_{ab} is the distance between a and b		

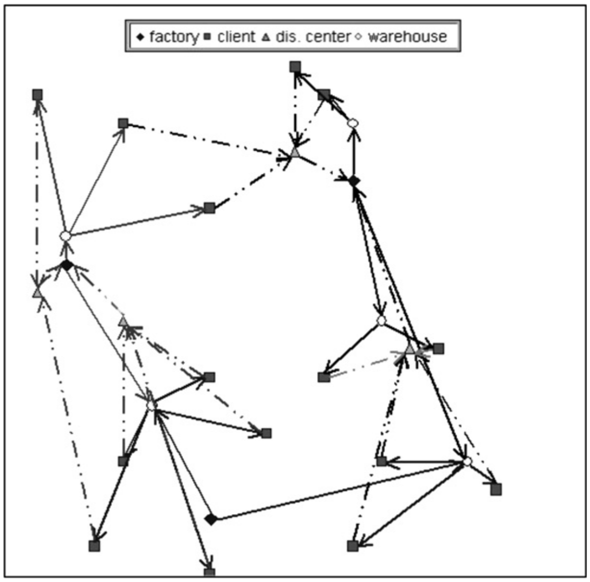

Fig. 1. Iberian case network – case 1.

Table 2
Iberian case computational results

Model	Total variables	Binary variables	Number of constraints	Number of LP's	CPU's (second)	Optimal value (10^3 m.u.)
RNM	1099	18	381	131	0.062	11 004

though this seems like a mixed open/closed loop situation, we are, in fact, in the presence of a closed loop network, since all factories belong to the same company.

The computational results are shown in Table 2. This and all values reported were obtained on a Pentium IV 3.065 GHz machine, running Windows XP Professional, using the commercial solver GAMS/CPLEX 8.1.

4. Model mathematical formulation

As described above, the RNM model (Fleischmann et al., 2001), although quite general, does not consider important aspects of actual logistics networks such as capacity limits, multi-product flows and uncertainty.

In this work, we have extended the proposed model to account for each of these aspects. First, the formulation of the recovery distribution network model with capacity constraints will be given. Then the new model will be modified to account for several products and finally uncertainty will be added. In each section an example based on the motivating case (Section 3) will be presented.

4.1. Recovery distribution network model with capacity constraints

When establishing a distribution network, managers have to take into account that facilities cannot process an unlimited volume of products. Their capacities can be limited for a variety of reasons such as shortage of space, machinery and staff. Capacity limits must therefore be considered in the modelling of factory production, warehouses storage and disassembly centres. Maximum and minimum bounds are considered in order to account for operational and physical restrictions.

Based on the problem description given in Section 2, the recovery distribution network model with capacity constraints involves the following sets, parameters and variables:

Sets
$I = \{1, \ldots, N_p\}$ potential factories,
$I_0 = I \cup \{0\}$ potential factories plus a disposal option,
$J = \{1, \ldots, N_w\}$ potential warehouses,
$K = \{1, \ldots, N_c\}$ potential customers,
$L = \{1, \ldots, N_r\}$ potential disassembly centres.

Parameters
d_k customer k demand in the reuse market, $k \in K$,
r_k customer k return of used products, $k \in K$,
γ minimal disposal fraction,
c^f_{ijk} unit variable cost of demand served by factory i, through warehouse j, to customer k (may include transportation, production costs, …) $i \in I$, $j \in J$, $k \in K$,
c^r_{kli} unit variable cost of customer k return, made trough disassembly centre l, to factory i (may include transportation costs, reprocessing costs, …) $k \in K$, $l \in L$, $i \in I$,
c^r_{kl0} unit variable cost of customer k return disposal, made through warehouse l, $k \in K$, $l \in L$,
c^u_k unit variable cost of customer k non-satisfied demand, $k \in K$,
c^w_k unit variable cost of customer k non-satisfied return, $k \in K$,
f^p_i fixed cost of opening factory i, $i \in I$,
f^w_j fixed cost of opening warehouse j, $j \in J$,
f^r_l fixed cost of opening disassembly centre l, $l \in L$,
g^p_i maximum capacity of factory i, $i \in I$,
g^w_j maximum capacity of warehouse j, $j \in J$,
g^r_l maximum capacity of disassembly centre l, $l \in L$,
t^p_i minimum capacity of factory i, $i \in I$,
t^w_j minimum capacity of warehouse j, $j \in J$,
t^r_l minimum capacity of disassembly centre l, $l \in L$.

Variables
X^f_{ijk} demand fraction served by factory i, through warehouse j, to customer k, $i \in I, j \in J, k \in K$ – forward flow,
X^r_{kli} return fraction of customer k, through disassembly centre l, to factory i, $k \in K$, $l \in L$, $i \in I_0$ – return flow,
U_k non-satisfied demand fraction of customer k, $k \in K$,
W_k non-satisfied return fraction of customer k, $k \in K$,
$Y^p_i = 1$ if factory i is opened, $i \in I$,
$Y^w_j = 1$ if warehouse j is opened, $j \in J$,
$Y^r_l = 1$ if disassembly centre l is opened, $l \in L$.

Using the above definitions, the model is formulated as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{i\in I} f^p_i Y^p_i + \sum_{j\in J} f^w_j Y^w_j + \sum_{l\in L} f^r_l Y^r_l + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} c^f_{ijk} d_k X^f_{ijk} + \sum_{k\in K}\sum_{l\in L}\sum_{i\in I_0} c^r_{kli} r_k X^r_{kli} \\ & + \sum_{k\in K} c^u_k d_k U_k + \sum_{k\in K} c^w_k r_k W_k \end{aligned} \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in I}\sum_{j\in J} X^f_{ijk} + U_k = 1, \quad \forall k \in K, \tag{2}$$

$$\sum_{l\in L}\left(\sum_{i\in I} X^r_{kli} + X^r_{kl0}\right) + W_k = 1, \quad \forall k \in K, \tag{3}$$

$$\sum_{k\in K}\sum_{l\in L} r_k X^r_{kli} \leqslant \sum_{j\in J}\sum_{k\in K} d_k X^f_{ijk}, \quad \forall i \in I, \tag{4}$$

$$\gamma \sum_{i \in I} X^r_{kli} \leqslant X^r_{kl0}, \quad \forall l \in L, k \in K, \tag{5}$$

$$\sum_{k \in K} \sum_{j \in J} d_k X^f_{ijk} \leqslant g^p_i Y^p_i, \quad \forall i \in I, \tag{6}$$

$$\sum_{k \in K} \sum_{j \in J} d_k X^f_{ijk} \geqslant t^p_i Y^p_i, \quad \forall i \in I, \tag{7}$$

$$\sum_{i \in I} \sum_{k \in K} d_k X^f_{ijk} \leqslant g^w_j Y^w_j, \quad \forall j \in J, \tag{8}$$

$$\sum_{i \in I} \sum_{k \in K} d_k X^f_{ijk} \geqslant t^w_j Y^w_j, \quad \forall j \in J, \tag{9}$$

$$\sum_{i \in I_0} \sum_{k \in K} r_k X^r_{kli} \leqslant g^r_l Y^r_l, \quad \forall l \in L, \tag{10}$$

$$\sum_{i \in I_0} \sum_{k \in K} r_k X^r_{kli} \geqslant t^r_l Y^r_l, \quad \forall l \in L, \tag{11}$$

$$0 \leqslant X^f_{ijk}, X^r_{kli}, U_k, W_k \leqslant 1, \quad Y^p_i, Y^w_j, Y^r_l \in \{0, 1\}. \tag{12}$$

As this model is built on the one proposed by Fleischmann et al. (2001), it minimizes costs, Eq. (1), with the first three terms representing respectively the cost of opening factories (i), warehouses (w) and disassembly centres (l). The fourth term represents the cost of satisfying demands, while the fifth term is related to the recovery of the end-of-life materials. Finally, the last two terms are related respectively to the non-satisfaction of demand and of return. Constraint (2) assures that the demand for all customers is taken into account, either by being supplied (first term of the equation) or by being allocated to the non-satisfied demand variable (second term). In terms of customers' return the same approach is adopted in Eq. (3), while the difference in the first term is due to the possibility that the customers' return may undergo disposal. Specifically, this equation considers three different return fractions: to factory, to disposal and non-satisfied. Constraint (4) assures the model balance, in that, for each factory, the return volume cannot exceed the demand volume. Constraint (5) models a minimum disposal fraction. Eqs. (6)–(11) are the capacity constraints, which make redundant the usual facilities open constraints. Each pair of constraints corresponds to a different kind of facility: (6) and (7) establish maximum and minimum limits for factories; (8) and (9) for warehouses and (10) and (11) for disassembly centres.

As shown by Fleischmann et al. (2001), this model incorporates some of the unique characteristics of the reverse chain. Both open and closed loop networks can be considered, as well as the return of products (this latter being dictated either by market forces or by legislation). To model open or closed loop networks, parameters d_k and r_k are employed. For $d_k r_k > 0$, i.e. both parameters positive, a closed loop supply chain is modelled, while for $d_k r_k = 0$ the supply chain operates in an open loop. In terms of return, the choice of a high value for the penalty cost c^u_k, which implies low values for W_k, reflects legislation enforcement; conversely, the choice of a low c^u_k reflects market awareness for recycling/remanufacture alternatives.

Please note that the market for remanufacture products may differ from the market of new products, thus for some customers k, $d_k = 0$.

Case 2: To analyse the effect of these new constraints, the motivating case was enlarged so as to encompass capacity limits imposed on all facilities. These values are shown in Table 3.

Table 4 and Fig. 2 show the results from solving the new case.

Table 3
Facility capacity limits for case 2

Site	Minimum capacity	Maximum capacity
Factories	50000	70000
Warehouses	20000	50000
Disassembly centres	20000	40000

Table 4
Capacity constraint model computational results: case 2

Model	Number of constraints	Number of LP's	CPU's (second)	Optimal value (10^3 m.u.)
Capacity	147	207	0.078	11115

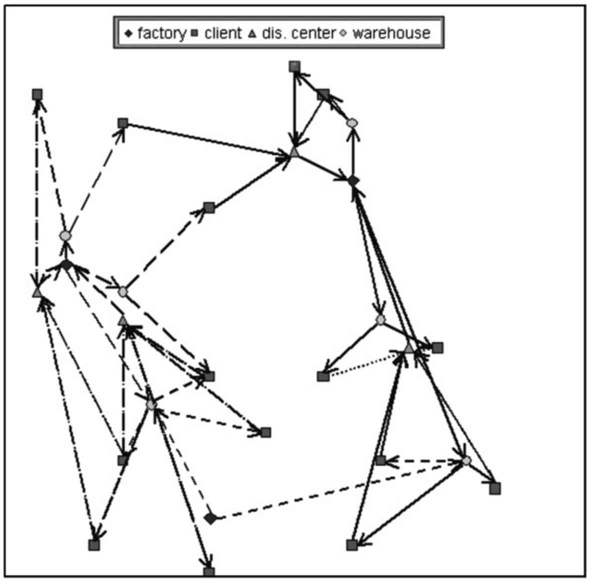

Fig. 2. Case 2 network.

In computational terms one should notice that the number of constraints has decreased substantially (147 as compared to 381). The main reason is that the new capacity constraints are smaller in number than the usual open constraints for facilities, which were made redundant, as stated above. The CPU time obtained when compared to case 1 was of the same order of magnitude (0.078 against 0.062).

In terms of the network, it can be seen that these capacity constraints resulted in the opening of one additional warehouse. The factories and disassembly centres remain the same as in Fig. 1. However, some customers are now served by different facilities. One may conclude that production/storage limitations do impact the design of the optimal distribution network.

4.2. Multi-product formulation

Another restriction in the RNM model is that it considers only a single product. Since this situation is unusual in a real distribution network, we propose a new formulation that allows for multiple products.

Consider a set $M = \{1,\ldots,N_v\}$ for the existing products. Almost all variables and parameters will be from now on product dependent. For instance, X^f_{ijk} is now replaced by X^f_{mijk} which is defined as the demand fraction of product m served by factory i, through warehouse j, to customer k. Constraints (2)–(5), which remained unchanged when capacity constraints were introduced, are now generalized to account for the multi-product situation:

$$\sum_{i\in I}\sum_{j\in J} X^f_{mijk} + U_{mk} = 1, \quad \forall m \in M,\ k \in K, \tag{13}$$

$$\sum_{l\in L}\left(\sum_{i\in I} X^r_{mkli} + X^r_{mkl0}\right) + W_{mk} = 1, \quad \forall m \in M,\ k \in K, \tag{14}$$

$$\sum_{k\in K}\sum_{l\in L} r_{mk}X^{r}_{mkli} \leqslant \sum_{j\in J}\sum_{k\in K} d_{mk}X^{f}_{mijk}, \quad \forall m\in M,\ i\in I, \tag{15}$$

$$\gamma\sum_{i\in I} X^{r}_{mkli} \leqslant X^{r}_{mkl0}, \quad \forall m\in M,\ l\in L,\ k\in K. \tag{16}$$

Constraints (6)–(11), which impose limits to facility capacities, will be replaced by constraints that will limit the total capacity of facilities. These limits refer to the total processing/warehousing of all products that run through the network.

As an example, the changes in factory capacity constraints are presented. Changes are similar for the remaining constraints. Constraints (6) and (7) are now rewritten as follows:

$$\sum_{m\in M}\sum_{k\in K}\sum_{j\in J} d_{mk}X^{f}_{mijk} \leqslant g^{p}_{i}Y^{p}_{i}, \quad \forall i\in I, \tag{17}$$

$$\sum_{m\in M}\sum_{k\in K}\sum_{j\in J} d_{mk}X^{f}_{mijk} \geqslant t^{p}_{i}Y^{p}_{i}, \quad \forall i\in I. \tag{18}$$

Furthermore, we can easily account for the possibility of limiting the production of each product:

$$\sum_{k\in K}\sum_{j\in J} d_{mk}X^{f}_{mijk} \leqslant g^{p}_{\mathrm{mi}}Y^{p}_{i}, \quad \forall m\in M,\ i\in I, \tag{19}$$

$$\sum_{k\in K}\sum_{j\in J} d_{mk}X^{f}_{mijk} \geqslant t^{p}_{\mathrm{mi}}Y^{p}_{i}, \quad \forall m\in M,\ i\in I, \tag{20}$$

where g^{p}_{mi} and t^{p}_{mi} are, respectively, the maximum and minimum production limits of product m for factory i.

Case 3: Case 2 is now implemented for two products. Product one has the same data used in the previous cases, while the characteristics of product two are presented in Table 5.

Table 6 gives the results for Case 3. Although the number of constraints and variables has increased (259 as compared to 147), the CPU times did not deteriorate significantly (0.156 as compared to 0.078 seconds).

Since two different products are considered, the final structure is associated with two different distribution networks (Fig. 3). Note that the global network structure is more intricate not only as a result of the two products, but also because of the introduction of capacity constraints.

Looking separately at each distribution network, we see that all customers' demands and returns were satisfied for product 1. This result was achieved through the use of more than one factory/warehouse or disassembly centre/factory. In contrast, only a single factory/warehouse and disassembly centre/factory were used in case 2 (Fig. 2).

Table 5
Product 2 parameters: case 3

Description	Parameter	Values (m.u.)
Transportation cost by distance and product		
Factory–warehouse	c^{pw}	4
Warehouse–costumer	c^{wc}	8
Costumer–disassembly centre	c^{cr}	6
Disassembly centre–factory	c^{rp}	2
Demand	d	$\sim$Unif[5000, 15000]
Return	r	$\sim$Unif[3000, 8000]
Not satisfied demand cost	c^{u}	$\sim$Unif[6000, 12000]
Not satisfied return cost	c^{w}	$\sim$Unif[5000, 8000]
Model coefficients		
$c^{f}_{ijk} = c^{pw}t_{ij} + c^{wc}t_{\mathrm{jk}}, \quad c^{r}_{kli} = c^{cr}t_{kl} + c^{rp}t_{li}$		
where t_{ab} is the distance between a and b		

Table 6
Case 3 computational results

Model	Total variables	Binary variables	Number of constraints	Number of LP's	CPU's (seconds)	Optimal value (10^3 m.u.)
Multi	2179	18	259	444	0.156	20420

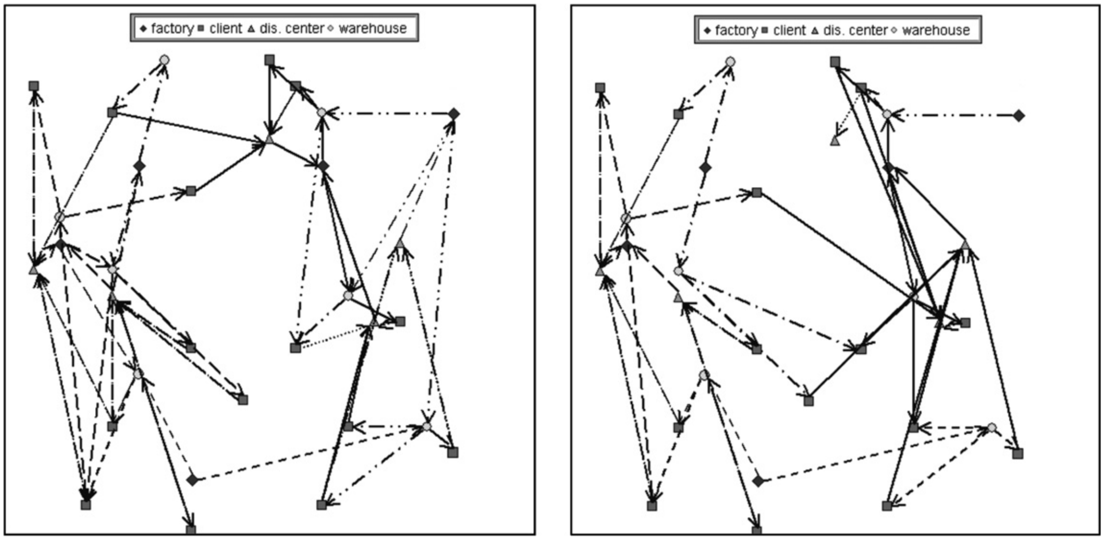

Fig. 3. Case 3 networks for product 1 and 2, respectively.

With regard to product 2, again all customers' demands and returns were satisfied. In Fig. 3 it seems that one disassembly centre was opened just to collect one customer's return sent totally for disposal (because there is no link between that centre and the factory located in Salamanca). However in the network for product 1, the same disassembly centre serves three more customers and sends the returned products to the factory located in Madrid.

It is important to note that the global structure is obtained for the two products and the distribution is made simultaneously for both products.

4.3. Uncertainty

Literature shows that one of the major problems in the reverse logistic chains is the uncertainty associated with demand and return. Hence, this problem should be addressed when trying to model such systems. In this paper we employ a scenario-based approach to model such conditions.

4.3.1. Uncertainty approach

Let Θ be the set of all possible scenarios and $\theta \in \Theta$ a particular scenario. Also, let all binary variables be included in vector y and all the continuous variables in vector x. Let f be the vector of the fixed costs related to the opening of the facilities and c the vector containing the remaining coefficients in the objective function.

For a particular scenario θ, the concise model can be stated as follows:

$$\begin{aligned} \min \quad & fy + c_\theta x \\ & A_\theta x \leqslant a_\theta, \\ & B_\theta x \leqslant Cy, \\ & y \in \{0,1\}, \quad x \geqslant 0, \end{aligned}$$

where A_θ, B_θ and C are matrices and a_θ is a vector.

Consider π_θ to be the probability of scenario θ. Since we only wish to model a finite number of discrete scenarios, the expected value function becomes an ordinary sum. The uncertainty model is then given by the following mixed integer formulation:

$$\begin{aligned} \min \quad & fy + \sum_\theta \pi_\theta c_\theta x_\theta \\ & A_\theta x_\theta \leqslant a_\theta, \\ & B_\theta x_\theta \leqslant Cy, \\ & y \in \{0,1\}, \quad x_\theta \geqslant 0. \end{aligned}$$

Note that the obtained solution for this model is not optimal for individual scenarios, but it gives the network structure for the worst possible scenario. For further details, please refer to the work of Birge and Louveaux (1997).

4.3.2. Detailed model

Assuming that only customers' demand and return are scenario-dependent, the remaining parameters have the same values in the three scenarios. This includes the binary variables, which translate the condition of whether or not facilities are opened. As a consequence, only the continuous variables will vary with the scenario.

Based on the uncertainty approach described above, the capacitated multi-product reverse network model can be defined as follows:

Sets

In addition to the sets described in Section 4.1, we add
$M = \{1, \ldots, N_v\}$ existing products,
$S = \{1, \ldots, N_s\}$ potential scenarios.

Parameters

The same parameters defined in Section 4.1 will be employed. However some will require generalization for product m and scenario s, and a scenario probability needs to be added, as follows:
d_{mks} demand of product m, of customer k in the reuse market, for scenario s, $m \in M$, $k \in K$, $s \in S$,
r_{mks} return of used product m, of customer k, for scenario s, $m \in M$, $k \in K$, $s \in S$,
c^f_{mijk} unit variable cost of demand of product m, served by factory i, through warehouse j, to customer k (may include transportation, production costs, ...) $m \in M$, $i \in I$, $j \in J$, $k \in K$,
c^r_{mkli} unit variable cost of product m return by customer k, through disassembly centre l, to factory i (may include transportation costs, reprocessing costs, ...), $m \in M$, $k \in K$, $l \in L$, $i \in I$,
c^r_{mkl0} unit variable cost of product m disposal collected in customer k, through disassembly centre l, $m \in M$, $k \in K$, $l \in L$,
c^u_{mk} unit variable cost for non-satisfied demand of product m for customer k, $m \in M$, $k \in K$,
c^w_{mk} unit variable cost for non-satisfied return of product m, of customer k, $m \in M$, $k \in K$,
π_s probability of scenario s, $s \in S$.

Variables

Similar to parameters, some variables used in Section 4.1 will be modified, as follows:
X^f_{mijks} demand fraction of product m, served by factory i, through warehouse j, to customer k, in scenario s, $m \in M$, $i \in I$, $j \in J$, $k \in K$, $s \in S$ – forward flow,
X^r_{mklis} return fraction of product m, made by customer k, through disassembly centre l, to factory i, in scenario s, $m \in M$, $k \in K$, $l \in L$, $i \in I_0$, $s \in S$ – reverse flow,
U_{mks} non-satisfied demand fraction of product m, of customer k, in scenario s, $m \in M$, $k \in K$, $s \in S$,
W_{mks} non-satisfied return fraction of product m of customer k, in scenario s, $m \in M$, $k \in K$, $s \in S$.

Formulation

$$\begin{aligned}
\text{Min}\quad & \sum_{m\in M}\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{s\in S}\pi_s c^f_{mijk} d_{mks} X^f_{mijks} + \sum_{m\in M}\sum_{k\in K}\sum_{l\in L}\sum_{i\in I_0}\sum_{s\in S}\pi_s c^r_{mkli} r_{mks} X^r_{mklis} \\
& + \sum_{m\in M}\sum_{k\in K}\sum_{s\in S}\pi_s c^u_{mk} d_{mks} U_{mks} + \sum_{m\in M}\sum_{k\in K}\sum_{s\in S}\pi_s c^w_{mk} r_{mks} W_{mks} \\
& + \sum_{i\in I} f^p_i Y^p_i + \sum_{j\in J} f^w_j Y^w_j + \sum_{l\in L} f^r_l Y^r_l \qquad (21)
\end{aligned}$$

$$\text{s.t.}\quad \sum_{i\in I}\sum_{j\in J} X^f_{mijks} + U_{mks} = 1, \quad \forall m\in M,\ k\in K,\ s\in S \qquad (22)$$

$$\sum_{l\in L}\left(\sum_{i\in I} X^r_{mklis} + X_{mkl0s}\right) + W_{mks} = 1, \quad \forall m\in M,\ k\in K,\ s\in S \qquad (23)$$

$$\sum_{k\in K}\sum_{l\in L} r_{mks} X^r_{mklis} \leqslant \sum_{j\in J}\sum_{k\in K} d_{mks} X^f_{mijks}, \quad \forall m\in M,\ i\in I,\ s\in S \qquad (24)$$

$$\gamma \sum_{i\in I} X^r_{mklis} \leqslant X^r_{mkl0}, \quad \forall m\in M,\ l\in L,\ k\in K,\ s\in S \qquad (25)$$

$$\sum_{m\in M}\sum_{k\in K}\sum_{j\in J} d_{mks} X^f_{mijks} \leqslant g^p_i Y^p_i, \quad \forall i\in I,\ s\in S \qquad (26)$$

$$\sum_{m\in M}\sum_{k\in K}\sum_{j\in J} d_{mks} X^f_{mijks} \geqslant t^p_i Y^p_i, \quad \forall i\in I,\ s\in S \qquad (27)$$

$$\sum_{m\in M}\sum_{i\in I}\sum_{k\in K} d_{mks} X^f_{mijks} \leqslant g^w_j Y^w_j, \quad \forall j\in J,\ s\in S \qquad (28)$$

$$\sum_{m\in M}\sum_{i\in I}\sum_{k\in K} d_{mks} X^f_{mijks} \geqslant t^w_j Y^w_j, \quad \forall j\in J,\ s\in S \qquad (29)$$

$$\sum_{m\in M}\sum_{i\in I_0}\sum_{k\in K} r_{mks} X^r_{mklis} \leqslant g^r_l Y^r_l, \quad \forall l\in L,\ s\in S \qquad (30)$$

$$\sum_{m\in M}\sum_{i\in I_0}\sum_{k\in K} r_{mks} X^r_{mklis} \geqslant t^r_l Y^r_l, \quad \forall l\in L,\ s\in S \qquad (31)$$

$$0 \leqslant X^f_{mijks}, X^r_{mklis}, U_{mks}, W_{mks} \leqslant 1, \quad Y^p_i, Y^w_j, Y^r_l \in \{0,1\}. \qquad (32)$$

This model not only maintains the characteristics of the model proposed by Fleischmann et al. (2001), where it is possible to model open or closed loop supply chains and demand push or pull drivers, but also allows the modelling of a multi-product network with capacity constraints.

Case 4: Since the previous networks were designed for new products, the data provided by the company were based on estimated values. Consequently, a model accounting for some uncertainty in the provided data was identified as a useful tool. A scenario-based model was then built, which can provide the decision makers with an adequate framework to investigate different patterns of product demand and return.

Again as data are confidential, different values were generated to run case 4. So the previous two products will again be considered, together with capacity constraints. Three scenarios for each product will be established.

The scenarios for demand and return are described in Table 7. The data from case 3 are preserved in scenario 1 for both products. Scenario 2 of product 1 models the most pessimistic situation and scenario 3 models an optimistic one. For product 2 similar economic situations are also modelled: scenario 2 is optimistic while scenario 3 is pessimistic.

For each scenario there is an associated probability value (10%, 75% and 15%, respectively). The example is solved with CPLEX version 8.1 and the results are presented in Table 8 and Figs. 4 and 5.

Analysing the computational results, it can be seen that, as expected, the model size increased significantly and the CPU time grew accordingly, together with the number of LP's needed to solve the case instance.

Fig. 4 presents the various scenario networks created for product 1. It is found that all customers are served and that 100% service, in both demand and return, is guaranteed. This is shown in Figs. 5–7.

Table 7
Demand and return scenarios for product 1 and 2: scenarios probabilities

Product	Scenario	Demand	Return	Scenario probability
1	1	∼Unif[7000, 20 000]	∼Unif[5000, 13 000]	0.10
	2	∼Unif[1000, 10 000]	∼Unif[500, 8000]	0.75
	3	∼Unif[15 000, 20 000]	∼Unif[5000, 15 000]	0.15
2	1	∼Unif[5000, 15 000]	∼Unif[3000, 8000]	0.10
	2	∼Unif[15 000, 25 000]	∼Unif[11 000, 6000]	0.75
	3	∼Unif[5000, 7000]	∼Unif[2000, 6000]	0.15

Table 8
Case 4 computational results

Model	Total variables	Binary variables	Number of constraints	Number of LP's	CPU's (second)	Optimal value (10^3 m.u.)
Scenarios	6499	18	769	5195	2.171	10 805

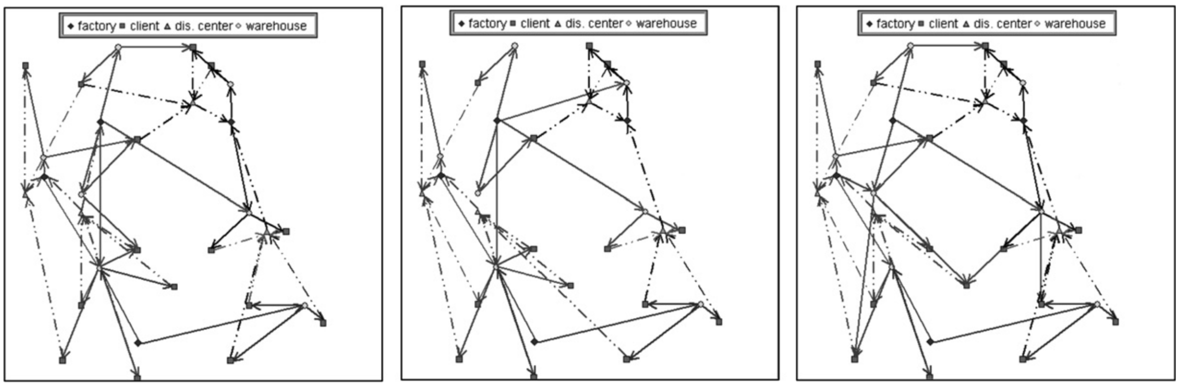

Fig. 4. Case 4 networks for product 1 in scenarios 1, 2 and 3, respectively.

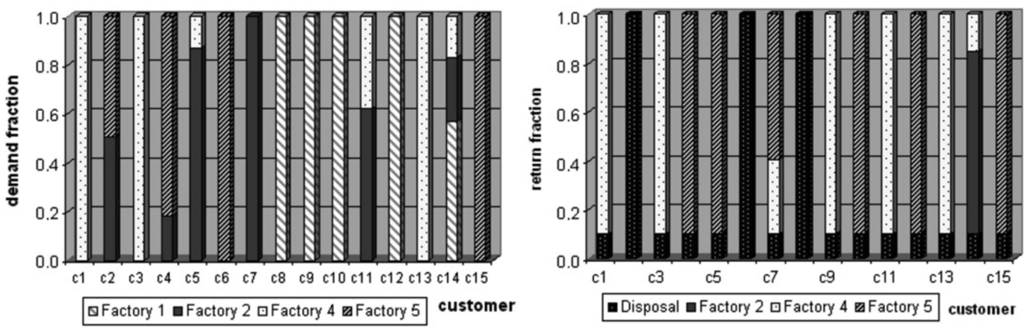

Fig. 5. Customers' demand and return for product 1, in scenario 1.

In Fig. 5, the graphic reveals the fraction of customers' demand and return that is served by each factory. For example, in scenario 1, customer c1 has his demand for product 1 totally supplied by the factory located in Viseu (factory 4), while customer c14 has his demand supplied by three different factories: Seville (factory 1), Salamanca (factory 2) and Viseu (factory 4). In terms of return, customer c2 return is entirely disposed, while for customer c14, only 0.1 of his return is disposed, a fraction slightly above 0.7 is sent to the factory in Saragossa and the remaining to the factory in Viseu.

It can be noted in Figs. 5–7, how customer c2 has his demand differently met in all three scenarios: in scenario 1 by the factories in Salamanca and Madrid (factory 5), in scenario 2 wholly by the Salamanca factory, and in scenario 3 wholly by the Madrid factory. The solution found for customer c15 is the opposite: supply is guaranteed by Madrid in all three scenarios. In terms of return, the solutions found for all the scenarios are not very different. The same solution is found for all customers, exception made for customers c7, c8, c10 and c14, which have their return taken care by different factories.

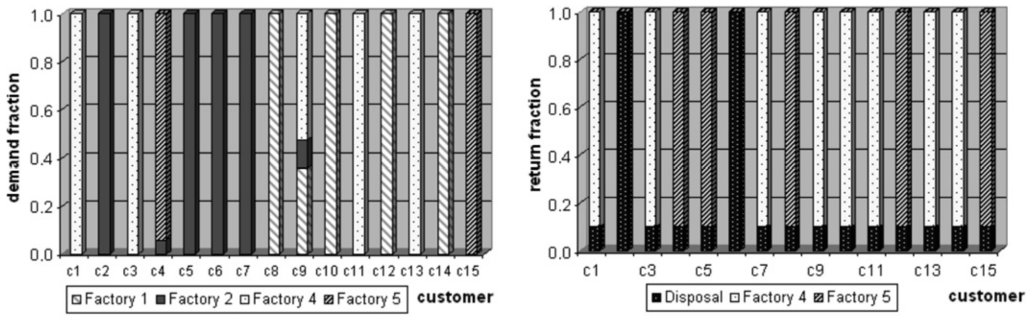

Fig. 6. Customers' demand and return for product 1, in scenario 2.

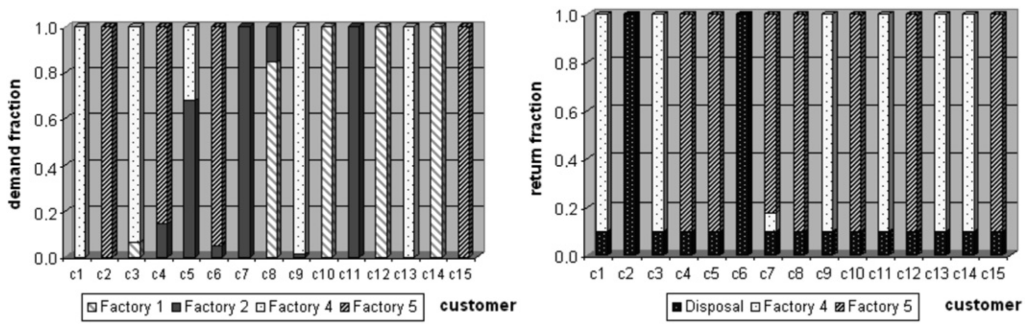

Fig. 7. Customers' demand and return for product 1, in scenario 3.

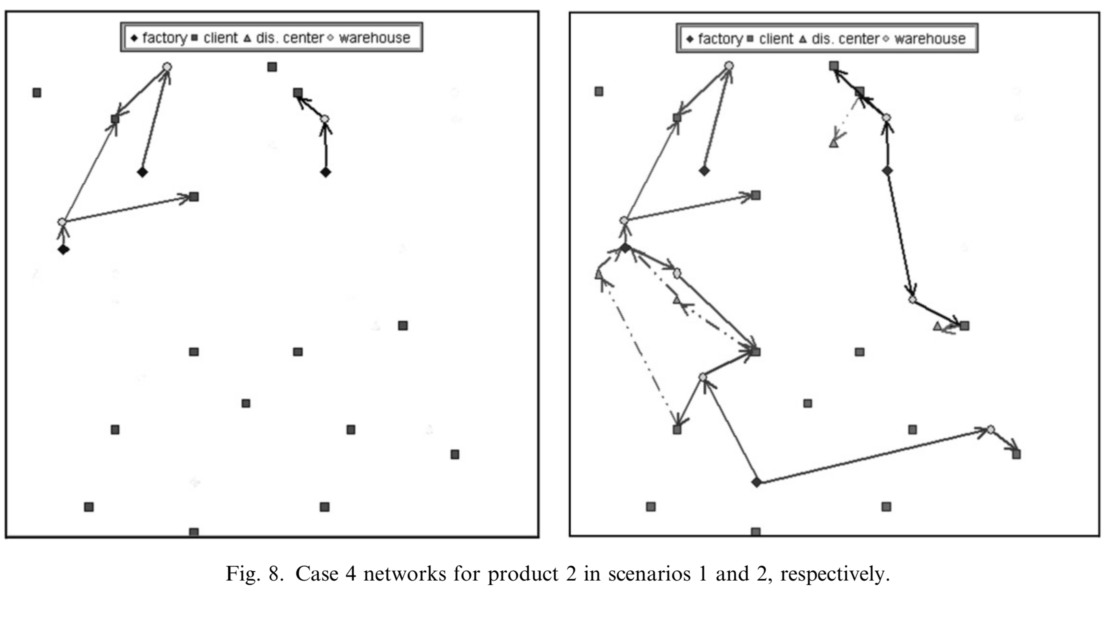

Fig. 8. Case 4 networks for product 2 in scenarios 1 and 2, respectively.

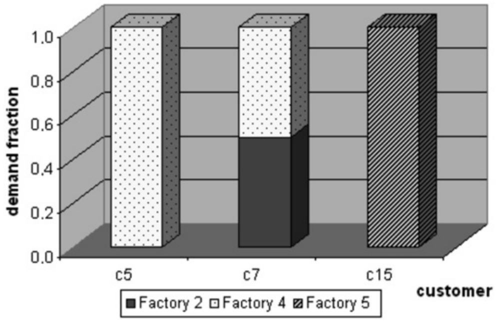

Fig. 9. Customers' demand and return for product 2, in scenario 1.

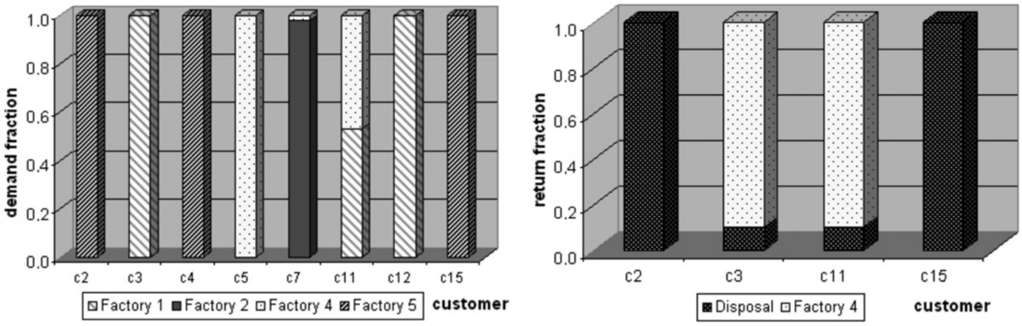

Fig. 10. Customers' demand and return for product 2, in scenario 2.

For product 2, two networks were obtained, scenario 1 and 2, since for scenario 3 no network was deemed economically viable (Fig. 8).

It can be noted in Fig. 8 that not all costumers were served. In scenario 1, customers c5, c7 and c15 have their demand met (Fig. 9). In scenario 2, the same three customers and an additional five also have their demand guaranteed. For some, their return is collected, while for others (c2 and c15) it is disposed (Fig. 10).

5. Conclusions

The number of scientific publications in the field of reverse logistics has been steadily growing, reflecting the increasing importance of this subject, mainly as a result of society's accrued environmental awareness. However, taking a closer look into the distribution planning area, one can notice that reported work is mostly case dependent and thus general models are missing, in particular models that relate reverse and forward chains.

This work proposes a generalised model for the design of reverse logistics networks. The model is based on the recovery network model (RNM) proposed by Fleischmann et al. (2001). While appearing to be the most general in the literature, the RNM still contains some important limitations. In particular, it assumes unlimited capacity of facilities, it deals with a single product formulation, and it does not account for any kind of uncertainty (e.g. in product demand and return). In order to address these drawbacks, this work extends the RNM model and develops a capacitated multi-product reverse logistics network model with uncertainty.

The capacity constraints are imposed on total production/storage capacity of the facilities, which may be factories, warehouses or distribution centres. Our expanded formulation allows for any number of products, establishing a network for each product while guaranteeing total capacities for each facility at a minimum cost. Furthermore, the general method was studied in the context of uncertainty in both product demands and returns, through the use of a multi-scenario approach.

The generality of the model was corroborated and very satisfactory computational times were obtained. However, it is important to note that as the problem size increases, the computational burden might be expected to grow accordingly.

Due to this foreseen limitation, future work by the authors includes the application of decomposition methods to the model. These methods can be generic as, for instance, benders decomposition, or methods that will explore directly the problem structure.

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