IE 554 Project Proposal

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Problem Description

Graph partitioning is a key topic in optimization. We study the **Dominator Partition Problem**, introduced by Hedetniemi and Haynes (2006) [1], and propose an exact integer programming model with potential improvements.

In graph $G = (V, E)$, a vertex v *dominates* set $S \subseteq V$ if it's adjacent to all $u \in S$. A **dominator partition** divides V into k blocks, each dominated by some $v \in V$. The goal is to find the smallest such k, called $\pi_d(G)$.

Our Plan

We gave the following IP model for the dominator partition problem for a fixed k:

Sets & Parameters
V: set of vertices in the graph, of size n
a_{vu}: 1 if vertices v and u are adjacent
k: number of blocks in the partition

Decision Variables
x_{vi}: 1 if vertex v is assigned to the ith block
d_{vi}: 1 if vertex v dominates block i

Objective Function & Constraints

$$\min \quad 0 \qquad \text{(no objective)}$$

$$\text{s.t.} \quad \sum_{i=1}^{k} x_{vi} = 1, \quad \forall v \in V \qquad \text{(each vertex is assigned to one block)}$$

$$\sum_{v \in V} x_{vi} \geq 1 \quad \forall i \in \{1, 2, \ldots, k\} \qquad \text{(no empty blocks)}$$

$$x_{ui} \leq a_{vu} + (1 - d_{vi}) \quad \forall u, v \in V, i \in \{1, 2, \ldots, k\} \qquad \text{(domination condition)}$$

$$\sum_{i=1}^{k} d_{vi} \geq 1 \quad \forall v \in V \qquad \text{(each vertex dominates at least one block)}$$

$$\sum_{v \in V} x_{vi} \geq \sum_{v \in V} x_{v,i+1} \quad \forall i \in \{1, 2, \ldots, k-1\} \qquad \text{(blocks are used in order)}$$

$$x_{vi}, d_{vi} \quad \forall v \in V, i \in \{1, 2, \ldots, k\} \qquad \text{(binary variables)}$$

The planned contributions are:

Exact IP Model: First IP formulation for dominator partitioning.

Model Strengthening: Add valid inequalities.

Experiments: Compare original vs. improved models on graphs of various sizes.

Literature Review

Most existing work is theoretical, focused on complexity and special graph classes [1]. Determining $\pi_d(G)$ is NP-complete, but no exact IP formulation exists in prior literature. Our study fills this gap.

References

[1] S. M. Hedetniemi et al., *Dominator Partitions of Graphs*, 2008.