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IE 554 Project – Mathematical Models

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1 Introduction

2 Mathematical Formulation of Dominator Partition Problem

Sets & Parameters

$G = (V, E)$: a graph with vertex set V and edge set E

π: dominator partition of size k of G

Decision Variables

x_{vi}: 1 if vertex v is assigned to the i^{th} block

d_{vi}: 1 if vertex v dominates block i

Objective Function & Constraints

$$
\begin{aligned}
\min \quad & 0 && \text{(no objective)} \\
\text{s.t.} \quad & \sum_{i=1}^{k} x_{vi} = 1, \quad \forall v \in V && \text{(each vertex is assigned to one block)} \\
& \sum_{v \in V} x_{vi} \geq 1 \quad \forall i \in \pi && \text{(no empty blocks)} \\
& x_{ui} + d_{vi} \leq 1 \quad \{\forall u, v \in V \mid \{u, v\} \notin E\}, i \in \pi && \text{(domination condition)} \\
& \sum_{i=1}^{k} d_{vi} \geq 1 \quad \forall v \in V && \text{(each vertex dominates at least one block)} \\
& \sum_{v \in V} x_{vi} \geq \sum_{v \in V} x_{v,i+1} \quad \forall i \in \{1, 2, \ldots, k-1\} && \text{(blocks are used in order)} \\
& x_{vi}, d_{vi} \quad \forall v \in V, i \in \pi && \text{(binary variables)}
\end{aligned}
$$

3 Valid Inequalities

4 Impact of Valid Inequalities to Performance

5 Conclusion