

IE 554 Project – Mathematical Models

Emek Irmak & Ömer Turan Şahinaslan

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1 Dominator Partition Model (variable k)

1.1 Sets & Parameters

V : set of vertices in the graph, of size n

a_{vu} : 1 if vertices v and u are adjacent

1.2 Decision Variables

x_{vi} : 1 if vertex v is assigned to the i^{th} block

y_i : 1 if the i^{th} block is used (non-empty)

d_{vi} : 1 if vertex v dominates block i

1.3 Objective Function & Constraints

$$\begin{aligned} \min \quad & \sum_{i=1}^n y_i && \text{(minimize the number of blocks used)} \\ \text{s.t.} \quad & \sum_{i=1}^n x_{vi} = 1, \quad \forall v \in V && \text{(each vertex is assigned to exactly one block)} \\ & x_{vi} \leq y_i \quad \forall v \in V, i \in \{1, 2, \dots, n\} && \text{(vertex assigned only if block used)} \\ & \sum_{v \in V} x_{vi} \geq y_i \quad \forall i \in \{1, 2, \dots, n\} && \text{(block used only if it is not empty)} \\ & x_{ui} \leq a_{vu} + (1 - d_{vi}) \quad \forall u, v \in V, i \in \{1, 2, \dots, n\} && \text{(domination condition)} \\ & \sum_{i=1}^n d_{vi} \geq 1 \quad \forall v \in V && \text{(each vertex dominates at least one block)} \\ & d_{vi} \leq y_i \quad \forall v \in V, i \in \{1, 2, \dots, n\} && \text{(cannot dominate an empty block)} \\ & x_{vi}, y_i, d_{vi} \in \{0, 1\} \quad \forall v \in V, i \in \{1, 2, \dots, n\} && \text{(binary variables)} \end{aligned}$$