IE 554 Project Proposal

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Problem Description

Graph partitioning is a key topic in optimization. We study the **Dominator Partition Problem**, introduced by Hedetniemi and Haynes (2006) [1], and propose an exact integer programming model with potential improvements for scalability.

In graph $G = (V, E)$, a vertex v *dominates* set $S \subseteq V$ if it's adjacent to all $u \in S$. A **dominator partition** divides V into k blocks, each dominated by some $v \in V$. The goal is to find the smallest such k, called $\pi_d(G)$.

Our Plan

We gave the following IP model for the dominator partition problem for a fixed k:

Sets & Parameters

V: set of vertices in the graph, of size n

a_{vu}: 1 if vertices v and u are adjacent

k: number of blocks in the partition

Decision Variables

x_{vi}: 1 if vertex v is assigned to the i^{th} block

d_{vi}: 1 if vertex v dominates block i

Objective Function & Constraints

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\begin{array}{lll}
\min & 0 & \text{(no objective)} \\
\text{s.t.} & \displaystyle\sum_{i=1}^{k} x_{vi} = 1, \quad \forall v \in V & \text{(each vertex is assigned to one block)} \\
& \displaystyle\sum_{v \in V} x_{vi} \geq 1 \quad \forall i \in \{1, 2, \dots, k\} & \text{(no empty blocks)} \\
& x_{ui} \leq a_{vu} + (1 - d_{vi}) \quad \forall u, v \in V, i \in \{1, 2, \dots, k\} & \text{(domination condition)} \\
& \displaystyle\sum_{i=1}^{k} d_{vi} \geq 1 \quad \forall v \in V & \text{(each vertex dominates at least one block)} \\
& \displaystyle\sum_{v \in V} x_{vi} \geq \sum_{v \in V} x_{v,i+1} \quad \forall i \in \{1, 2, \dots, k-1\} & \text{(blocks are used in order)} \\
& x_{vi}, d_{vi} \quad \forall v \in V, i \in \{1, 2, \dots, k\} & \text{(binary variables)}
\end{array}
$$

The planned contributions are:

Exact IP Model: First IP formulation for dominator partitioning.

Model Strengthening: Add valid inequalities.

Experiments: Compare original vs. improved models on graphs of various sizes.

Literature Review

Most existing work is theoretical, focused on complexity and special graph classes [1]. Determining $\pi_d(G)$ is NP-complete, but no exact IP formulation exists in prior literature. Our study fills this gap.

References

[1] S. M. Hedetniemi et al., *Dominator Partitions of Graphs*, 2008.