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IE 554 Project – Mathematical Models

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CONTENTS

1 Dominator Partition Model (variable k)

Sets & Parameters

V: set of vertices in the graph, of size n

a_{vu}: 1 if vertices v and u are adjacent

Decision Variables

x_{vi}: 1 if vertex v is assigned to the i^{th} block

y_i: 1 if the i^{th} block is used (non-empty)

d_{vi}: 1 if vertex v dominates block i

Objective Function & Constraints

$$\min \quad \sum_{i=1}^{n} y_i \qquad \text{(minimize the number of blocks used)}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{vi} = 1, \quad \forall v \in V \qquad \text{(each vertex is assigned to one block)}$$

$$x_{vi} \le y_i \quad \forall v \in V, i \in \{1, 2, \ldots, n\} \qquad \text{(vertex assigned only if block used)}$$

$$\sum_{v \in V} x_{vi} \ge y_i \quad \forall i \in \{1, 2, \ldots, n\} \qquad \text{(block used only if it is not empty)}$$

$$x_{ui} \le a_{vu} + (1 - d_{vi}) \quad \forall u, v \in V, i \in \{1, 2, \ldots, n\} \qquad \text{(domination condition)}$$

$$\sum_{i=1}^{n} d_{vi} \ge 1 \quad \forall v \in V \qquad \text{(each vertex dominates at least one block)}$$

$$d_{vi} \le y_i \quad \forall v \in V, i \in \{1, 2, \ldots, n\} \qquad \text{(cannot dominate an empty block)}$$

$$\sum_{v \in V} x_{vi} \ge \sum_{v \in V} x_{v,i+1} \quad \forall i \in \{1, 2, \ldots, n-1\} \qquad \text{(blocks are used in order)}$$

$$x_{vi}, y_i, d_{vi} \quad \forall v \in V, i \in \{1, 2, \ldots, n\} \qquad \text{(binary variables)}$$

2 Dominator Partition Model (fixed k)

Sets & Parameters

V: set of vertices in the graph, of size n

a_{vu}: 1 if vertices v and u are adjacent

k: number of blocks in the partition

Decision Variables

x_{vi}: 1 if vertex v is assigned to the i^{th} block

d_{vi}: 1 if vertex v dominates block i

Objective Function & Constraints

$$\begin{aligned}
\min \quad & 0 && \text{(no objective)} \\
\text{s.t.} \quad & \sum_{i=1}^{k} x_{vi} = 1, \quad \forall v \in V && \text{(each vertex is assigned to one block)} \\
& \sum_{v \in V} x_{vi} \geq 1 \quad \forall i \in \{1, 2, \ldots, k\} && \text{(no empty blocks)} \\
& x_{ui} \leq a_{vu} + (1 - d_{vi}) \quad \forall u, v \in V, i \in \{1, 2, \ldots, k\} && \text{(domination condition)} \\
& \sum_{i=1}^{k} d_{vi} \geq 1 \quad \forall v \in V && \text{(each vertex dominates at least one block)} \\
& \sum_{v \in V} x_{vi} \geq \sum_{v \in V} x_{v,i+1} \quad \forall i \in \{1, 2, \ldots, k-1\} && \text{(blocks are used in order)} \\
& x_{vi}, d_{vi} \quad \forall v \in V, i \in \{1, 2, \ldots, k\} && \text{(binary variables)}
\end{aligned}$$