Yes Bank Forecate for 123 days (Started from 31 Aug 2019 to 31 Dec 2019)

Using Time Series Analysis

```
In [ ]: import numpy as np
import pandas as pd
```

Importing File

```
In [46]: YB_df=pd.read_csv(r'C:\Users\admin\Desktop\Yesbank.csv',header=0)
In [47]: YB_df.head()
```

Out[47]:

	Date	Open Price	High Price	Low Price	Close Price	WAP	No.of Shares	No. of Trades	Total Turnover (Rs.)	Deliverable Quantity	% Do Qty Trad
0	12- Jul- 05	65.00	70.00	60.15	12.42	62.113290	26141270	53078	1623720286	7123756.0	27
1	13- Jul- 05	61.45	62.35	57.40	12.03	60.126246	5756848	13173	346137657	1799410.0	31
2	14- Jul- 05	59.40	59.80	57.00	11.61	58.062454	1362144	4998	79089423	481702.0	35
3	15- Jul- 05	57.80	64.40	56.25	12.31	61.528808	7338637	18200	451537589	1725030.0	23
4	18- Jul- 05	62.80	65.65	62.35	12.88	64.378865	6084805	14645	391732838	1717998.0	28
4											•

```
In [48]: YB_df.dtypes
Out[48]: Date
                                        object
         Open Price
                                       float64
         High Price
                                       float64
         Low Price
                                       float64
         Close Price
                                       float64
         WAP
                                       float64
         No.of Shares
                                         int64
         No. of Trades
                                          int64
         Total Turnover (Rs.)
                                         int64
         Deliverable Quantity
                                       float64
         % Deli. Qty to Traded Qty
                                       float64
         Spread High-Low
                                       float64
         Spread Close-Open
                                       float64
         dtype: object
In [49]: YB_df.shape
Out[49]: (3504, 13)
In [50]: YB_df.isnull().sum()
Out[50]: Date
                                       0
         Open Price
                                       0
         High Price
                                        0
         Low Price
                                        0
         Close Price
         WAP
         No.of Shares
                                        0
         No. of Trades
                                       0
         Total Turnover (Rs.)
                                       0
         Deliverable Quantity
                                       1
         % Deli. Qty to Traded Qty
                                       1
         Spread High-Low
                                       0
         Spread Close-Open
                                       0
         dtype: int64
```

Use Drop function

	Date	Close	Price
0	12-Jul-05		12.42
1	13-Jul-05		12.03
2	14-Jul-05		11.61
3	15-Jul-05		12.31
4	18-Jul-05		12.88
5	19-Jul-05		12.79
6	20-Jul-05		12.73
7	21-Jul-05		12.62
8	22-Jul-05		12.50
9	25-Jul-05		12.36
10	26-Jul-05		12.51
11	27-Jul-05		12.70
12	29-Jul-05		12.60
13	1-Aug-05		12.70
14	2-Aug-05		14.05
15	3-Aug-05		14.13
16	4-Aug-05		14.41
17	5-Aug-05		14.37
18	8-Aug-05		14.26
19	9-Aug-05		13.95
20	10-Aug-05		14.39
21	11-Aug-05		14.42
22	12-Aug-05		14.23
23	16-Aug-05		13.99
24	17-Aug-05		13.95
25	18-Aug-05		13.80
26	19-Aug-05		13.63
27	22-Aug-05		13.38
28	23-Aug-05		12.99
29	24-Aug-05		12.90
	• • • •		
3474	18-Jul-19		85.80
3475	19-Jul-19		83.25
3476	22-Jul-19		91.15
3477	23-Jul-19		90.70
3478	24-Jul-19		89.15
3479	25-Jul-19		87.65
3480	26-Jul-19		96.10
3481	29-Jul-19		94.75
3482	30-Jul-19		86.10
3483	31-Jul-19		91.30
3484	1-Aug-19		88.40
3485	2-Aug-19		88.30
3486	5-Aug-19		81.10
	6-Aug-19		
3487			85.40
3488	7-Aug-19		86.85
3489	8-Aug-19		89.15
3490	9-Aug-19		82.10
3491	13-Aug-19		73.60
3492	14-Aug-19		76.55
3493	16-Aug-19		79.45
3494	19-Aug-19		76.70
3495	20-Aug-19		71.25
3496	21-Aug-19		65.40
3497	22-Aug-19		56.30
3498	23-Aug-19		59.25

```
3499 26-Aug-19
                                63.00
         3500 27-Aug-19
                                64.30
         3501 28-Aug-19
                                59.50
         3502 29-Aug-19
                                57.35
         3503 30-Aug-19
                                59.50
         [3504 rows x 2 columns]
In [52]: YB_df.info()
         <class 'pandas.core.frame.DataFrame'>
         RangeIndex: 3504 entries, 0 to 3503
         Data columns (total 2 columns):
                        3504 non-null object
         Date
         Close Price
                        3504 non-null float64
         dtypes: float64(1), object(1)
         memory usage: 54.8+ KB
```

Importing matplot, seaborn & datatime for data visualisation

Out[135]:

Close Price

Date	
2005-07-12	12.42
2005-07-13	12.03
2005-07-14	11.61
2005-07-15	12.31
2005-07-18	12.88

```
In [56]: YB_df.tail()
```

Out[56]:

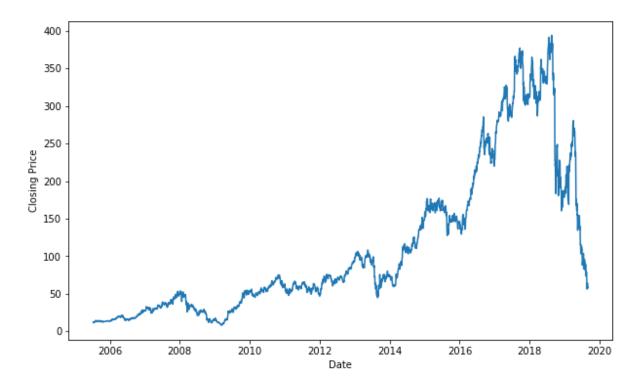
Close Price

Date	
2019-08-26	63.00
2019-08-27	64.30
2019-08-28	59.50
2019-08-29	57.35
2019-08-30	59.50

```
In [57]: import matplotlib.pyplot as plt
import seaborn as sns

plt.xlabel('Date')
plt.ylabel('Closing Price')
plt.plot(YB_df)
```

Out[57]: [<matplotlib.lines.Line2D at 0xbe7c668>]



Using Rolling Mean & Rolling STD, to find out Stationarity

```
In [59]: rolmean = YB_df.rolling(window=365).mean()
rolstd = YB_df.rolling(window=365).std()
print(rolmean, rolstd)
```

C.	Los	e	Pr	١i	ce	

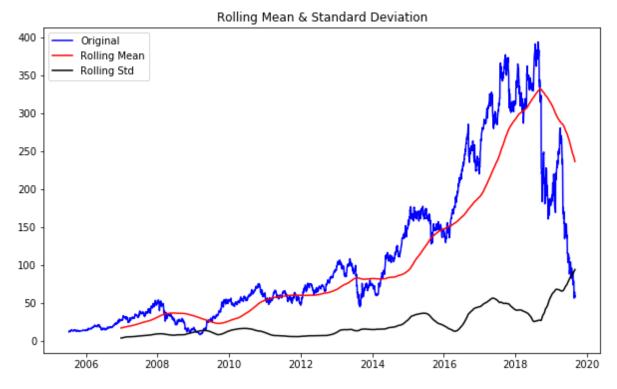
	Close Price
Date	
2005-07-12	NaN
2005-07-13	NaN
2005-07-14	NaN
	_
2005-07-15	NaN
2005-07-18	NaN
2005-07-19	NaN
2005-07-20	NaN
2005-07-21	NaN
2005-07-22	NaN
2005-07-25	NaN
2005-07-26	NaN
2005-07-27	NaN
2005-07-29	NaN
2005-08-01	NaN
2005-08-02	NaN
2005-08-03	
	NaN
2005-08-04	NaN
2005-08-05	NaN
2005-08-08	NaN
2005-08-09	NaN
2005-08-10	NaN
2005-08-11	NaN
2005-08-12	NaN
2005-08-16	NaN
2005-08-17	NaN
2005-08-18	NaN
2005-08-19	NaN
2005-08-22	NaN
2005-08-23	NaN
2005-08-24	NaN
2005-06-24	IValv
• • •	•••
2019-07-18	256.341507
2019-07-19	255.570137
2019-07-22	254.823973
2019-07-23	254.092877
2019-07-24	253.368767
2019-07-25	252.637808
	251.917123
2019-07-26	
2019-07-29	251.217808
2019-07-30	250.510959
2019-07-31	249.833562
2019-08-01	249.162877
2019-08-02	248.486849
2019-08-05	247.817123
	247.133425
2019-08-06	
2019-08-07	246.494110
2019-08-08	245.861781
2019-08-09	245.232192
2019-08-13	244.577397
2019-08-14	243.939726
2019-08-16	243.302192
2019-08-19	242.645479
2019-08-20	241.954110
2019-08-21	241.241781
2019-08-22	240.500000

			,		
2019-08-23		239.779452			
2019-08-26					
2019-08-27					
2019-08-28					
2019-08-29					
2019-08-30		236.316027			
F2504				61	ъ.
[3504 rows	Х	1 columns]		Close	Price
Date					
2005-07-12		NaN			
2005-07-13		NaN			
2005-07-14		NaN			
2005-07-15		NaN			
2005-07-18		NaN			
2005-07-19		NaN			
2005-07-20		NaN			
2005-07-21		NaN			
2005-07-22		NaN			
2005-07-25		NaN			
2005-07-26		NaN			
2005-07-27		NaN			
2005-07-29		NaN			
2005-08-01		NaN			
2005-08-02		NaN			
2005-08-03		NaN			
2005-08-04		NaN			
2005-08-05		NaN			
2005-08-08		NaN			
2005-08-09		NaN			
2005-08-05		NaN			
2005-08-10		NaN			
2005-08-11		NaN			
2005-08-12		NaN			
2005-08-10		NaN			
2005-08-17		NaN			
2005-08-19		NaN			
2005-08-19		NaN			
2005-08-22		NaN			
2005-08-23		NaN			
2003-00-24					
2019-07-18		84.745187			
2019-07-18		85.036128			
2019-07-22		85.281015			
2019-07-23		85.541374			
2019-07-24		85.816214			
2019-07-25		86.088699			
2019-07-26		86.295394			
2019-07-29		86.532287			
2019-07-30		86.824741			
2019-07-31		87.100055			
2019-08-01		87.398346			
2019-08-02		87.686016			
2019-08-05		88.028666			
2019-08-06		88.318687			
2019-08-07		88.632346			
2019-08-08		88.929649			
2019-08-09		89.273623			

```
2019-08-13
              89.653829
2019-08-14
              90.019203
2019-08-16
              90.358173
2019-08-19
              90.695871
2019-08-20
              91.038243
2019-08-21
              91.399993
              91.799480
2019-08-22
2019-08-23
              92.187392
2019-08-26
              92.551920
2019-08-27
              92.920133
2019-08-28
              93.307515
2019-08-29
              93.699613
2019-08-30
              94.084712
```

[3504 rows x 1 columns]

```
In [87]: orig = plt.plot(YB_df, color='blue', label='Original')
    mean = plt.plot(rolmean, color='red', label='Rolling Mean')
    std = plt.plot(rolstd, color='black', label='Rolling Std')
    plt.legend(loc='best')
    plt.title('Rolling Mean & Standard Deviation')
    plt.show(block=False)
```



Dickey Fuller Test for Stationarity

```
In [88]: from statsmodels.tsa.stattools import adfuller

print ('Results of Dickey-Fuller Test:')
Dickey_fuller_test = adfuller(YB_df['Close Price'], autolag='AIC')

dfoutput = pd.Series(Dickey_fuller_test[0:4], index=['Test Statistics', 'p-val ue', '#Lags Used', 'Number of Observation Used'])
for key,value in Dickey_fuller_test[4].items():
    dfoutput['Critical Value (%s)'%key] = value

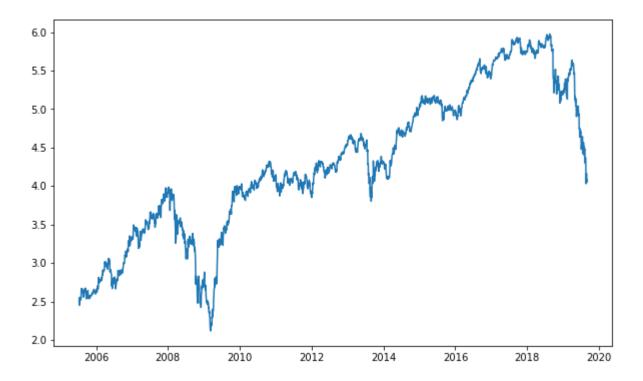
print(dfoutput)
```

Results of Dickey-Fuller Test: Test Statistics -1.597308 p-value 0.484946 #Lags Used 25.000000 Number of Observation Used 3478.000000 Critical Value (1%) -3.432232 Critical Value (5%) -2.862371 Critical Value (10%) -2.567213 dtype: float64

For Estimating trend using Log-Scale

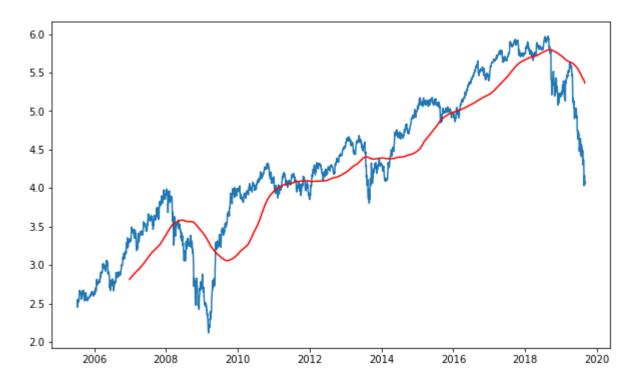
```
In [89]: YB_df_logscale = np.log(YB_df)
    plt.plot(YB_df_logscale)
```

Out[89]: [<matplotlib.lines.Line2D at 0x10453080>]



```
In [90]: movingAverage = YB_df_logscale.rolling(window=365).mean()
    movingStd = YB_df_logscale.rolling(window=365).std()
    plt.plot(YB_df_logscale)
    plt.plot(movingAverage, color='red')
```

Out[90]: [<matplotlib.lines.Line2D at 0x104e56a0>]



In [91]: YB_Logscale_minus_movingaverage = YB_df_logscale - movingAverage
YB_Logscale_minus_movingaverage.head(12)

Out[91]:

Close Price

Date	
2005-07-12	NaN
2005-07-13	NaN
2005-07-14	NaN
2005-07-15	NaN
2005-07-18	NaN
2005-07-19	NaN
2005-07-20	NaN
2005-07-21	NaN
2005-07-22	NaN
2005-07-25	NaN
2005-07-26	NaN
2005-07-27	NaN

```
In [92]: YB_Logscale_minus_movingaverage.dropna(inplace=True)
    YB_Logscale_minus_movingaverage.head(12)
```

Out[92]:

Date	
2006-12-26	0.494501
2006-12-27	0.502135
2006-12-28	0.478693
2006-12-29	0.476746
2007-01-02	0.520696
2007-01-03	0.503641

2007-01-04

2007-01-05

2007-01-08

2007-01-09

2007-01-10

2007-01-11

Close Price

0.500431

0.506477

0.490349

0.491041

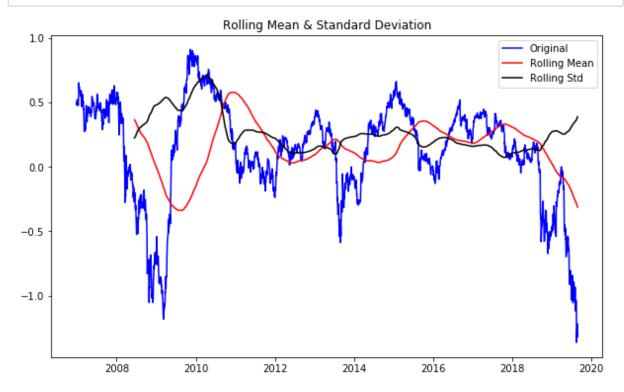
0.475095

0.475125

Using def function, where call come together in Dickey Fuller

In [93]: from statsmodels.tsa.stattools import adfuller def test_Stationarity(timeseries): movingAverage = timeseries.rolling(window=365).mean() movingSTD = timeseries.rolling(window=365).std() orig = plt.plot(timeseries, color='blue', label='Original') mean = plt.plot(movingAverage, color='red', label='Rolling Mean') std = plt.plot(movingSTD, color='black', label='Rolling Std') plt.legend(loc='best') plt.title('Rolling Mean & Standard Deviation') plt.show(block=False) print ('Results of Dickey-Fuller Test:') Dickey_fuller_test = adfuller(timeseries['Close Price'], autolag='AIC') dfoutput = pd.Series(Dickey_fuller_test[0:4], index=['Test Statistics', 'p -value', '#Lags Used', 'Number of Observation Used']) for key,value in Dickey fuller test[4].items(): dfoutput['Critical Value (%s)'%key] = value print(dfoutput)

In [94]: | test_Stationarity(YB_Logscale_minus_movingaverage)



Results of Dickey-Fuller Test:

Test Statistics	-1.132803
p-value	0.701815
#Lags Used	2.000000
Number of Observation Used	3137.000000
Critical Value (1%)	-3.432436
Critical Value (5%)	-2.862462
Critical Value (10%)	-2.567261

dtype: float64

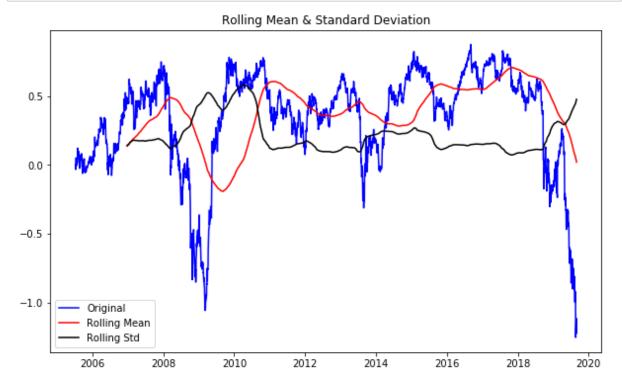
Using EWA to find trend

```
In [95]: EWA = YB_df_logscale.ewm(halflife=365, min_periods=0, adjust=True).mean()
    plt.plot(YB_df_logscale)
    plt.plot(EWA, color='red')
```

Out[95]: [<matplotlib.lines.Line2D at 0x115ba710>]



```
In [96]: LEWA = YB_df_logscale - EWA
  test_Stationarity(LEWA)
```



Results of Dickey-Fuller Test:

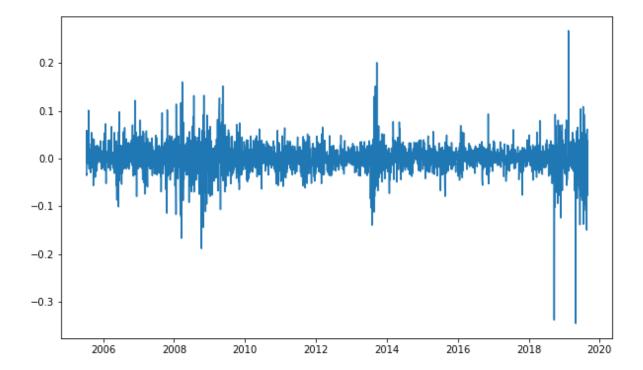
Test Statistics -1.030771
p-value 0.741876
#Lags Used 7.000000
Number of Observation Used 3496.000000
Critical Value (1%) -3.432222
Critical Value (5%) -2.862367
Critical Value (10%) -2.567210

dtype: float64

Log diff is nonthing but (d)

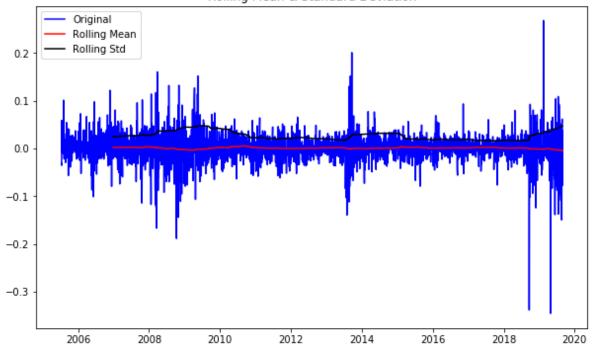
```
In [97]: log_diff_shifting = YB_df_logscale - YB_df_logscale.shift()
    plt.plot(log_diff_shifting)
```

Out[97]: [<matplotlib.lines.Line2D at 0x116447f0>]



In [102]: log_diff_shifting.dropna(inplace=True)
 test_Stationarity(log_diff_shifting)





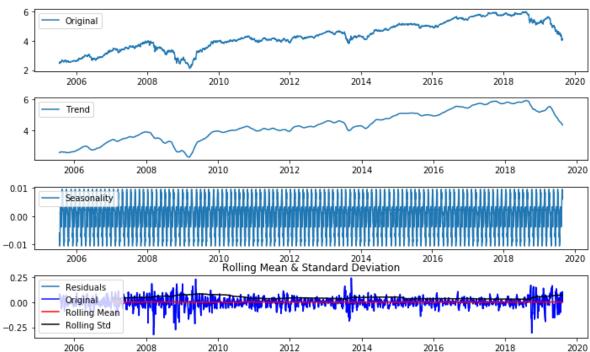
Results of Dickey-Fuller Test:

Test Statistics -21.898607
p-value 0.000000
#Lags Used 6.000000
Number of Observation Used 3496.000000
Critical Value (1%) -3.432222
Critical Value (5%) -2.862367
Critical Value (10%) -2.567210

dtype: float64

Using Decompose function to find out Trend, Seasonal & Residual

```
from statsmodels.tsa.seasonal import seasonal decompose
In [111]:
          decomposition = seasonal decompose(YB df logscale, freq=30)
          trend = decomposition.trend
          seasonal = decomposition.seasonal
          residual = decomposition.resid
          plt.subplot(411)
          plt.plot(YB_df_logscale, label='Original')
          plt.legend(loc='best')
          plt.subplot(412)
          plt.plot(trend, label='Trend')
          plt.legend(loc='best')
          plt.subplot(413)
          plt.plot(seasonal, label='Seasonality')
          plt.legend(loc='best')
          plt.subplot(414)
          plt.plot(residual, label='Residuals')
          plt.legend(loc='best')
          plt.tight layout()
          decomposedlogdata = residual
          decomposedlogdata.dropna(inplace=True)
          test Stationarity(decomposedlogdata)
```



```
Results of Dickey-Fuller Test:

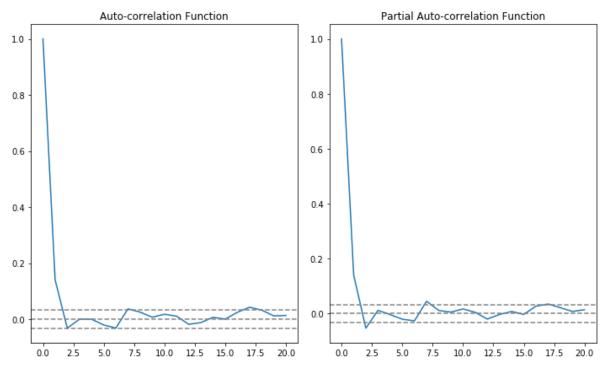
Test Statistics -1.535076e+01
p-value 3.722924e-28
#Lags Used 2.500000e+01
Number of Observation Used 3.448000e+03
Critical Value (1%) -3.432248e+00
Critical Value (5%) -2.862379e+00
```

Critical Value (5%) -2.862379e+00 Critical Value (10%) -2.567216e+00

dtype: float64

ACF And PACF { Is nonthing but (p) & (q) }

```
In [113]:
          from statsmodels.tsa.stattools import acf, pacf
          lag acf = acf(log diff shifting, nlags=20)
          lag_pacf = pacf(log_diff_shifting, nlags=20, method='ols')
          #plot ACF
          plt.subplot(121)
          plt.plot(lag_acf)
          plt.axhline(y=0,linestyle='--',color='gray')
          plt.axhline(y=-1.96/np.sqrt(len(log_diff_shifting)),linestyle='--',color='gra
          y')
          plt.axhline(y=1.96/np.sqrt(len(log_diff_shifting)),linestyle='--',color='gray'
          plt.title('Auto-correlation Function')
          #plot PACF
          plt.subplot(122)
          plt.plot(lag pacf)
          plt.axhline(y=0,linestyle='--',color='gray')
          plt.axhline(y=-1.96/np.sqrt(len(log_diff_shifting)),linestyle='--',color='gra
          plt.axhline(y=1.96/np.sqrt(len(log_diff_shifting)),linestyle='--',color='gray'
          plt.title('Partial Auto-correlation Function')
          plt.tight layout()
```



Finally Using ARIMA Model to Predict the 123 days

In [121]: from statsmodels.tsa.arima_model import ARIMA #AR model model = ARIMA(YB_df_logscale, order=(2,1,2)) results_AR = model.fit(disp=-1) plt.plot(log_diff_shifting) plt.plot(results_AR.fittedvalues, color='red') plt.title('RSS: %.4f'% sum((results_AR.fittedvalues-log_diff_shifting["Close Price"])**2)) # Residual sum of sq (RSS) print('Plotting AR model')

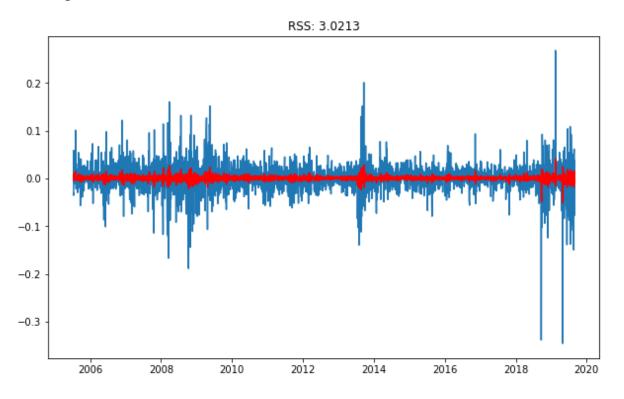
C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
225: ValueWarning: A date index has been provided, but it has no associated f
requency information and so will be ignored when e.g. forecasting.

' ignored when e.g. forecasting.', ValueWarning)

C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 225: ValueWarning: A date index has been provided, but it has no associated f requency information and so will be ignored when e.g. forecasting.

' ignored when e.g. forecasting.', ValueWarning)

Plotting AR model



In [122]: #MA model model = ARIMA(YB_df_logscale, order=(2,1,2)) results_MA = model.fit(disp=-1) plt.plot(log_diff_shifting) plt.plot(results_MA.fittedvalues, color='red') plt.title('RSS: %.4f'% sum((results_MA.fittedvalues-log_diff_shifting["Close P rice"])**2)) # Residual sum of sq (RSS) print('Plotting MA model')

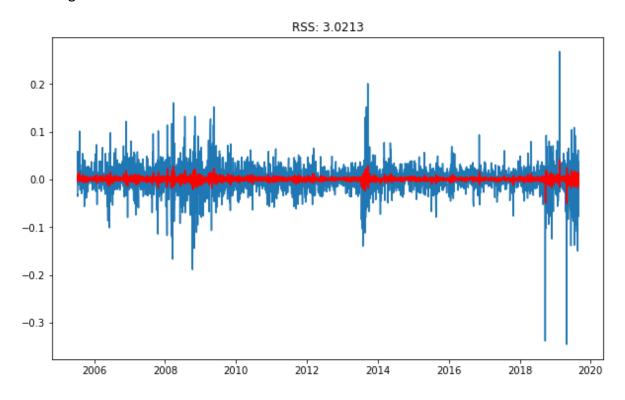
C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 225: ValueWarning: A date index has been provided, but it has no associated f requency information and so will be ignored when e.g. forecasting.

' ignored when e.g. forecasting.', ValueWarning)

C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
225: ValueWarning: A date index has been provided, but it has no associated f
requency information and so will be ignored when e.g. forecasting.

'ignored when e.g. forecasting.', ValueWarning)

Plotting MA model



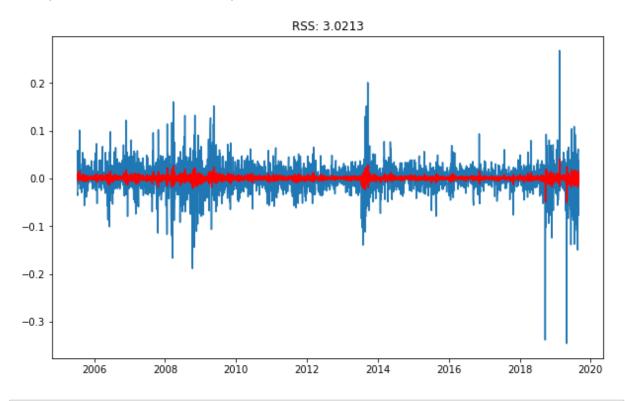
C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
225: ValueWarning: A date index has been provided, but it has no associated f
requency information and so will be ignored when e.g. forecasting.

' ignored when e.g. forecasting.', ValueWarning)

C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
225: ValueWarning: A date index has been provided, but it has no associated f
requency information and so will be ignored when e.g. forecasting.

'ignored when e.g. forecasting.', ValueWarning)

Out[123]: Text(0.5, 1.0, 'RSS: 3.0213')



In [126]: # Now joining all together

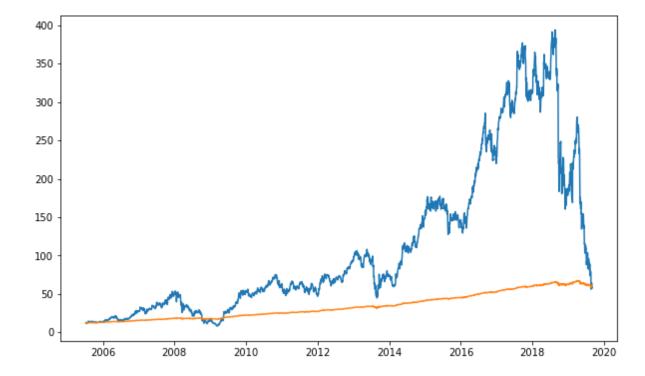
predictions_ARIMA_diff = pd.Series(results_ARIMA.fittedvalues, copy=True)
print(predictions_ARIMA_diff.head())

dtype: float64

```
In [128]:
          # Convert to cumalative sum
          predictions ARIMA diff cumsum = predictions ARIMA diff.cumsum()
          print(predictions ARIMA diff cumsum.head())
          Date
          2005-07-13
                        0.000448
          2005-07-14
                       -0.003647
          2005-07-15
                       -0.006796
          2005-07-18
                        0.003917
          2005-07-19
                        0.007455
          dtype: float64
In [129]: | predictions_ARIMA_log = pd.Series(YB_df_logscale['Close Price'].ix[0], index=Y
          B df logscale.index)
          predictions ARIMA log = predictions ARIMA log.add(predictions ARIMA diff cumsu
          m,fill_value=0)
          predictions ARIMA log.head()
          C:\Users\admin\Anaconda3\lib\site-packages\ipykernel_launcher.py:1: Deprecati
          onWarning:
          .ix is deprecated. Please use
          .loc for label based indexing or
          .iloc for positional indexing
          See the documentation here:
          http://pandas.pydata.org/pandas-docs/stable/indexing.html#ix-indexer-is-depre
            """Entry point for launching an IPython kernel.
Out[129]: Date
          2005-07-12
                         2.519308
          2005-07-13
                        2.519756
                        2.515661
          2005-07-14
          2005-07-15
                         2.512512
          2005-07-18
                         2.523225
          dtype: float64
          # converting data into original form
 In [ ]:
```

```
In [130]: prediction_ARIMA = np.exp(predictions_ARIMA_log)
    plt.plot(YB_df)
    plt.plot(prediction_ARIMA)
```

Out[130]: [<matplotlib.lines.Line2D at 0x14730da0>]



In [136]: YB_df

Out[136]:

	Close Price
Date	
2005-07-12	12.42
2005-07-13	12.03
2005-07-14	11.61
2005-07-15	12.31
2005-07-18	12.88
2005-07-19	12.79
2005-07-20	12.73
2005-07-21	12.62
2005-07-22	12.50
2005-07-25	12.36
2005-07-26	12.51
2005-07-27	12.70
2005-07-29	12.60
2005-08-01	12.70
2005-08-02	14.05
2005-08-03	14.13
2005-08-04	14.41
2005-08-05	14.37
2005-08-08	14.26
2005-08-09	13.95
2005-08-10	14.39
2005-08-11	14.42
2005-08-12	14.23
2005-08-16	13.99
2005-08-17	13.95
2005-08-18	13.80
2005-08-19	13.63
2005-08-22	13.38
2005-08-23	12.99
2005-08-24	12.90
2019-07-18	85.80
2019-07-19	83.25
2019-07-22	91.15

Close Price

90.70
89.15
87.65
96.10
94.75
86.10
91.30
88.40
88.30
81.10
85.40
86.85
89.15
82.10
73.60
76.55
79.45
76.70
71.25
65.40
56.30
59.25
63.00
64.30
59.50
57.35
59.50

3504 rows × 1 columns

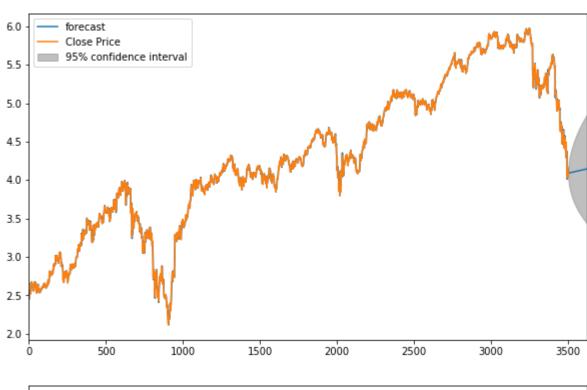
In [137]: results_ARIMA.plot_predict(1,3627)

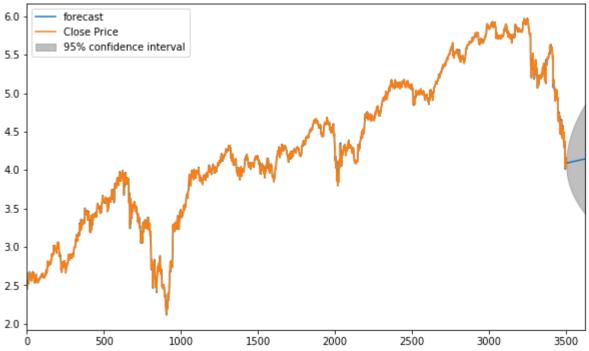
C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
531: ValueWarning: No supported index is available. Prediction results will b
e given with an integer index beginning at `start`.
 ValueWarning)

C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
531: ValueWarning: No supported index is available. Prediction results will b
e given with an integer index beginning at `start`.
 ValueWarning)

C:\Users\admin\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
531: ValueWarning: No supported index is available. Prediction results will b
e given with an integer index beginning at `start`.
 ValueWarning)

Out[137]:





In [134]: results_ARIMA.forecast(steps=123)

```
Out[134]: (array([4.09303664, 4.09206857, 4.09238572, 4.09297463, 4.09339644,
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In []: