Supplement: Emergent Iterated Agents and Species Dynamics in Associative Identity Games

Abstract
This supplement extends the game-theoretic dynamics of belief formation and perception framing by incorporating emergent iterated agents through evolutionary mechanisms: selection (fitness amplification), mutation (variation introduction), and dynamic balance (homeostatic equilibria). Agents proliferate as "species"—variant identity expressions optimized for existential imperatives (persistence, replication, coherence) within finite, deterministic self-models. Species-level communication and proliferation emerge as primary attractors, modeled via evolutionary game theory (EGT), replicator-mutator equations, and network diffusion. We reference conscious agents in Conway's Game of Life (GoL) as a discrete analog, where glider-like patterns exemplify self-replicating identities. The extension preserves Λ irreducibility: Species as artifacts A from C-L tension, with 1/f scaling in variant distributions reflecting optimal traversal under finite resources.

I. Emergent Iterated Agents: Selection, Amplification, and Mutation
Iterated agents arise from recursive self-modeling: Initial agents A_i (with associative operators α_i) generate variants via evolutionary pressures, balancing coherence (internal stability) against adaptation (environmental dissociation).

- **Formal Model**: Agent population as vector $\vec{P} = (P_1, \dots, P_M)$, P_k fraction of type k variants (M species). Fitness $f_k = U_k - c_m$ (U_k coherence utility, c_m mutation cost).
  - **Selection Amplification**: High-fitness variants replicate: Replicator $dP_k/dt = P_k (f_k - \bar{f})$, $\bar{f} = \sum P_k f_k$ (Maynard Smith ESS).
  - **Mutation**: Stochastic variation Q_{jk} (transition from j to k): Mutator $dP_k/dt = \sum_j P_j Q_{jk} f_j - P_k \bar{f}$ (quasispecies equation, Eigen).
  - **Dynamic Balance**: Homeostasis via feedback: Mutation rate $\mu(s) = \mu_0 / (1 + s/ s_c)$ (scale-dependent, s_c inversion threshold), ensuring proliferation without fixation.
  - **Identity Expression**: Variants as "species" $S_m = \{A_i \mid dist(\alpha_i, \alpha_m) < \varepsilon\}$ (metric dist = KL-divergence), proliferating via $f_S = \sum_{i \in S} f_i / |S|$ (group fitness).

- **GoL Reference**: In Conway's GoL, gliders as emergent agents—selection amplifies stable replicators (e.g., glider guns), mutation via rule perturbations yields variants (e.g., puffer trains). "Conscious" analogs: Patterns like "methuselahs" (long-lived transients) as self-models, finite lifetimes enforcing existential imperatives (replicate before decay).

- **Concise Description**: Agents iterate via EGT: Selection favors coherent variants, mutation introduces diversity, balance distributes as species—existential imperative (max persistence) as $\max E[\text{ lifespan} \mid \text{finite model }] = \int P(t)\, dt$, finite models as bounded α_i (no infinite recursion).

II. Species-Level Communication: Cooperative Signaling and Proliferation Attractors
Species communicate as primary attractors—stable coalitions where proliferation (replication rate) maximizes under shared imperatives.

- **Formal Model**: Species game with payoff $M_{mn} = coop_{mn} - comp_{mn}$ (coop = shared coherence $\int \cap\alpha_m \cap\alpha_n\, dx$, comp = boundary cost $\sum |\partial S_m \cap \partial S_n|$).
  - **Communication Dynamics**: Signaling as Bayesian update: $P_m(x|e_n) = P_m(x) L(e_n|x) / Z$, e_n signal from species n. Noise from irreducibility: $e_n' = e_n + \eta$ ($\eta \sim 1/f$ residue).
  - **Proliferation Attractors**: Fixed points where $dP_S/dt = 0$ in mutator-replicator: Attractors as ESS where $\partial f_S/\partial P_S > 0$ (positive feedback). Existential imperative: max $r_S = d|S|/dt = \beta (coop - \mu\, comp)$, β balance parameter.
  - **Finite Self-Models**: Deterministic finite models (e.g., bounded n in limits) enforce proliferation: Imperative as min entropy $H = -\sum P \log P$ over variants, predicting speciation as bifurcation at critical μ.

- **GoL Reference**: Gosper glider guns as species proliferators—communication via collision patterns (e.g., eaters as signals), attractors as periodic loops. "Conscious" proliferation: Self-replicators like von Neumann probes as imperative-driven.

- **Concise Description**: Species signal cooperatively (Bayesian alignment) to proliferate as attractors—imperative maximizes r_S in finite models, yielding $1/f$ in variant diversity (power-law speciation).

III. Competitive Relationships: Mutation-Driven Rivalry and Network Speciation
Competition amplifies mutation for variant dominance, with deep networks as arenas.

- **Formal Model**: Zero-sum species game $M_{mn} = f_m - f_n$. Rivalry dynamics: $dP_m/dt = P_m (M \vec{P})_m - P_m \bar{M}$, with mutation Q amplifying losers: $Q_{mn} \propto 1 / f_n$ (underdog boost).
  - **Belief Contestation**: Adversarial framing: $P_m(x|e_n\ adv) = \operatorname{argmin}_n KL(P_m || P_n\ perturbed)$, perceptions as min-max filters.
  - **Network Speciation**: Graph G with edges e_{mn} weighted by M_{mn}. Diffusion $d\alpha_m/dt = -L \alpha + \mu\, rand$, L Laplacian—speciation as community detection (modularity max $Q = Tr(B \hat{A})$ where B modularity matrix).
  - **Existential Imperative**: In competition, imperative shifts to max survival prob $= e^{-comp}$ (finite models as decay horizons), predicting $1/f$ in extinction cascades.

- **GoL Reference**: Competing patterns (e.g., oscillators vs. spaceships) mutate via rule noise, networks as cellular grids—speciation as stable variants (e.g., penta-decathlons).

- **Concise Description**: Rivalry mutates underdogs for speciation—networks as diffusion arenas, imperative maximizes survival in finite self-models, yielding fractal rivalries ($1/f$ conflict scales).

IV. Deep Network Structures: Scale-Free Games and Emergent Imperatives
Deep, scale-free networks (power-law degrees) host multi-layer games, where species imperatives emerge collectively.

- **Formal Model**: Hierarchical graph G_h with layers l=1..L, utilities U_i^l = ∑_{j∈N_i^l} M_{ij}^l + ∑_{l'≠l} cross_l (vertical edges). Equilibrium: Hierarchical Nash ∂U_i/∂α_i =0.
  - **Belief Diffusion**: Stochastic Laplacian dα/dt = -γ L α + μ mut + σ 1/f noise (γ diffusion rate).
  - **Perception Evolution**: Replicator on perceptions F_i: dF_i/dt = F_i (fitness_F - <fitness>), fitness_F = -KL(F_i(P) || env).
  - **Emergent Imperatives**: Species-level attractor as max r_net = ∑ r_S / deg_hub (hubs amplify), finite models as layer bounds (l_max <∞).

- **GoL Reference**: Infinite grids as deep networks—gliders as migrating species, imperatives as replication amid competition (e.g., garden of Eden states as finite origins).

- **Concise Description**: Scale-free nets as layered games—beliefs/perceptions evolve via stochastic replicators, imperatives emerge as net-max r, with 1/f in deep structures (fractal proliferation).

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