Maximum Likelihood Estimation with Push-Pull Noise Models

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In this set of notes, we describe the maximum likelihood estimation techniques used to fit our push-pull amplifier models to experimental data.

$$P(\log(GFP)) = \frac{1}{\sqrt{2\pi\sigma_{\mathrm{GFP}}^2}} \exp\left(-\frac{(\log(GFP) - \mu_{GRP})^2}{2\sigma_{\mathrm{GFP}}^2}\right) \tag{1}$$

$$P(\log(anti)) = \frac{1}{\sqrt{2\pi\sigma_{\mathrm{anti}}^2}} \exp\left(-\frac{(\log(anti) - \mu_{anti})^2}{2\sigma_{\mathrm{anti}}^2}\right) \tag{2}$$

$$P(\log(anti) \quad \text{and} \quad \log(GFP)) = \frac{1}{\sqrt{2\pi\left(\sigma_{\mathrm{anti}}^2\sigma_{\mathrm{GFP}}^2 - \sigma_{\mathrm{anti,GFP}}^2\right)}} \tag{3}$$

$$\times \exp\left(\frac{\sigma_{\mathrm{GFP}}^2(\log(anti) - \mu_{anti})^2 + \sigma_{\mathrm{anti}}^2(\log(GFP) - \mu_{GFP})^2 - 2\sigma_{\mathrm{anti,GFP}}(\log(anti) - \mu_{anti})(\log(GFP) - \mu_{GFP})}{2\left(\sigma_{\mathrm{anti}}^2\sigma_{\mathrm{GFP}}^2 - \sigma_{\mathrm{anti,GFP}}^2\right)}\right) \tag{4}$$

$$P(\log(anti)|\log(GFP)) = \frac{P(\log(anti) \quad \text{and} \quad \log(GFP))}{P(\log(GFP))} \tag{5}$$

$$= \sqrt{\frac{\sigma_{\mathrm{GFP}}^2}{\left(\sigma_{\mathrm{anti}}^2\sigma_{\mathrm{GFP}}^2 - \sigma_{\mathrm{anti,GFP}}^2\right)}} \exp\left(-\frac{\left(\sigma_{\mathrm{GFP}}^2(\log(anti) - \mu_{anti}) - \sigma_{\mathrm{anti,GFP}}(\log(GFP) - \mu_{GFP})\right)^2}{2\sigma_{\mathrm{GFP}}^2\left(\sigma_{\mathrm{anti}}^2\sigma_{\mathrm{GFP}}^2 - \sigma_{\mathrm{anti,GFP}}^2\right)}\right) \tag{6}$$

$$= \frac{1}{\sqrt{\Sigma^2}} \exp\left(-\frac{\left(\log(anti) - A\log(GFP) - B\right)^2}{2\Sigma^2}\right) \tag{7}$$

$$\mathcal{L} = -\frac{1}{N}\sum_i \log\left(P(\log(anti_i)|\log(GFP_i))\right) \tag{8}$$

$$= \frac{1}{2\Sigma^2 N}\sum_i \left(\log(anti_i) - A\log(GFP_i) - B\right)^2 + \frac{1}{2}\log\left(\Sigma^2\right) \tag{9}$$