§ 4 Shellsort ---- by Donald Shell

〖Example〗 Sort:

	81	94	11	96	12	35	17	95	28	58	41	75	15
5-sort	35	17	11	28	12	41	75	15	96	58	81	94	95
3-sort	28	12	11	35	15	41	58	17	94	75	81	96	95
1-sort	11	12	15	17	28	35	41	58	75	81	94	95	96

- Define an *increment sequence* $h_1 < h_2 < \ldots < h_t$ ($h_1 = 1$)
- Define an h_k-sort at each phase for $k = t, t - 1, \ldots, 1$

Note: An h_k-sorted file that is then h_{k-1}-sorted remains h_k-sorted.

☞ Shell's increment sequence:

$$h_t = \lfloor N/2 \rfloor, \; h_k = \lfloor h_{k+1}/2 \rfloor$$

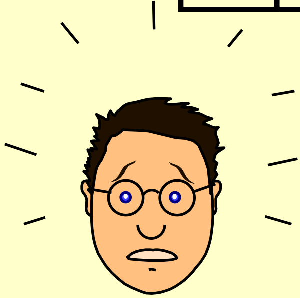
```
void Shellsort( ElementType A[ ], int N )
{
    int  i, j, Increment;
    ElementType  Tmp;
    for ( Increment = N / 2; Increment > 0; Increment /= 2 )
        /*h sequence */
        for ( i = Increment; i < N; i++ ) { /* insertion sort */
            Tmp = A[ i ];
            for ( j = i; j >= Increment; j - = Increment )
                if( Tmp < A[ j - Increment ] )
                    A[ j ] = A[ j - Increment ];
                else
                    break;
                A[ j ] = Tmp;
        } /* end for-I and for-Increment loops */
}
```

☞ Worst-Case Analysis:

【Theorem】The worst-case running time of Shellsort, using Shell's increments, is $\Theta(N^2)$.

〖Example〗 A bad case:

	1	9	2	10	3	11	4	12	5	13	6	14	7	15	8	16
8-sort	1	9	2	10	3	11	4	12	5	13	6	14	7	15	8	16
4-sort	1	9	2	10	3	11	4	12	5	13	6	14	7	15	8	16
2-sort	1	9	2	10	3	11	4	12	5	13	6	14	7	15	8	16
1-sort	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

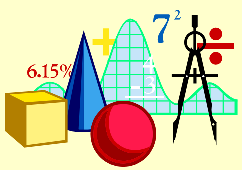


Pairs of increments are not necessarily relatively prime.
Thus the smaller increment can have little effect.

☞ Hibbard's Increment Sequence:

$h_k = 2^k - 1$ ---- consecutive increments have no common factors.

【Theorem】 The worst-case running time of Shellsort, using Hibbard's increments, is $\Theta(N^{3/2})$.



Conjectures:

- $T_{\text{avg – Hibbard}}(N) = O(N^{5/4})$

Shellsort is a very simple algorithm, yet with an extremely complex analysis. It is good for sorting up to moderately large input (tens of thousands).

- Sedgewick's best sequence is $\{1, 5, 19, 41, 109, \ldots\}$ in which the terms are either of the form $9\times4^i - 9\times2^i + 1$ or $4^i - 3\times2^i + 1$. $T_{\text{avg}}(N) = O(N^{7/6})$ and $T_{\text{worst}}(N) = O(N^{4/3})$.

§ 5 Heapsort

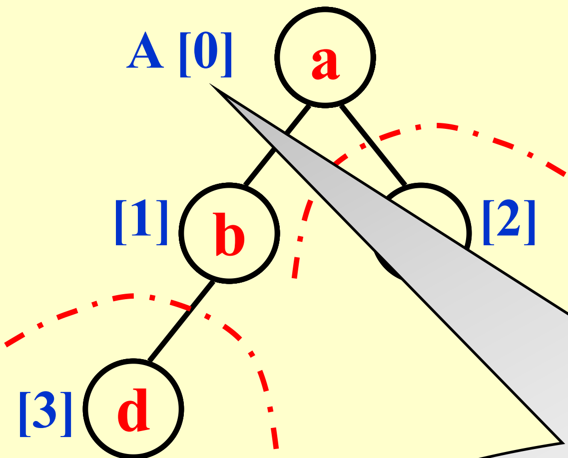
Algorithm 1:

```
{
    BuildHeap( H );                        /* O( N ) */
    for ( i=0; i<N; i++ )
        TmpH[ i ] = DeleteMin( H );        /* O( log N ) */
    for ( i=0; i<N; i++ )
        H[ i ] = TmpH[ i ];                /* O( 1 ) */
}
```

$$T(N) = O(N \log N)$$

The space requirement is doubled.

Algorithm 2:




```
void Heapsort( ElementType A[ ], int N )
{  int i;
   for ( i = N / 2; i >= 0; i - - ) /* BuildHeap */
      PercDown( A, i, N );
   for ( i = N - 1; i > 0; i - - ) {
      Swap( &A[ 0 ], &A[ i ] ); /* DeleteMax */
      PercDown( A, 0, i );
   }
```

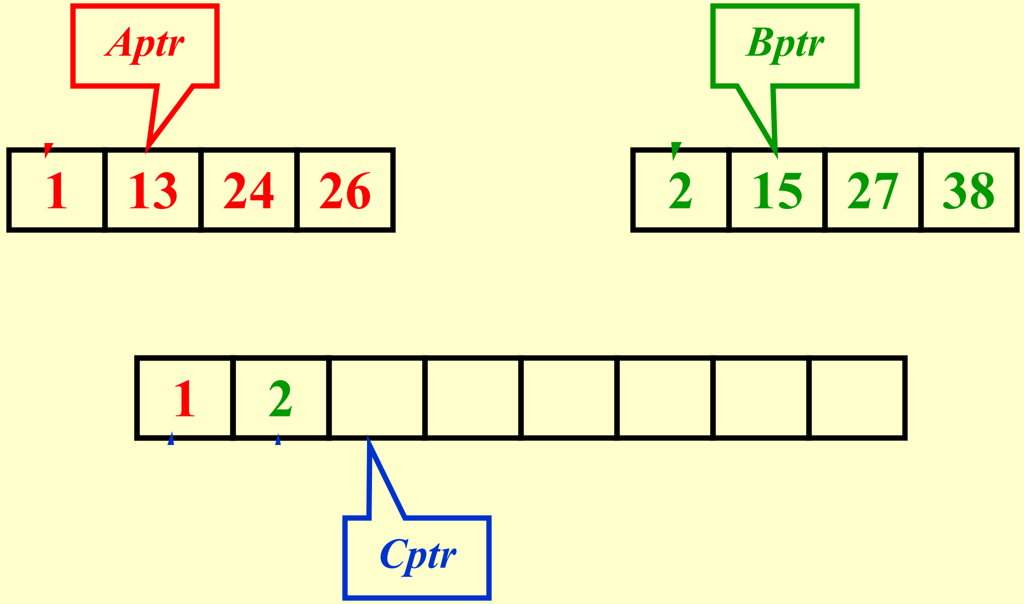
Heapsort data start from position 0.

【Theorem】 The average number of comparisons used to heapsort a random permutation of N distinct items is $2N \log N - O(N \log \log N)$.

Note: Although Heapsort gives the best average time, in practice it is slower than a version of Shellsort that uses Sedgewick's increment sequence.

§ 6 Mergesort

1. Merge two sorted lists



$T(N) = O(N)$ where N is the total number of elements.

2. Mergesort

```
void MSort( ElementType A[ ], ElementType TmpArray[ ],
                        int Left, int Right )
{   int  Center;
    if ( Left < Right ) {  /* if there are elements to be sorted */
           Center = ( Left + Right ) / 2;
           MSort( A, TmpArray, Left, Center );              /* T( N / 2 ) */
           MSort( A, TmpArray, Center + 1, Right );     /* T( N / 2 ) */
           Merge( A, TmpArray, Left, Center + 1, Right );  /* O( N ) */
    }
}

void Merge
{   Elem
    Tm
    if (

           free(
    }
    else  FatalError( "No space for tmp array!!!" );
}
```

If a TmpArray is declared locally for each call of Merge, then $S(N) = O(\ N \log N\ )$

```
/* Lpos = start of left half, Rpos = start of right half */
void Merge( ElementType A[ ], ElementType TmpArray[ ],
                int Lpos, int Rpos, int RightEnd )
{   int  i, LeftEnd, NumElements, TmpPos;
    LeftEnd = Rpos - 1;
    TmpPos = Lpos;
    NumElements = RightEnd - Lpos + 1;
    while( Lpos <= LeftEnd && Rpos <= RightEnd ) /* main loop */
        if ( A[ Lpos ] <= A[ Rpos ] )
            TmpArray[ TmpPos++ ] = A[ Lpos++ ];
        else
            TmpArray[ TmpPos++ ] = A[ Rpos++ ];
    while( Lpos <= LeftEnd ) /* Copy rest of first half */
        TmpArray[ TmpPos++ ] = A[ Lpos++ ];
    while( Rpos <= RightEnd ) /* Copy rest of second half */
        TmpArray[ TmpPos++ ] = A[ Rpos++ ];
    for( i = 0; i < NumElements; i++, RightEnd - - )
         /* Copy TmpArray back */
        A[ RightEnd ] = TmpArray[ RightEnd ];
}
```

3. Analysis

$$T(1) = 1$$

$$\begin{aligned} T(N) &= 2T(N/2) + O(N) \\ &= 2^k\, T(N/2^k) + k * O(N) \\ &= N * T(1) + \log N * O(N) \\ &= O(N + N \log N) \end{aligned}$$

Note: Mergesort requires **linear extra memory**, and copying an array is slow. It is hardly ever used for internal sorting, but is quite useful for **external sorting**.

Iterative version :