2. Quadratic Probing

$f(i) = i^2$; /* a quadratic function */

【Theorem】If quadratic probing is used, and the table size is prime, then a new element can always be inserted if the table is at least half empty.

Proof: Just prove that the first ⌊TableSize/2⌋ alternative locations are all distinct. That is, for any $0 < i \neq j \leq \lfloor \text{TableSize}/2 \rfloor$, we have

$$(h(x) + i^2) \% \text{TableSize} \neq (h(x) + j^2) \% \text{TableSize}$$

Suppose: $h(x) + i^2 = h(x) + j^2 \pmod{\text{TableSize}}$

then: $i^2 = j^2 \pmod{\text{TableSize}}$

$(i+j)(i-j) = 0 \pmod{\text{TableSize}}$

TableSize is prime ➡ either $(i+j)$ or $(i-j)$ is divisible by TableSize

Contradiction !

For any x, it has ⌈TableSize/2⌉ distinct locations into which it can go. If at most ⌊TableSize/2⌋ positions are taken, then an empty spot can always be found. ∎

Note: If the table size is a prime of the form $4k + 3$, then the quadratic probing $f(i) = \pm i^2$ can probe the entire table.

Read Figures 7.15 - 7.16 for detailed representations and implementations of initialization.

```
Position  Find ( ElementType Key, HashTable H )
{  Position  CurrentPos;
   int  CollisionNum;
   CollisionNum = 0;
   CurrentPos = Hash( Key, H->TableSize );
   while( H->TheCells[ CurrentPos ].Info != Empty &&
          H->TheCells[ CurrentPos ].Element != Key ) {
          CurrentPos += 2 * ++CollisionNum – 1;
          if ( CurrentPos >= H->TableSize )  CurrentPos -= H->TableSize;
   }
   return CurrentPos;
}
```

What is returned?

```
void  Insert ( ElementType Key, HashTable H )
{
   Position  Pos;
   Pos = Find( Key, H );
   if ( H->TheCells[ Pos ].Info != Legitimate ) { /* OK to insert here */
         H->TheCells[ Pos ].Info = Legitimate;
         H->TheCells[ Pos ].Element = Key; /* Probably need strcpy */
   }
}
```

Question: How to delete a key?

Note: ① Insertion will be seriously slowed down if there are too many deletions intermixed with insertions.

② Although primary clustering is solved, *secondary clustering* occurs – that is, keys that hash to the same position will probe the same alternative cells.

3. Double Hashing

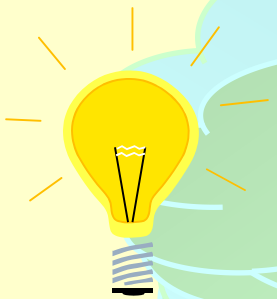
$f(i) = i * \text{hash}_2(x)$; /* $\text{hash}_2(x)$ is the 2nd hash function */

ⓘ $\text{hash}_2(x) \not\equiv 0$; ⓘ make sure that all cells can be probed.

☝ Tip: $\text{hash}_2(x) = R - (x \% R)$ with R a prime smaller than TableSize, will work well.

Note: ① If double hashing is correctly implemented, simulations imply that the expected number of probes is almost the same as for a random collision resolution strategy.
② Quadratic probing does not require the use of a second hash function and is thus likely to be simpler and faster in practice.

§ 5 Rehashing



- Build another table that is about twice as big;
- Scan down the entire original hash table for non-deleted elements;
- Use a new function to hash those elements into the new table.

Then what can we do? ng…

If there are N keys in the table, then $T(N) = O(N)$

Question: When to rehash?

Answer:

① As soon as the table is half full

② When an insertion fails

③ When the table reaches a certain load factor

Note: Usually there should have been $N/2$ insertions before rehash, so $O(N)$ rehash only adds a constant cost to each insertion.

However, in an interactive system, the unfortunate user whose insertion caused a rehash could see a slowdown.

Read Figures 7.23
for detailed implementation of rehashing.