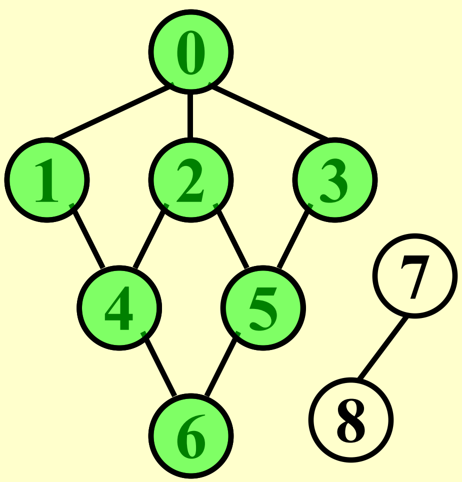
§ 6 Applications of Depth-First Search

/* a generalization of preorder traversal */

```
void DFS ( Vertex V )  /* this is only a template */
{   visited[ V ] = true;  /* mark this vertex to avoid cycles */
    for ( each W adjacent to V )
        if ( !visited[ W ] )
             DFS( W );
} /* T = O( |E| + |V| ) as long as adjacency lists are used */
```

1. Undirected Graphs

DFS (0)




```
void ListComponents ( Graph G )
{   for ( each V in G )
        if ( !visited[ V ] ) {
             DFS( V );
             printf(“\n“);
        }
}
```

0 1 4 6 5 2 3
7 8

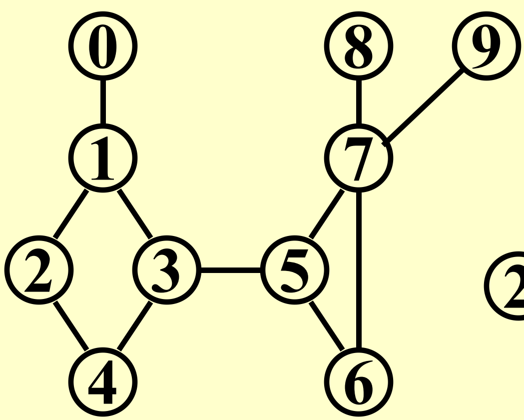
2. Biconnectivity

- *v* is an **articulation point** if G' = DeleteVertex(G, *v*) has **at least 2** connected components.
- G is a **biconnected graph** if G is connected and has no articulation points.
- A **biconnected component** is a maximal biconnected subgraph.

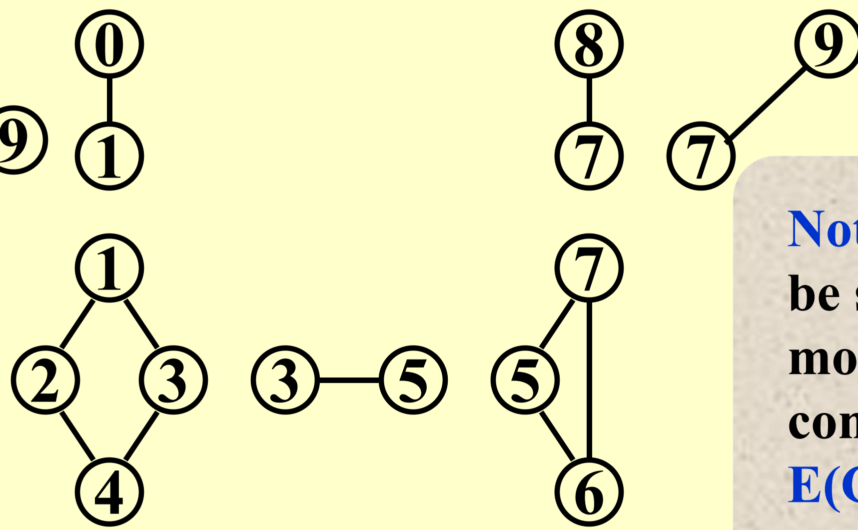
Biconnected graph

iculation point

〖Example〗



Connected graph



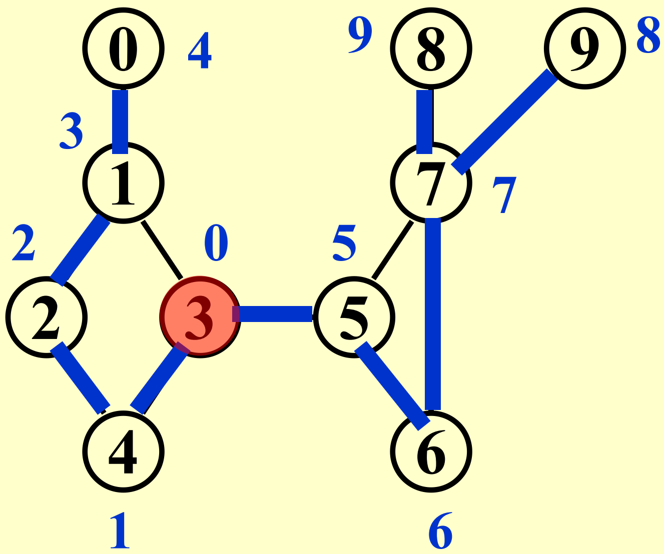
Biconnected components

> **Note:** No edges can be shared by two or more biconnected components. Hence **E(G) is partitioned** by the biconnected components of G.

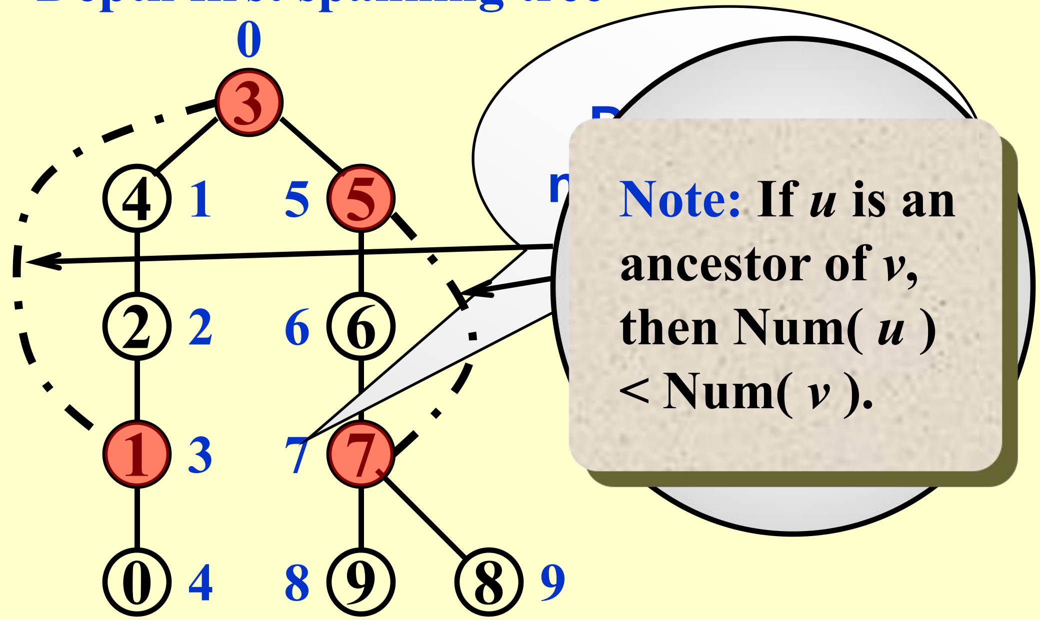
Finding the biconnected components of a connected undirected G

- Use **depth first search** to obtain a spanning tree of G

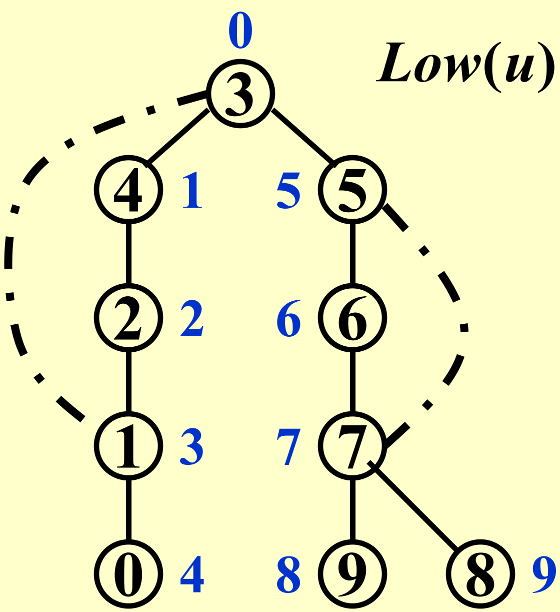
DFS (3)



Depth first spanning tree



- Find the **articulation points** in G
  - The root is an articulation point iff it has at least 2 children
  - Any other vertex u is an articulation point iff u has at least 1 child, and it is impossible to move down at least 1 step and then jump up to u's ancestor.



$$Low(u) = \min\{ Num(u), \min\{ Low(w) \mid w \text{ is a child of } u \}, \min\{ Num(w) \mid (u, w) \text{ is a back edge} \} \}$$

vertex	0	1	2	3	4	5	6	7	8	9
Num	4	3	2	0	1	5	6	7	9	8
Low	4	0	0	0	0	5	5	5	9	8

Therefore, *u* is an articulation point iff

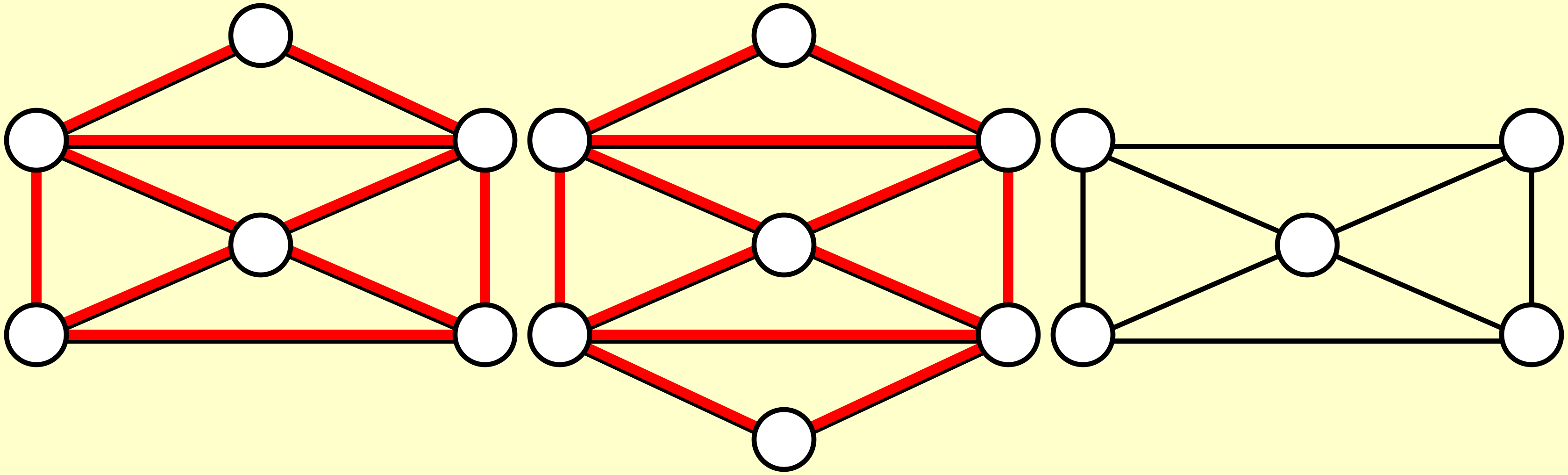
(1) *u* is the root and has at least 2 children; or

(2) *u* is not the root, and has at least 1 child such that

$Low(\, child \,) \geq Num(\, u \,)$.

Please read the pseudocodes on p.327 and p.329 for more details.

3. Euler Circuits



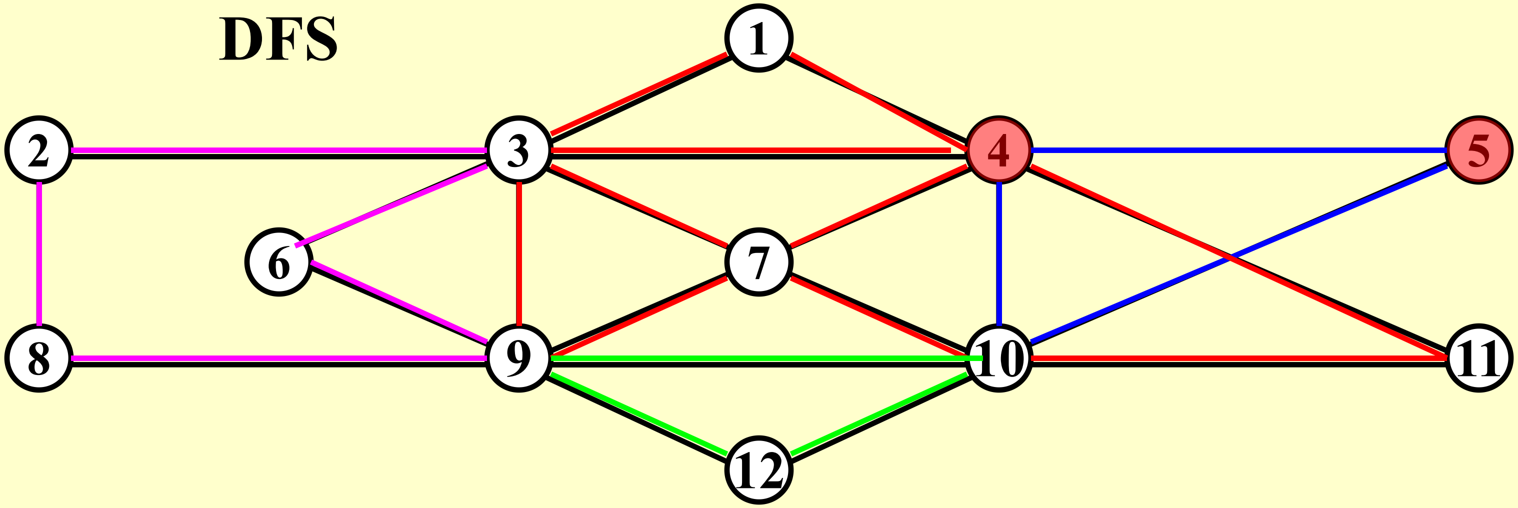
Draw each line exactly once without lifting your pen from the paper – *Euler tour*

Draw each line exactly once without lifting your pen from the paper, AND finish at the starting point – *Euler curcuit*

〖Proposition〗 An Euler circuit is possible only if the graph is connected and each vertex has an even degree.

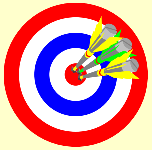
〖Proposition〗 An Euler tour is possible if there are exactly two vertices having odd degree. One must start at one of the odd-degree vertices.

DFS



Note:

- **The path should be maintained as a linked list.**
- **For each adjacency list, maintain a pointer to the last edge scanned.**
- $T = O(|E| + |V|)$



Find a simple cycle in an undirected graph that visits every vertex – *Hamilton cycle*