Learning Dynamics : Assignment 2

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1 Part 1: Evolutionary dynamics in infinite populations

1.1 How does the increase in the number of rounds affect the gradient of selection when only Copycat and Always Cheat are present in the population? Does it change the equilibria of the game?

We can theorize the following expected payoffs when Cooperators are extinct :

	Defector	Copycat
Defector	0	0.3
Copycat	-0.1	2

We can observe that the more rounds are played, the higher the expected average payoffs the copycats gets. When Cooperators become extinct, the Defectors's average expected payoffs drastically lowers. The copycats's average expected payoffs also lowers, but stays considerably high considering two copycats matching are guaranteed to have a big payoff. They will however lose on average less the more rounds are played as they only lose the first round, then they defect against defectors. This means that copycats have higher average expected payoff once Cooperators become extinct, explaining the gradient selection moving from the cooperators to the defectors, then to the copycats, because defectors lose their payoff once there are no cooperators to exploit.

1.2 Indicate all equilibria and their stability (stable, unstable or saddle point) of the game for which R = 10.

Because we are in an infinite population scenario, we have equal 1/3 odds of playing the trust game against a certain strategy. The expected payoffs can be described by the table below :

	Cooperator	Defector	Copycat
Cooperator	2	-1	2
Defector	3	0	0.3
Copycat	2	-0.1	2

We can then observe the following 2-simplex plot of the replicator dynamics for the 3 strategies over 10 rounds against a random opponent.

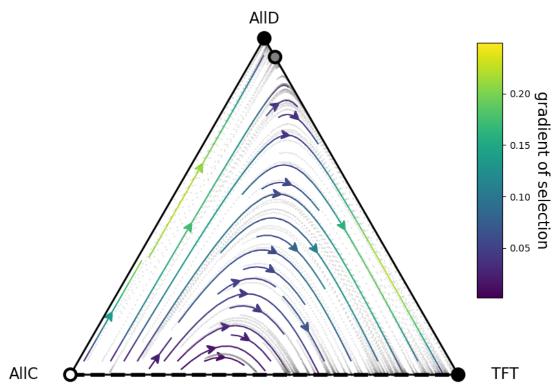

Figure 1: Replicator Dynamics for 3 strategies for 10 Rounds played per match

From this data we can notice 2 stable equilibria and 1 unstable equilibrium. There are no saddle points. The 2-simplex shows that a pure Defector and a pure Copycat stratgey are both the stable equilibria. The unstable equilibria is close to a pure Defector strategy. This can be further explained as seen from the expected payoffs table in a 10 rounds game that the average payoff for a Cooperator is 1, the expected average payoff for a Defector is 1.1 and the average payoff for a Copycat is 1.3. The payoffs will affirm our observations in the answer of the next question presented.

1.3 Is there a risk dominant strategy?

A risk dominant strategy is characterized by a Nash Equilibrium with minimal risk. In other words, it is a Nash equilibrium that presents the least worst-case amongst all expected payoffs of a strategy. We can find such risk dominant strategy for the Defector, who has the highest minimal possible payoff, equal to 0 against another Defector, as we can see on the upper table.

2 Part 2: Evolutionary dynamics in finite populations

2.1 What is effect of the intensity of selection β ? Are there any differences in the invasion diagram if $\beta = 0.1$ in comparison to when $\beta = 10$?

The effect of intensity of selection determines the portion of the population deviating from a strategy to another. In other words, the higher the intensity of selection is, the higher the intensity of invasion of a superior stratgey amongst a population of a different strategy will be. For example, we can observe the two following invasion diagram, where each match is 100 rounds :

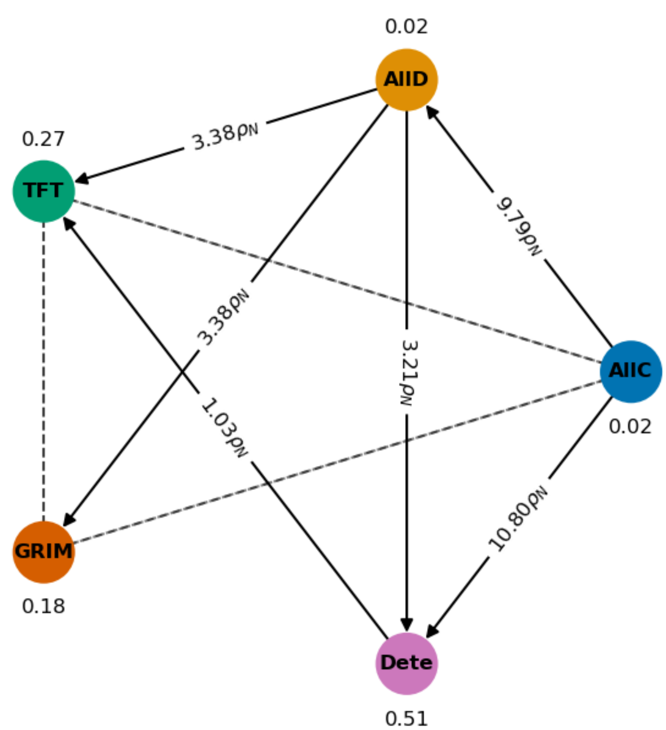

Figure 2: Invasion Diagram with parameters $\beta = 0.1$ and $Z = 100$

We can clearly see that the rate of invasion is much higher on figure 3 than

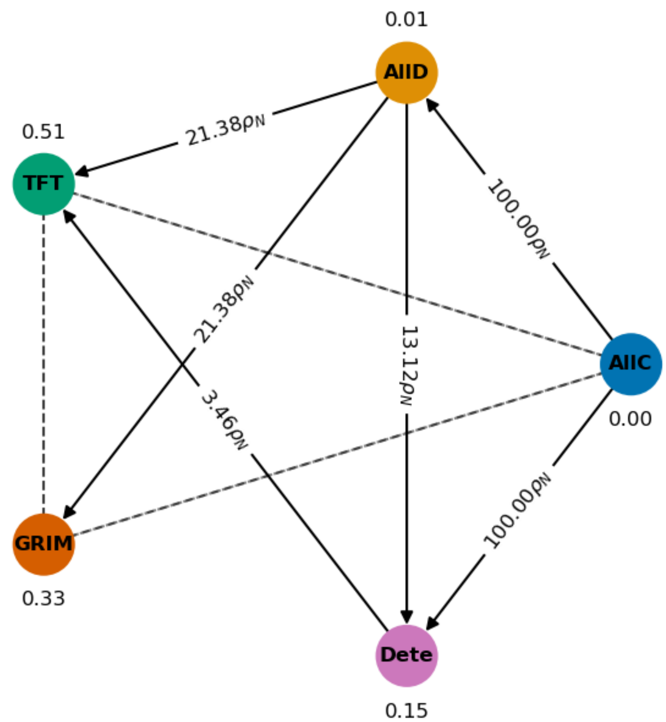

Figure 3: Invasion Diagram with parameters $\beta = 10$ and $Z = 100$

it is on figure 2, so much so that the Cooperator population reached extinction in the second case compared to the first. Interestingly, we can observe that the Copycat population has considerably increased (from 27 to 51) when we increased the intensity of selection, while the Detective population has considerably decreased (from 51 to 15). By considering the following table of expected payoffs, we can further justify our observations :

	Cooperator	Defector	Copycat	Grudger	Detective	Average Payoff
Cooperator	2	-1	2	2	-0.91	0.818
Defector	3	0	0.03	0.03	0.09	0.63
Copycat	2	-0.1	2	2	1.98	1.576
Grudger	2	-0.01	2	2	0.07	1.212
Detective	2.97	-0.03	1.98	0.03	1.98	1.386

We need to take into account that a strategies' effectiveness depends on the different population spreads. A higher intensity of invasion allows us to increase the impact of the size of a population. An invasion occurs when comparing the payoffs of two strategies matching, and its intensity is influenced by the difference of payout and β. This means that from the table alone, we can affirm the Cooperator population will shift to mostly Defectors or Detectives, as their payoffs when matching against cooperators are significantly higher. The decrease in Cooperators will incite Detectives and Defectors to change their strategies for a better payoff, increasing the population of copycats and grudgers.

By comparing the values of $\beta = 0.1$ and $\beta = 10$, we can observe a state where the detectives take full advantage of the remaining cooperators, while in figure 3, we can see them being invaded by copycats as there are no more cooperators to exploit and as such no reason to probe-cheat the other strategies, especially if all it does is risk losing payoff when matching against a Grudger that will stop cooperating. We can observe that the quantity of Grudger directly affects the shift of Detectives into Copycats.

2.2 What is the effect of population size Z ? Are there any changes to the invasion diagram when Z = 10 and Z = 100?

Let us observe the following invasion diagrams when the value of Z changes :

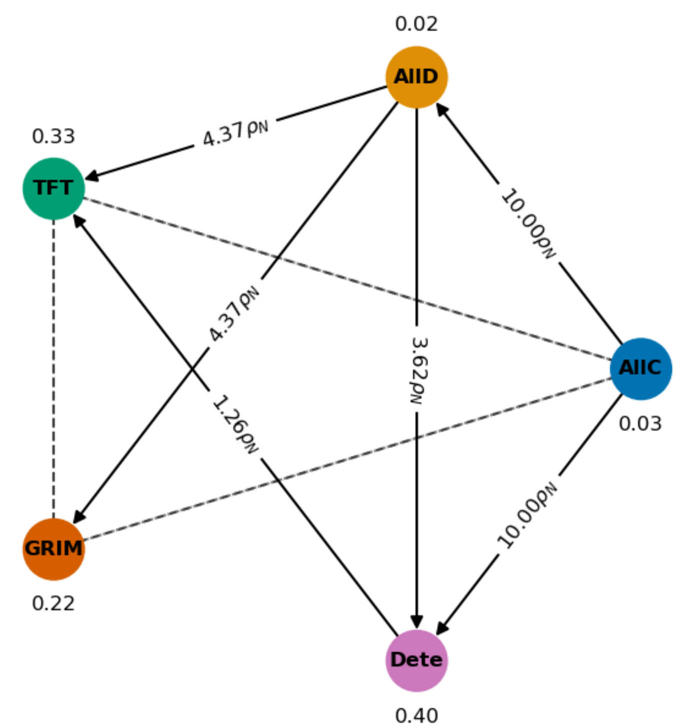

Figure 4: Invasion Diagram with parameters $\beta = 10$ and $Z = 10$

We can observe similar results when changing the population size and the intensity of invasion. When the population reaches a high enough number, the frequency at which a population changes will at least be one, which won't necessarily be the case for a population of 10 despite the intensity of invasion being the same. This means what we are observing is a butterfly effect, or a snowball effect, where by allowing one parameter to thrive, we observe greatly different results. Our observations lead us to the following conclusion : In a finite population, the Evolutionary dynamics is highly influenced by the fluidity

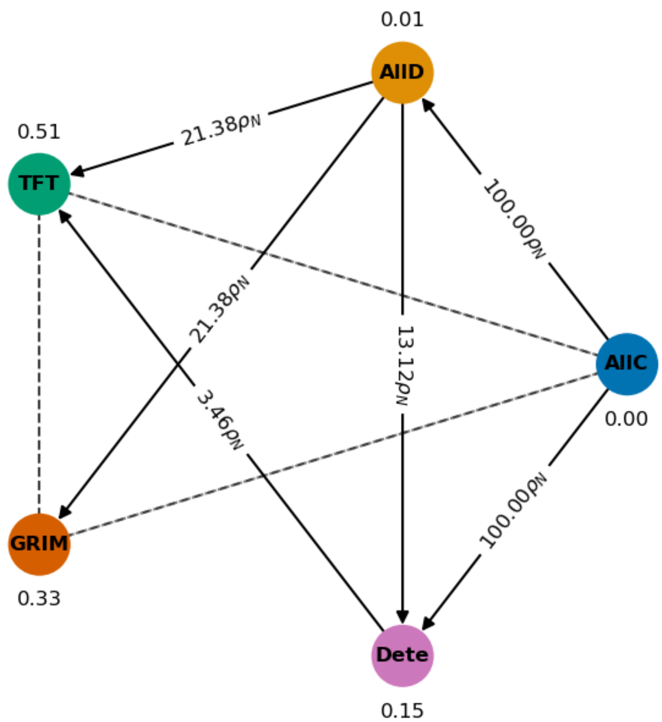

Figure 5: Invasion Diagram with parameters $\beta = 10$ and $Z = 100$

of transitions between strategies of a population.

2.3 Is there a dominant strategy (a strategy which is not invaded by any other) when $\beta = 10$ and Z = 50?

The following invasion diagram is described as follows :

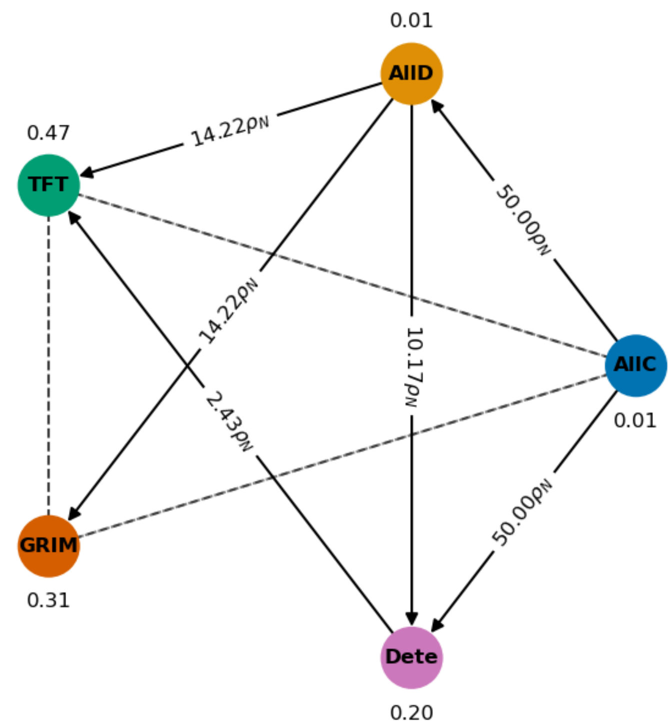

Figure 6: Invasion Diagram with parameters $\beta = 10$ and $Z = 50$

As per our previous observation, the Copycat strategy is always dominant, regardless of parameters selection. It is never invaded by any strategies, so in the case of $\beta = 10$ and $Z = 50$ and every other case, it is a dominant strategy.

3 Part 3: Monte-Carlo simulations – well-mixed populations

Our Monte-Carlo simulation produces the following result :

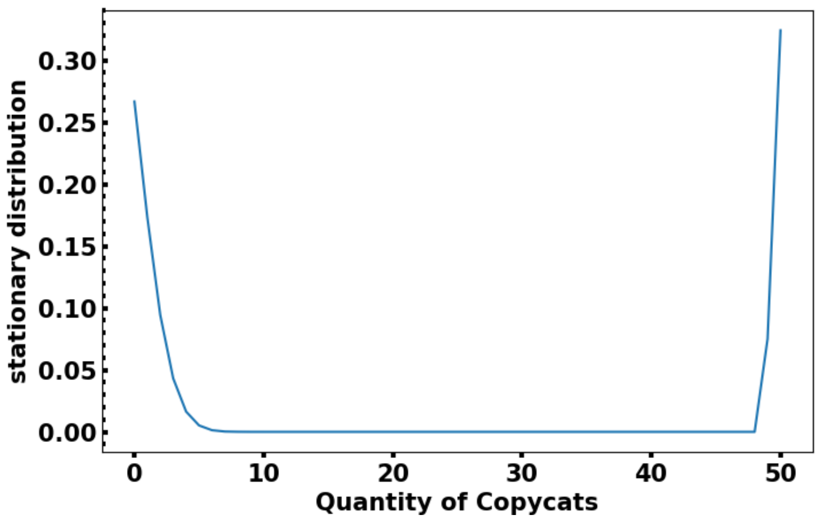

Figure 7: Stationary Distribution of all possible population states of size 50

This simulation introduces a population comprised of 50 individuals, opting either for the Defector startegy or the Copycat strategy. We can observe that there are 2 points of equilibria, respectively at 0 and 50 copycats. This observation match with the observations made in section 1.2 of this report. We can not observe any recorded states where the copycat population is between 9 and 48 out of 50. This means that during the transitory phase, the population stabilizes to either equilibria, and never transition from one point to the other, confirming that the recorded equilibria are stable points and not saddle points. We can however observe that the stationary distribution increases more sharply when reaching 50 than when reaching 0, indicating that it is much easier to reach equilibria towards a full copycat population than a full defector one. This means that for the specific parameters used in our experiment, there are roughly equal probabilities for the population to stabilizes towards a population of only Defectors or towards a population of only Copycats.

4 Conclusion

From our observations, we can conclude as to how the evolutionnary dynamics of the game of trust works in both infinite and finite populations. The Copycat strategy is undeniably the most rewarding but is not necessarily the one that always dominates. In the case where there are only defectors and copycats, there is a chance that the Defector strategy dominates and that only defectors remain in the population.