Elementary Charge: $e = 1.602 \times 10^{-19}\mathrm{C}$

Mass of Electron: $m = 9.109 \times 10^{-31}\mathrm{kg}$

Mass of Proton: $m = 1.672 \times 10^{-27}\mathrm{kg}$

Vacuum Permittivity: $\epsilon_0 = 8.854 \times 10^{-12}$

Coulomb Constant: $k = 9.0 \times 10^9 \frac{Nm^2}{C^2}$

Flux Equations

Flux through a surface

$\Phi_e = E_\perp A = EA\cos(\theta)$ where θ is the angle between the normal line and the flux lines or the angle between $\hat{n}$ and $\vec{E}$.

Flux Integral

$\Phi_e = \oint \vec{E} \cdot d\vec{A}$

When using a Gaussian sphere:

$\Phi_e = \frac{q}{4\pi\epsilon_0 r^2} \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$

Gauss Law Combined

$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$

Most General Use

$$\Phi_e = EA_{sphere} = 4\pi r^2 E$$

when using the surface area of a sphere:

$$4\pi r^2 E = \frac{Q_{in}}{\epsilon_0}$$

Flux of Infinite Plane of Charge

$Q_{in} = \eta A$

$\Phi_e = 2EA = \frac{Q_{in}}{\epsilon_0} = \frac{\eta A}{\epsilon_0}$

Area divides out revealing:

$E_{plane} = \frac{\eta}{2\epsilon_0}$

Uniform Electric Field

$U_{elec} = U_0 + qEs$

U_0 is the potential at the negative plate (often $U_0 = 0$)

s is the distance from the negative plate

Difference for Negative and Positive Charges

In general, movement against the electric field is negative energy.

Positive

- Field direction is "downhill." Potential energy decreases as the charge speeds up.
- Potential goes down and kinetic goes up as it moves to the negative.

Negative

- Field direction is "uphill." Potential energy increases as the charge slows.
- Potential goes up and kinetic goes down as it moves toward the negative.
- The movement of an electron towards the positive (natural movement) is *negative* mechanical energy: $E_{mech} = K_i + (-e)Ed$. K_i is zero.

Potential Equations

Potential Energy of Point Charges

$U_{elec} = K\frac{q_q q_2}{r}$ or $\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

Escape speed

$K_f + U_f = K_i + U_i$

$v_i = \sqrt{\frac{Kq_1q_2}{mr_i}}$ ← Unclear when the usage of this is.

Potential of a Dipole

$U_{dipole} = -pE\cos(\phi) = -\vec{p} \cdot \vec{E}$

Where $\vec{p}$ is the dipole vector

Electric Potential Energy from Voltage

$U_q = qV$

Where 1 volt = 1 J/C

Example: A proton would loose $-100e$ moving through $100V$. Potential is negative even though the charge is positive:

$$K_f + qV_f = K_i + qV_i$$
$$K_f = K_i - q\Delta V$$

Electric Potential inside a Parallel-Plate Capacitor

$V = Es$, where s is the distance form the *negative* electrode

Field can also be defined by $E = \frac{\Delta V_C}{d}$. This is more common.

Electric Potential from a Point Charge

$V = k\frac{q}{r}$

Electric Potential from a Ring

$V_{ring\ on\ axis} = k\frac{Q}{\sqrt{R^2+z^2}}$, R is radius, z is distance.

Electric Potential from Charged Disk

$V_{disk\ on\ axis} = \frac{Q}{2\pi\epsilon_0 R^2}(\sqrt{R^2+z^2} - z)$

$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s ds$

Electric Field from Potential

$E_s = -\frac{dV}{ds} \leftarrow$ Ensuring that a Δ is always $final - initial$ is important. The this equation is bad for varying electric fields.

Dielectric Constant

$\kappa = \frac{E_0}{E}$

Capacitance is then increased proportionally:

$C = \kappa C_0$

This is designed to let air be $\kappa = 1$

Change in Voltage

$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s ds$

Concentric Cylinders

$C = \frac{2\pi\epsilon_0 l}{\ln(\frac{R_2}{R_1})}$

Concentric Spheres

$C = \frac{4\pi\epsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}}$

Energy from Electric Field or Potential

Because $U_{elec} = U_0 + qEs$, the integral above ($-\int_{s_i}^{s_f} E_s ds$) can be multiplied by charge to get field because Voltage $= J/C$.

Field of radial rod and shell:

$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$, λ is equal to the charge density of all enclosed charges.

Charge on Capacitor

$Q = C\Delta V$

Parallel Plate Capacitor

$E = \frac{Q}{\epsilon_0 A}$ or $C = \frac{Q}{\Delta V_c} = \frac{\epsilon_0 A}{d}$

Energy of Capacitor

$U_C = \frac{1}{2}C(\Delta V)^2$