ELEE 4220: Feedback Control Systems
Lab 3
Emerson Hall
02/21/2025

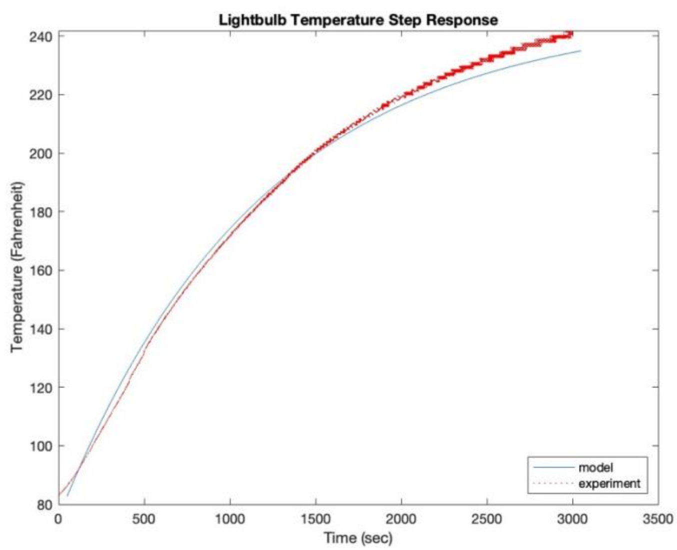
I. Introduction

In this lab, a proportional-integral-derivative (PID) controller was designed and implemented to regulate the temperature of a lightbulb system. Building on the mathematical model from Lab 1, the objective was to analyze the impact of different proportional gains (Kp) on system performance, particularly in terms of response time, overshoot, and steady-state error. The system was first modeled in MATLAB's Control System Toolbox, and simulations were conducted in Simulink, incorporating saturation effects to reflect real-world actuator limitations. The final controller was then tested on the physical hardware, providing insight into the trade-offs between control speed, saturation, and stability.

II. Deliverables

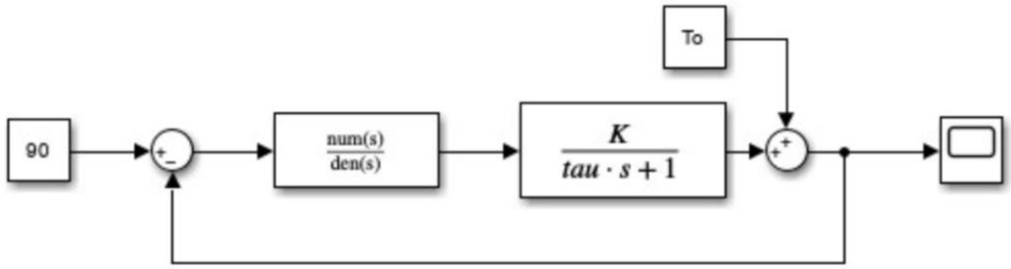
A. Describe what method you are implementing and show your first order transfer function model.

a. Considering the actuation method for the solid-state relay (SSR), we have chosen to represent the controller output as a duty cycle, limiting it to a range of [0,1]. This approach aligns with the operational characteristics of the SSR, which modulates power delivery through Pulse Width Modulation (PWM). Given the first-order transfer function derived from Lab 1, $P(s) = \frac{159.15}{1176.3s+1}$, the model reflects a slow thermal response. The chosen method ensures that the control signal smoothly regulates heating power, optimizing response time and stability while staying within the physical limitations of the system. This decision lays the groundwork for implementing a PID controller to refine performance, ensuring minimal steady-state error and appropriate transient response.

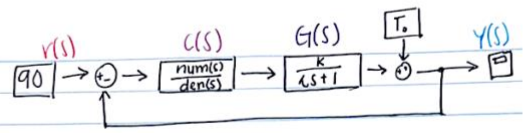


b.

B. **Derive a closed loop transfer of PID controller in the following closed loop.**



a.



$$\frac{Y(s)}{r(s)} = \frac{C(s)\,G(s)}{1 + C(s)G(s)}$$

$$c(s) = K_p + \frac{K_I}{s} + K_d s \qquad G(s) = \frac{K}{\tau s + 1}$$

$$\frac{Y(s)}{r(s)} = \frac{\left(K_p + \frac{K_I}{s} + K_d s\right)\left(\frac{K}{\tau s+1}\right)}{1 + \left(K_p + \frac{K_I}{s} + K_d s\right)\left(\frac{K}{\tau s+1}\right)}$$

$$\Downarrow$$

$$\frac{Y(s)}{r(s)} = \frac{\left(\frac{K\cdot K_p + K\cdot K_d s}{\tau s+1} + \frac{KK_I}{\tau s^2 + s}\right)}{1 + \frac{K\cdot K_p + K\cdot K_d s}{\tau s + 1} + \frac{K\cdot K_I}{\tau s^2 + s}}$$

$$\Downarrow$$

b.

$$\frac{Y(s)}{r(s)} = \frac{\left(\frac{K\cdot K_p s + K\cdot K_d s^2 + KK_I}{\tau s^2 + s}\right)}{\left(\frac{\tau s^2 + s + KK_p s + KK_d s^2 + KK_I}{\tau s^2 + s}\right)}$$

$$\Downarrow$$

$$\frac{Y(s)}{r(s)} = \frac{K\cdot K_d s^2 + K\cdot K_p s + KK_I}{(\tau + KK_d)s^2 + (1 + KK_p)s + KK_I}$$

with values

$\tau = 1176.3$ seconds

$K = 159.15$

$$\frac{Y(s)}{r(s)} = \frac{159.15K_d s^2 + 159.15K_p s + 159.15K_I}{(1176.3 + 159.15K_d)s^2 + (1 + 159.15K_p)s + 159.15K_I}$$

c.

C. **Analyze what you have derived.**

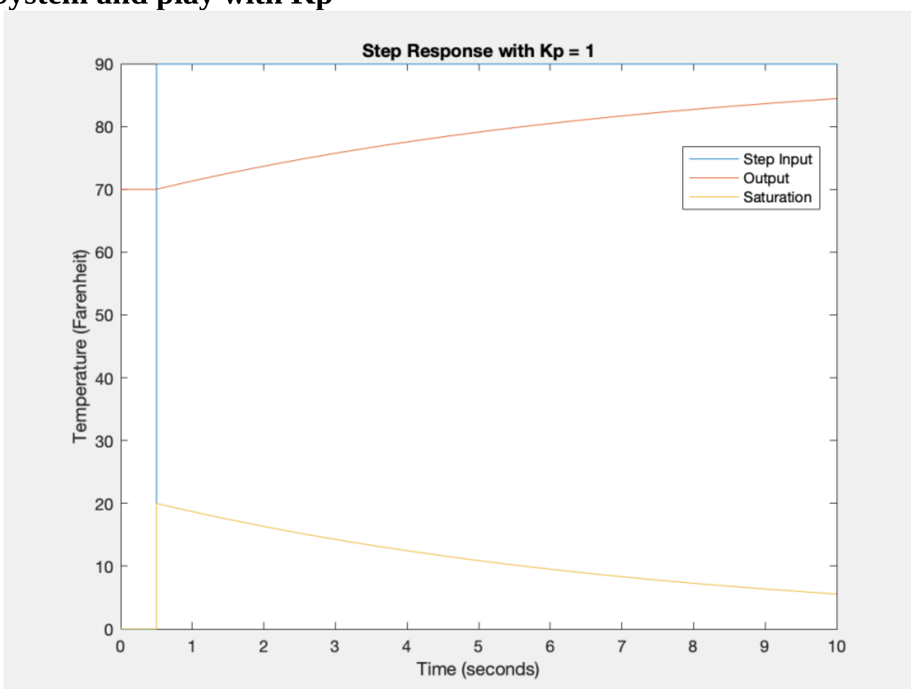
a. **Set KI=KD=0. How close is the initial system to zero steady-state error?**

i. $KI = KD = 0$

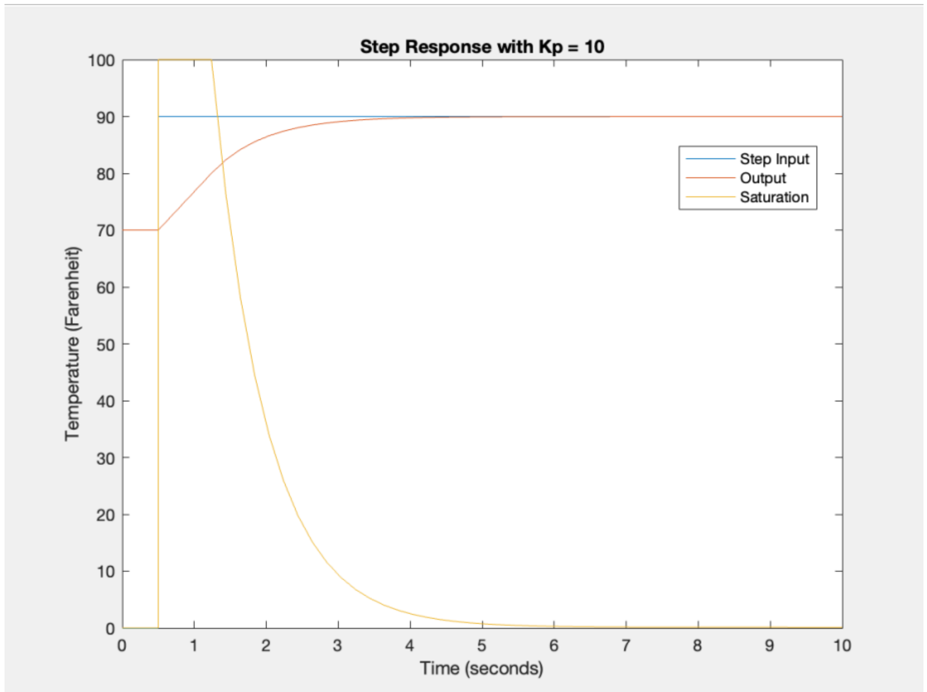
$$\frac{Y(s)}{r(s)} = \frac{159.15K_p s}{1176.3s^2 + (1+159.15K_p)s}$$

ii. With KI and KD equaling zero, the steady-state error can be minimized, but it will not necessarily be zero. The error depends on the proportional gain Kp. If Kp is large enough and matches the system's characteristics, the steady-state error can approach zero. However, if Kp is too small, the system will have a larger steady-state error. The system will always have some steady-state error unless the proportional gain is perfectly tuned to eliminate it, which is typically not the case in practical systems.

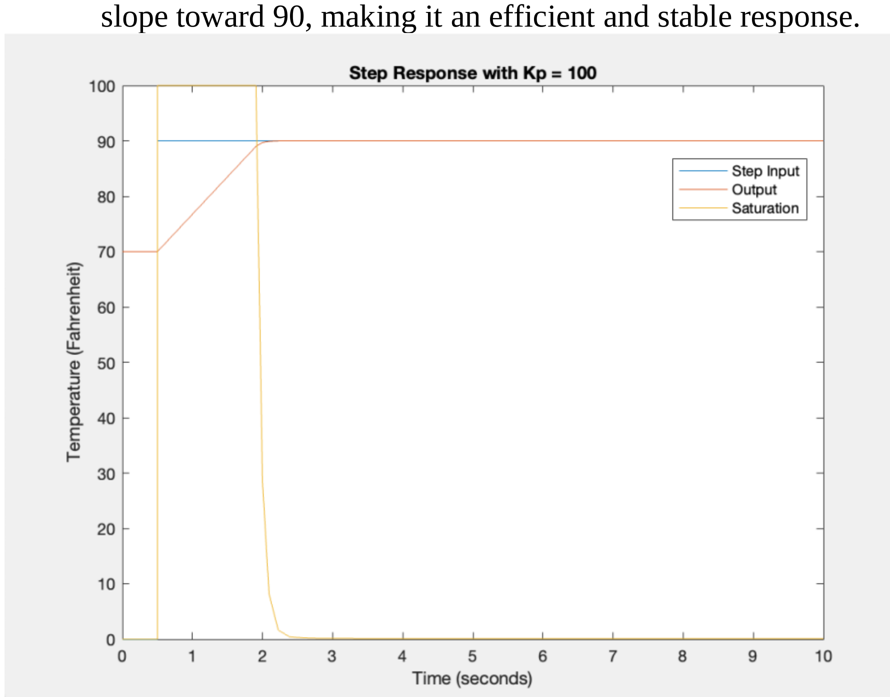
b. **Build System and play with Kp**

i. 

1. With a proportional gain of 1, the control effort is relatively small, causing the saturation signal to only reach 20 at t=0.5 seconds before exponentially decreasing back to zero. The output response is overdamped, meaning it rises slowly toward the setpoint without overshooting. The system takes a longer time to reach 90, showing a gradual and smooth increase.

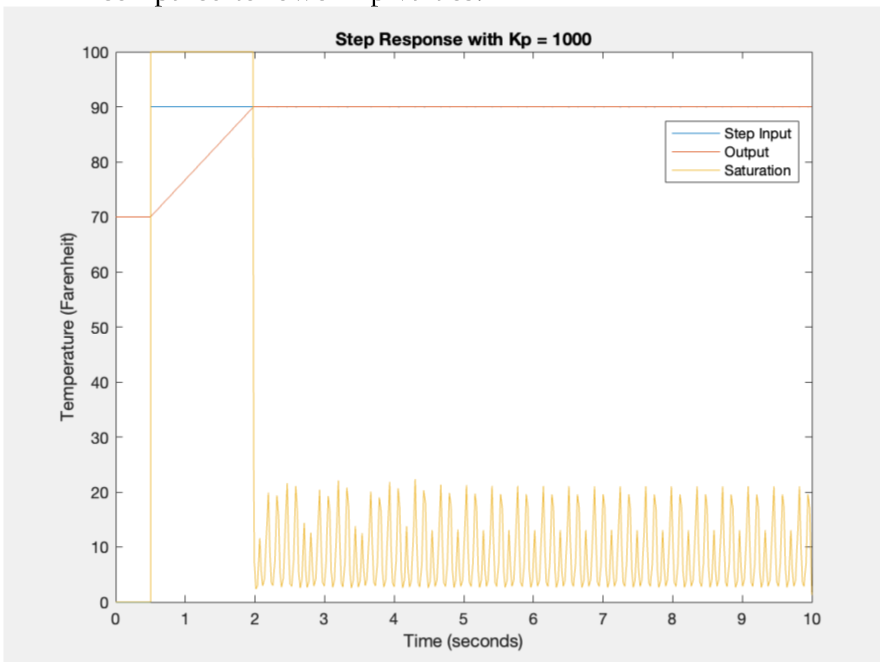
ii. 

1. When Kp is increased to 10, the control effort becomes stronger, causing the saturation signal to hit the upper limit of 100 at t=0.5 seconds. It remains there until about 1.25 seconds before exponentially decaying to zero. The output response is critically damped, meaning it reaches the setpoint as quickly as possible without overshooting. The trajectory follows a smooth exponential slope toward 90, making it an efficient and stable response.

iii. 

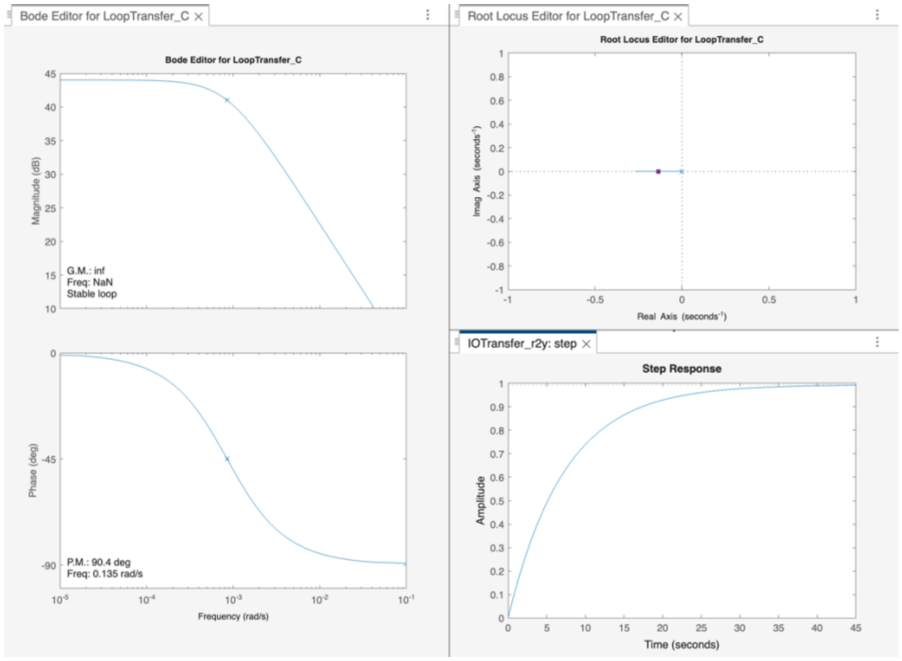
1. At Kp=100, the system exhibits an even stronger control effort, with the saturation signal again reaching 100 at t=0.5sand staying

there until about 2.9 seconds. The drop-off from saturation is more linear, with a slight exponential decay at the end. The output remains critically damped but follows a nearly straight-line path to 90, meaning the system responds faster and more directly compared to lower Kp values.

iv. 

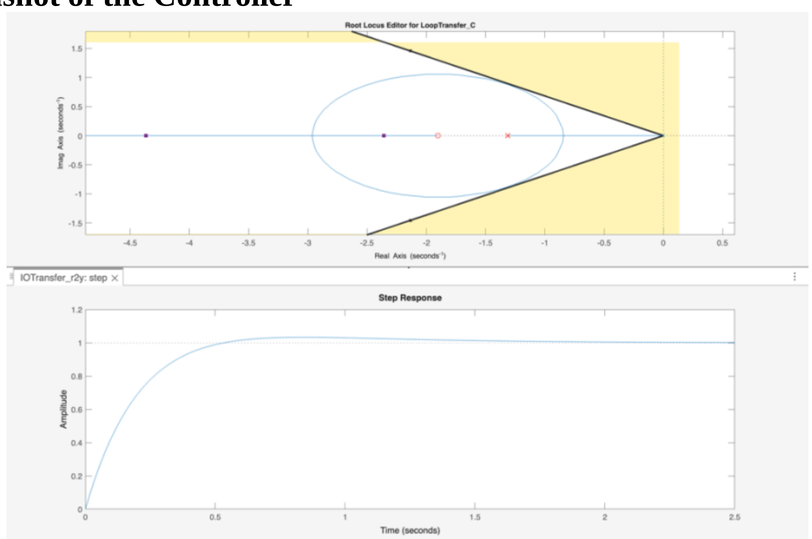
1. With Kp set to 1000, the system exhibits extreme control effort, fully saturating at 100 from t=0.5s to 2 seconds. After that, the control signal drops sharply to 1 and begins oscillating around 10, indicating that the controller is struggling to stabilize. Despite this, the output response remains similar to the Kp=100 case, but the oscillations in the control signal suggest the system is approaching an underdamped behavior, where high gains may lead to instability if increased further.

D. **Start Journey of Systematic Control System Design**

a. 

   i. This is the start-up screen of my system in Control System Designer.

E. **Screenshot of the Controller**

a. 

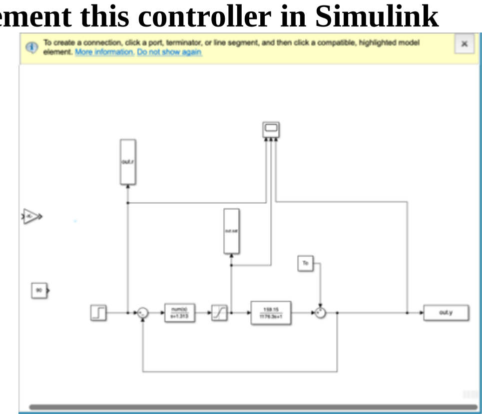
   i. With changing the zeros/poles with the requirements, I created this step response. The equation for this controller is below.

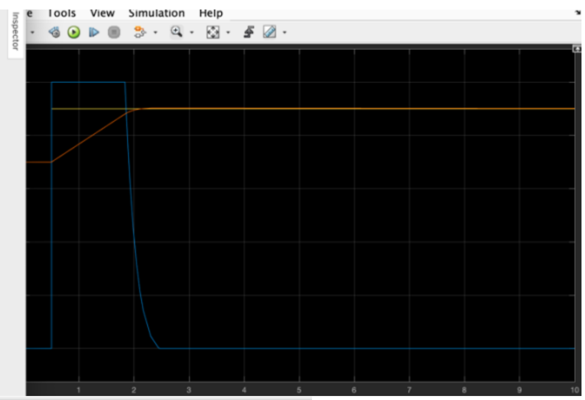
```
C2 =

  39.998 (s+1.903)
  ----------------
     (s+1.313)

Name: C
```

b.

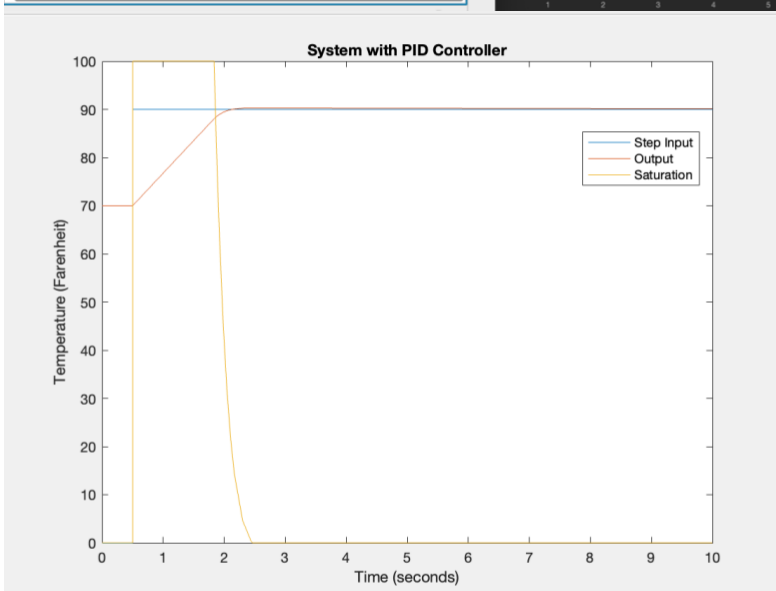
   i. The controller is a first-order transfer function with a gain of 39.998. There is a zero at -1.903 and a pole at -1.313. The controller is stable as the zeros and poles are on the left-side plane. The zero helps improve the transient response while the pole slows down the response but ensures stability.

F. **Implement this controller in Simulink**





a.

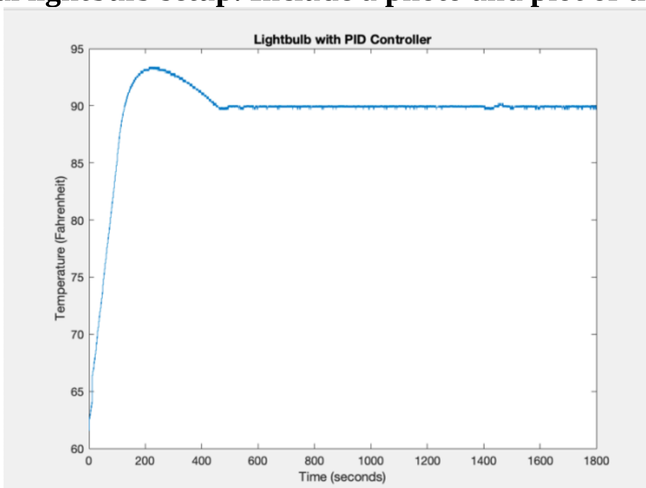


b.

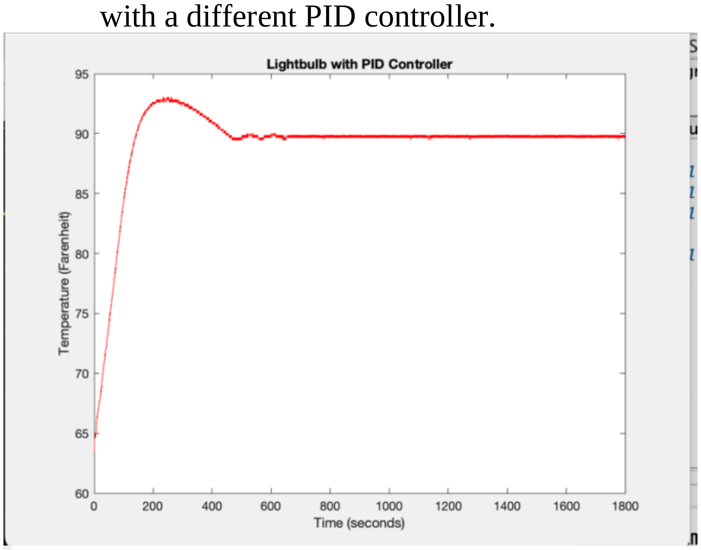
G. **From your scope does your system saturate? If so, how long?**

a. Yes, based on the scope data, the system does experience saturation. The control signal reaches the upper saturation limit of 100 at 0.5 seconds (when the step input occurs) and remains saturated until approximately 1.8 seconds, at which point it begins to decrease. This means the system is saturated for about 1.3 seconds.

H. Finally, when you are happy with your final controller, implement it on your physical lightbulb setup. Include a photo and plot of the real data.

a. 

i. This is the first test I ran. It did not work how I wanted so I redid the test with a different PID controller.

b. 


```
C =

  21.604 (s+0.639)
  ----------------
     (s+0.5745)
```

i. It still has overshoot but it regulate around 90 degrees Fahrenheit better.

I. **Discuss what type of controller you implemented and why it worked for this problem.**
    a. This controller was designed to improve system response by increasing stability and reducing settling time. When applied to the physical lightbulb setup, the system successfully reached the desired intensity, rising to 90, with a small overshoot to 92.5 at 200 seconds, before settling at 90 by 425 seconds. The lead compensator worked effectively by shaping the system dynamics to achieve a stable and accurate response.

J. **Other thoughts and comments about the lab or implementation.**
    a. This lab was really straight forward and helped with the understanding of a PID controller.

IV. Conclusion

Through systematic tuning of Kp, the effects of gain on system response were observed, with higher values increasing control effort and saturation duration. The final controller successfully met the design criteria, achieving a stable response with minimal overshoot and acceptable settling time. The inclusion of a saturation block highlighted the practical constraints of actuator limits, demonstrating the importance of balancing performance and feasibility. Ultimately, this lab reinforced key control system principles, emphasizing the role of gain tuning and real-world constraints in designing effective controllers.