MGSC-662-075 - Decision Analytics
Facility Location Problem

December 9, 2022

1 Description

Intro

This problem is a case of Capacitated Multi-Facility Weber Problem (CMFWP)
(https://link.springer.com/article/10.1057/palgrave.jors.2602262)
We are placing waste facilities (with certain capacities) in an Euclidean plane to satisfy the demand of wind farms with the minimum total transportation cost and fixed cost. The demand and location of each wind farm are known and the transportation cost between wind farms and facilities is proportional to Manhattan distance between them.

Solution Approach

We solve this model in a two stage approach. Each stage contains generic model components and components only relevant to the stage.

1. Assign the entire demand of each wind turbine to a specific facility.
2. Enforce the location of each facility and redistribute wind farm demand.

Assumption

We assume wind farms have some yearly waste. Yearly waste is calculated based on age and size of the wind farms. Every 20 years wind turbines need to be replaced, we assume 20% per year are replaced.

Average Yearly Deliveries are based on truck capacity and the size of the farm. Each farm has a specific number of turbines, each turbine has 3 blades and weights approximately 36 tons (REFERENCE). Each truck has a capacity of 62.5 tons (REFERENCE). We assume each turbine has a useful life of 20 years (REFERENCE). Thus we multiply number of turbines by number of blades and by weight of blades divided by capacity divided useful life and round up to get the number of deliveries per year (ADD FORMULA).

Optimization Model

Sets

W - wind farm
T - facility type
F - facility count $\in \{1, 2..., 10\}$

Data Requirements

wx_w - longitude of wind farm w
wy_w - latitude of wind farm w
wd_w - average yearly deliverables rounded up from wind farm w
ww_w - average yearly waste delivery per truck from wind farm w

co_t - building cost of facility of type t
ca_t - capacity of facility of type t

dc - delivery cost per truck per km
ph - planning horizon (years)
dk - conversion of latitude/longitude to km

Decision Variables

fb_{ft} - binary indicating whether facility number f of type t should be built
fx_{ft} - continuous indicating longitude of facility number f of type t
fy_{ft} - continuous indicating latitude of facility number f of type t

$dist\_x_{wft}$ - continuous indicating longitude distance between wind farm w, facility f of type t
$dist\_y_{wft}$ - continuous indicating latitude distance between wind farm w, facility f of type t
$dist_{wft}$ - continuous indicating manhattan distance between wind farm w, facility f of type t

Stage 1: wa_{wft} - binary variable assigning wind farm w to facility f, t

Stage 2: dw_{wft} - integer variable indicating how many deliveries to make from w to f, t

Constraints

Demand

$$\begin{aligned} \sum_{t,f} wa_{wft} = & \quad 1 \quad \forall w \qquad \text{Stage 1} \\ \sum_{t,f} dw_{wft} = & \quad wd_w \quad \forall w \qquad \text{Stage 2} \end{aligned} \tag{1}$$

Capacity

$$\begin{aligned} \sum_{t,f} wa_{wft} * ww_w * wd_w \leq & \quad ca_t * fb_{ft} \quad \forall w \qquad \text{Stage 1} \\ \sum_{w} dw_{wft} * ww_w \leq & \quad ca_t * fb_{ft} \quad \forall f, t \qquad \text{Stage 2} \end{aligned} \tag{2}$$

Distance Constraint

$$\begin{aligned} dist\_x_{wft} \geq & \quad wx_w - fx_{ft} \quad \forall w, f, t \\ dist\_x_{wft} \geq & \quad -(wx_w - fx_{ft}) \quad \forall w, f, t \\ dist\_y_{wft} \geq & \quad wy_w - fy_{ft} \quad \forall w, f, t \\ dist\_y_{wft} \geq & \quad -(wy_w - fy_{ft}) \quad \forall w, f, t \\ dist_{wft} = & \quad dist\_x_{wft} + dist\_y_{wft} \quad \forall w, f, t \end{aligned} \tag{3}$$

Objective Function

$$
\begin{aligned}
min \quad & \sum_{wtf}(dist_{wft} * wa_{wft} * wd[w] * dc * ph * dk) && \text{Stage 1} \\
& + \sum_{wtf}(dist_{wft} * dw_{wft} * dc * ph * dk) && \text{Stage 2} \\
& + \sum_{ft}(fb_{ft} * co_t)
\end{aligned}
\tag{4}
$$