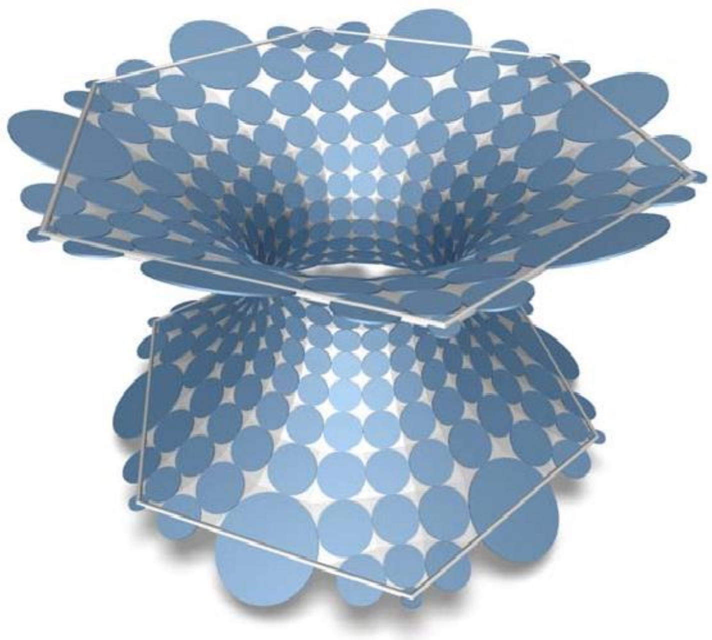
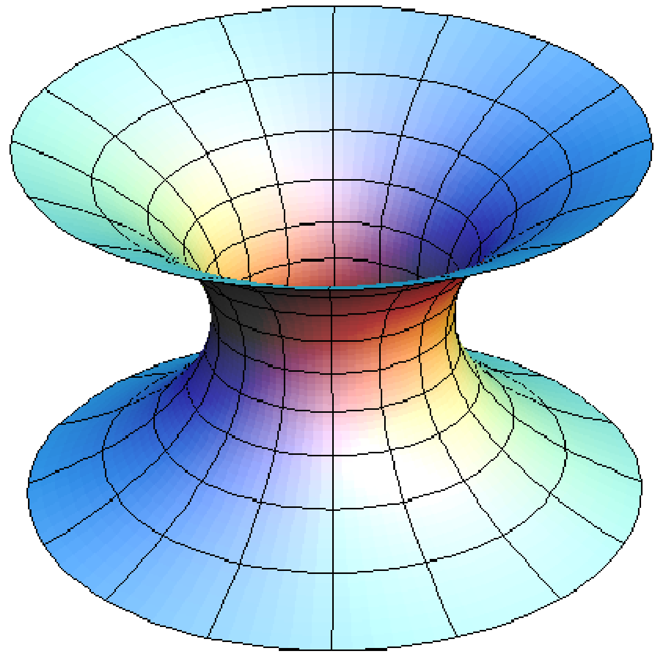
(Discrete) Differential Geometry





Motivation

- Understand the structure of the surface

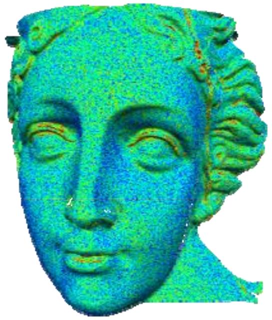
  Properties: smoothness, « curviness », important directions

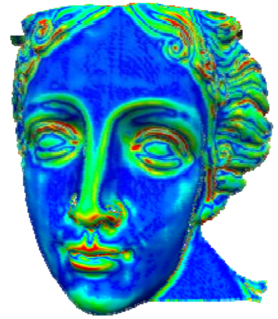
- How to modify the surface to change these properties
- What properties are preserved for different modifications

- The math behind the scenes for many geometry processing applications

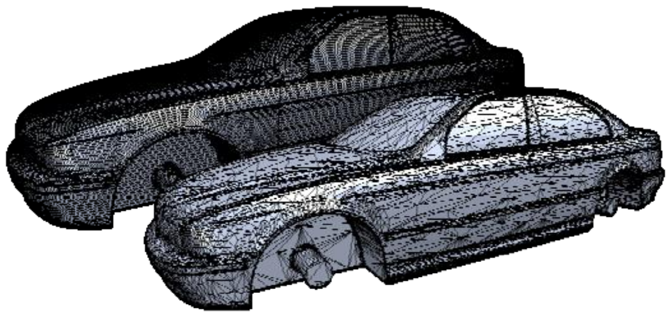
Motivation

- Smoothness
  - ➡ Mesh smoothing

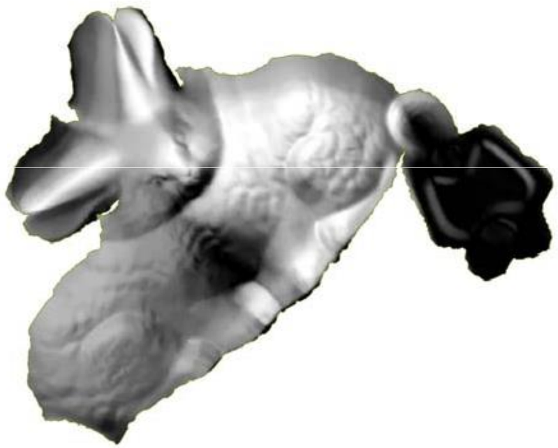




- Curvature
  - ➡ Adaptive simplification



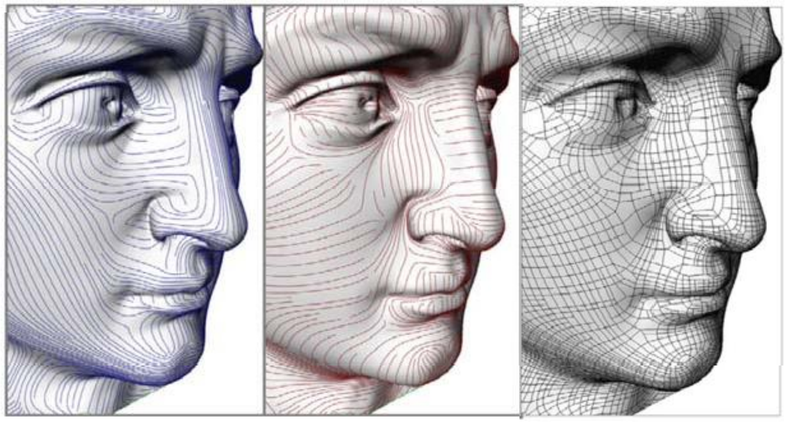
  - ➡ Parameterization



Motivation

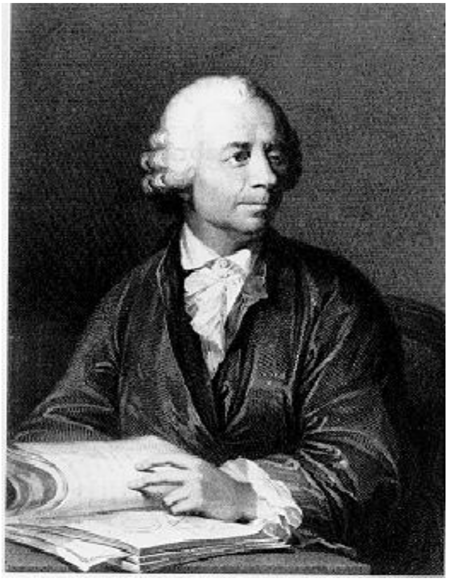
- Triangle shape
  - ➡ Remeshing

- Principal directions
  - ➡ Quad remeshing



Differential Geometry

- M.P. do Carmo: ***Differential Geometry of Curves and Surfaces***, Prentice Hall, 1976



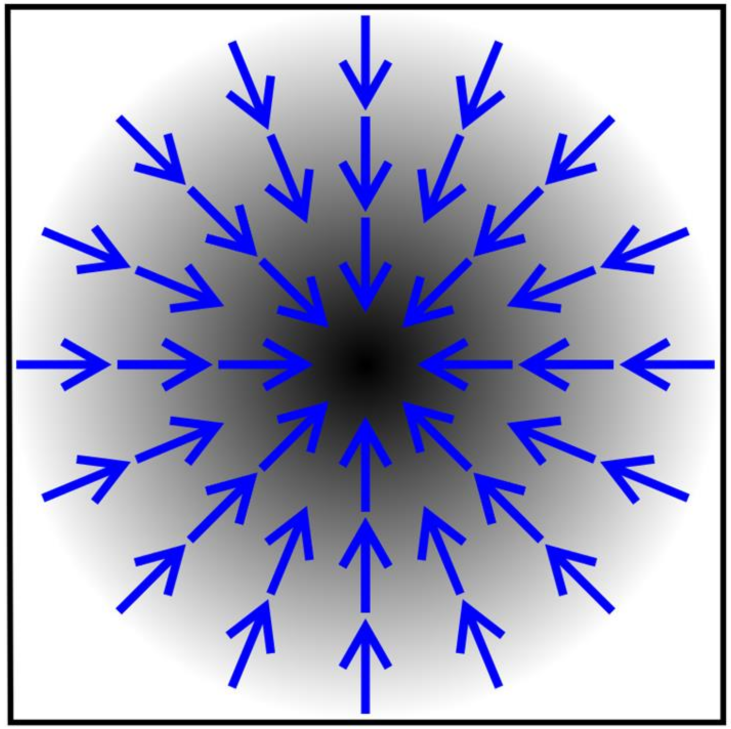
Leonard Euler (1707 - 1783)

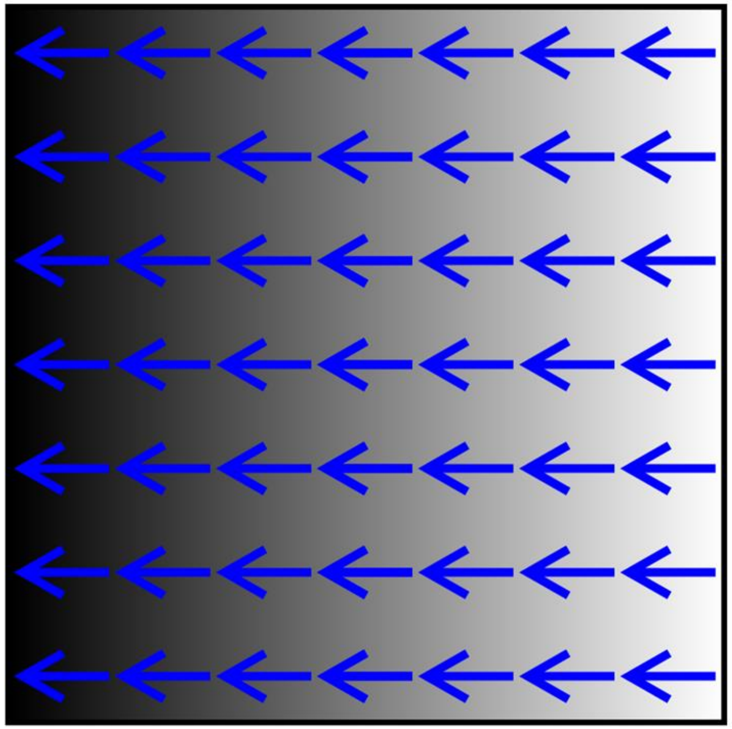
Carl Friedrich Gauss (1777 - 1855)

Functions and operators on a triangle mesh

Gradient in the Euclidean space

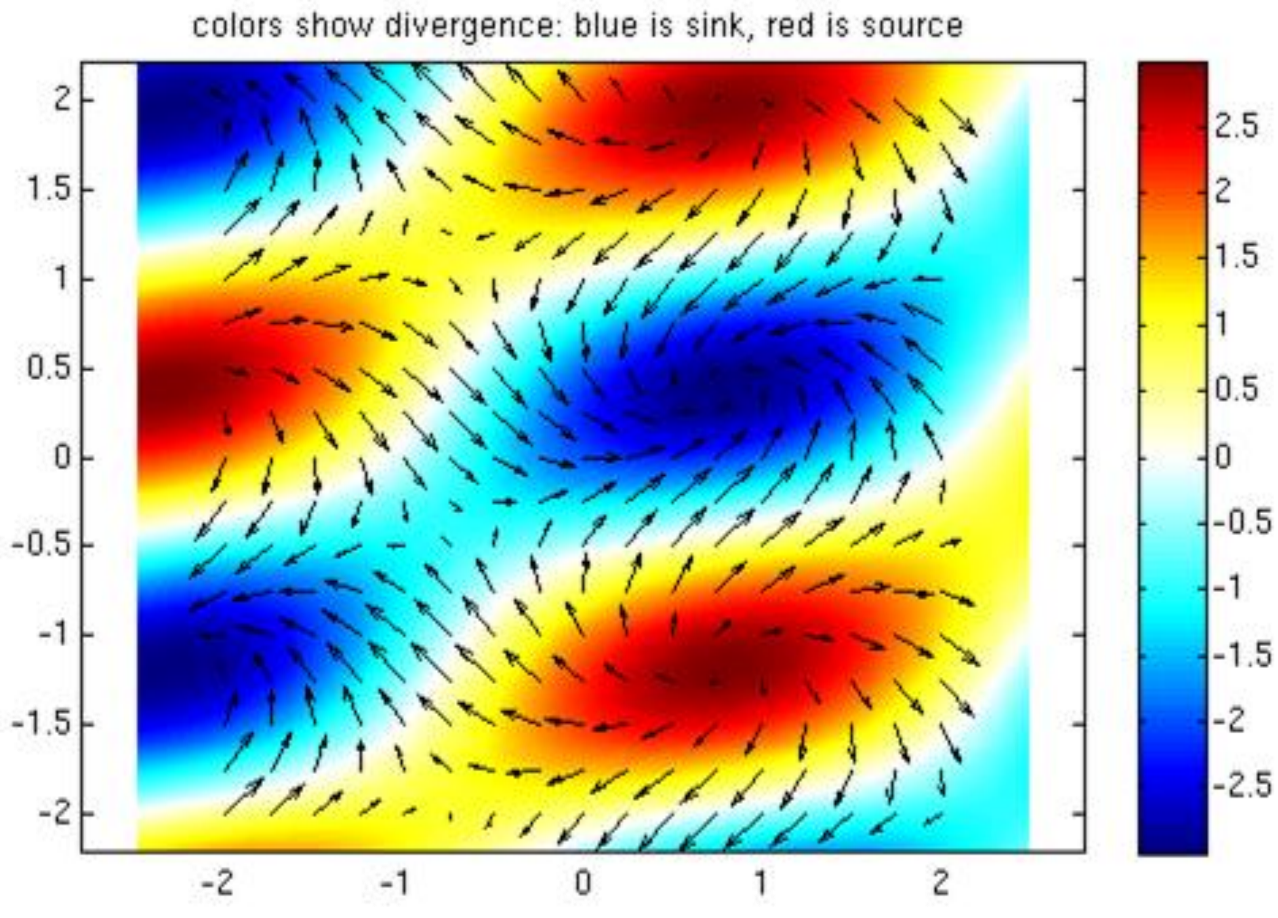
$$\vec{\nabla} f(x_1, x_2, \ldots, x_n) = \begin{bmatrix} \partial f / \partial x_1 \\ \partial f / \partial x_2 \\ . \\ \partial f / \partial x_n \end{bmatrix}$$





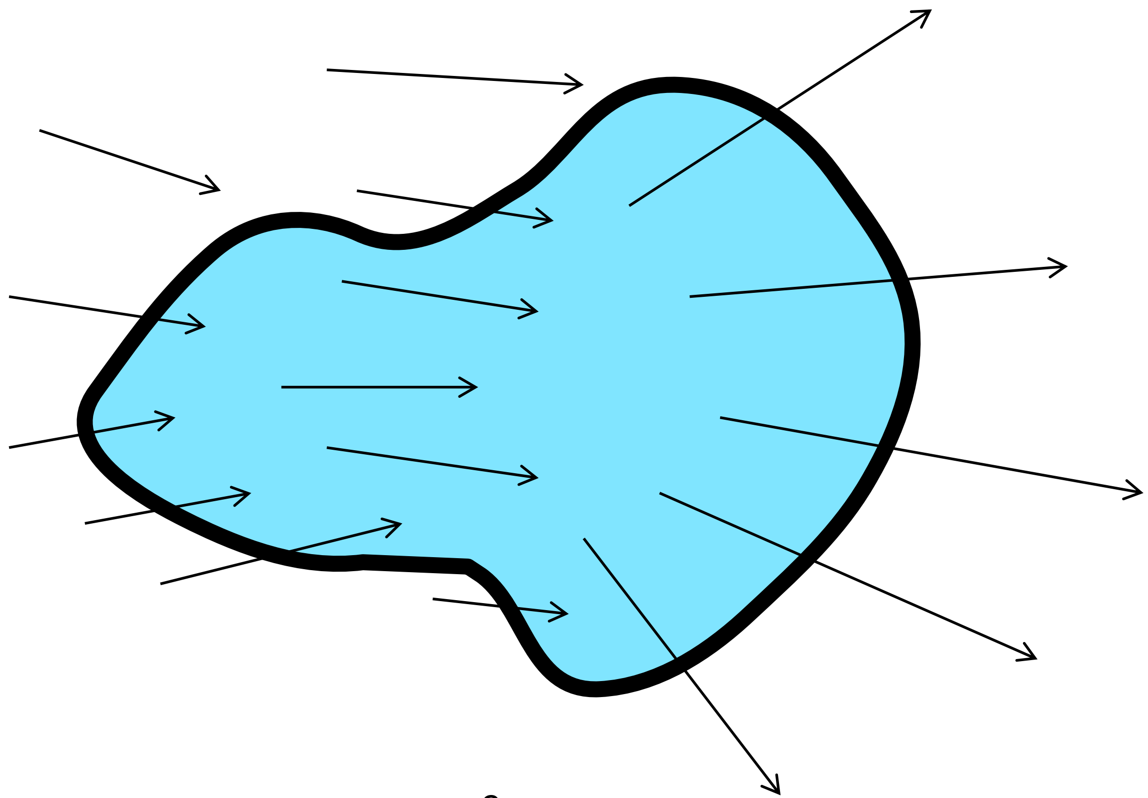
Divergence of a vector field in the Euclidean space

$$\nabla.\vec{f}(x_1,x_2,\ldots,x_n) = \partial f_1/\partial x_1 + \partial f_2/\partial x_2 + \ldots + \partial f_n/\partial x_n = \begin{bmatrix}\partial/\partial x_1\\ \partial/\partial x_2\\ .\\ \partial/\partial x_n\end{bmatrix}.\begin{bmatrix}f_1\\ f_2\\ .\\ f_n\end{bmatrix}$$



Green-Ostrogradski theorem

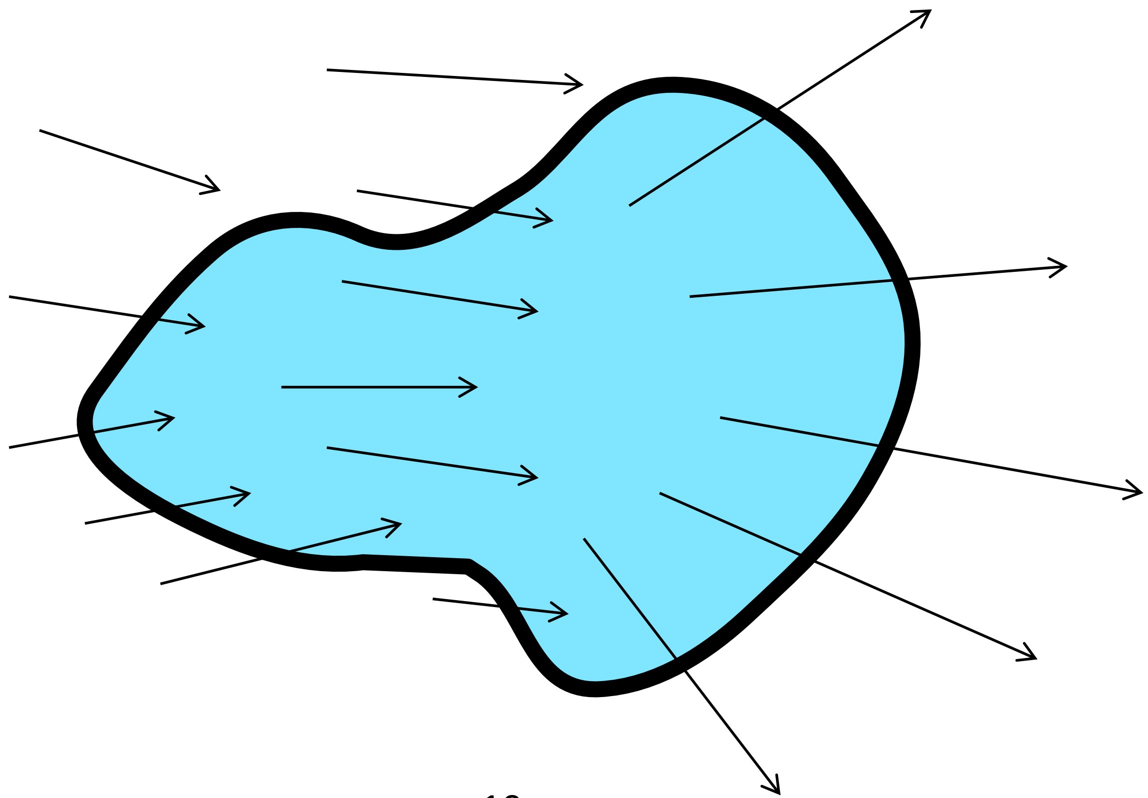
$$\int_{p \in B} \nabla . \vec{f}(p) dV(p) = \int_{p \in \partial B} \vec{f}(p) . N(p) dS(p)$$



Green-Ostrogradski theorem

$$\nabla . \vec{f}(q) = \frac{\lim_{vol(B)\to 0} \int_{p\in B} \nabla . \vec{f}(p) dV(p)}{vol(B)} = \lim_{vol(B)\to 0} \frac{\int_{p\in \partial B} \vec{f}(p) . N(p) dS(p)}{vol(B)}$$

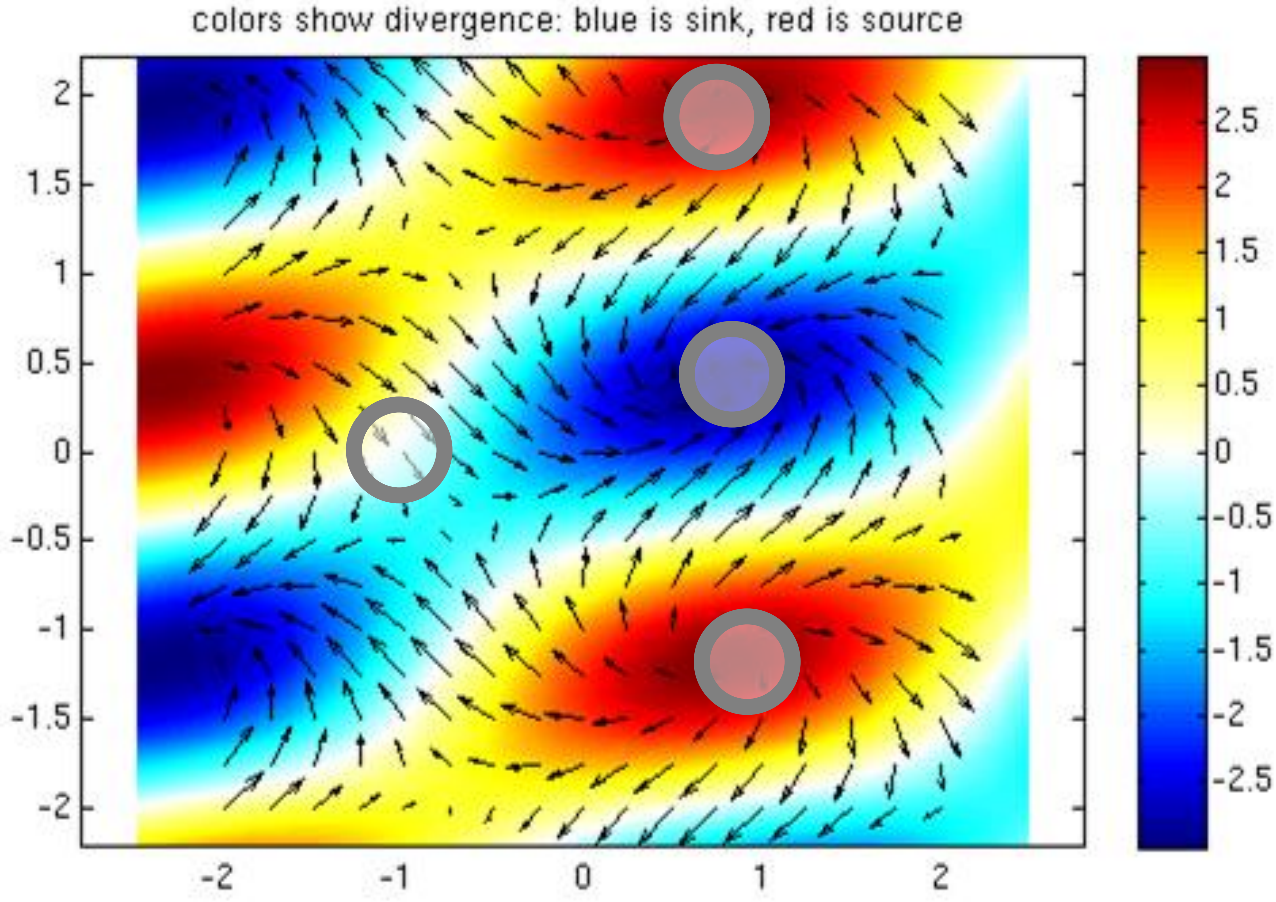
(with $q \in B$)



Green-Ostrogradski theorem

$$\nabla.\vec{f}(q) = \frac{\lim_{vol(B)\to 0} \int_{p\in B} \nabla.\vec{f}(p)dV(p)}{vol(B)} = \lim_{vol(B)\to 0} \frac{\int_{p\in\partial B} \vec{f}(p).N(p)dS(p)}{vol(B)}$$

(with $q \in B$)



Laplacian in the Euclidean space

$$\nabla^2 f(x_1, x_2, \ldots, x_n) = \partial^2\, f/\partial^2\, x_1 + \partial^2\, f/\partial^2\, x_2 + \ldots + \partial^2\, f/\partial^2\, x_n$$

$$\nabla^2 f(x_1, x_2, \ldots, x_n) = \nabla . \vec{\nabla} f(x_1, x_2, \ldots, x_n)$$

Laplacian(f) = Div(Grad(f))

Why do we care ?

$$\boxed{\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0}$$

Heat equation

Why do we care ?

Incompressible Navier-Stokes equations (*convective form*)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \nabla^2 \mathbf{u} = -\nabla w + \mathbf{g}.$$

Navier-Stokes momentum equation (*convective form*)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla \bar{p} + \nu \nabla^2 \mathbf{u} + \tfrac{1}{3}\nu \nabla(\nabla \cdot \mathbf{u}) + \mathbf{g}.$$

Navier-Stokes equations (fluids)

Why do we care ?

Name	Integral equations	Differential equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_\Omega \rho \, \mathrm{d}V$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	The electric flux leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$	$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux through a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t} \iint_\Sigma \mathbf{B} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage induced in a closed circuit is proportional to the rate of change of the magnetic flux it encloses.
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_\Sigma \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \iint_\Sigma \mathbf{E} \cdot \mathrm{d}\mathbf{S}$	$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$	The magnetic field induced around a closed loop is proportional to the electric current plus displacement current (rate of change of electric field) it encloses.

Maxwell equations (electro-magnetism)

Laplace Operator

$f : R^3 \to R \qquad \Delta f : R^3 \to R$

Laplace operator

gradient operator

2nd partial derivatives

$$\Delta f = \operatorname{div} \nabla f = \sum_i \frac{\partial^2 f}{\partial x_i^2} \in R$$

scalar function in Euclidean space

divergence operator

Cartesian coordinates

$$\operatorname{grad} f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Laplace-Beltrami Operator

- Extension to functions on manifolds

$$f : S \rightarrow R$$

Laplace-Beltrami

gradient operator

$$\Delta_S f = \mathrm{div}_S \nabla_S f \qquad \in R$$

scalar function on manifold S

divergence operator

Laplace-Beltrami Operator

- For coordinate function(s)

$$f(x, y, z) = \mathbf{x}$$

Laplace-Beltrami

gradient operator

mean curvature

$$\Delta_{\mathcal{S}} \boxed{\mathbf{x}} = \operatorname{div}_{\mathcal{S}} \nabla_{\mathcal{S}} \boxed{\mathbf{x}} = -2H\mathbf{n} \quad \in R^3$$

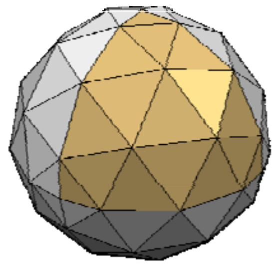
coordinate function

divergence operator

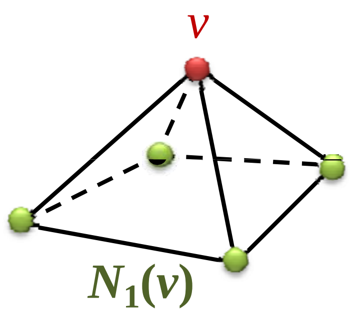
surface normal

Discrete Differential Operators

- **Assumption:** Meshes are piecewise linear approximations of smooth surfaces



- **Approach:** Approximate differential properties at point v as finite differences over local mesh neighborhood $N(v)$
  - v = mesh vertex
  - $N_d(v)$ = d-ring neighborhood

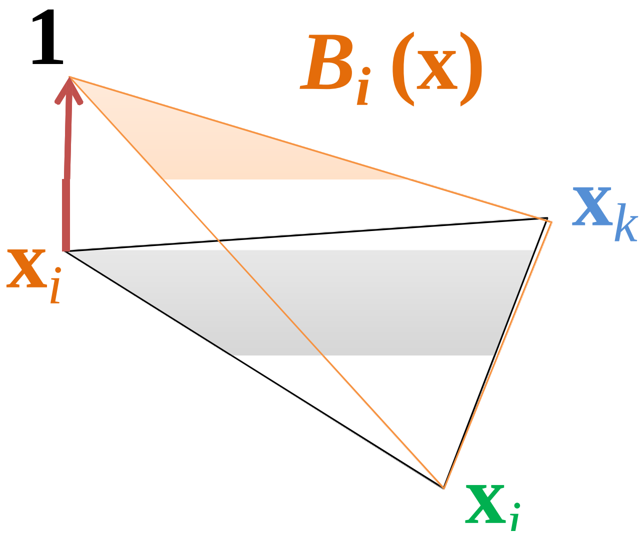


- **Disclaimer:** many possible discretizations, none is “perfect”

Functions on Meshes

- Function f given at mesh vertices $f(v_i) = f(\mathbf{x}_i) = f_i$
- Linear interpolation to triangle $\mathbf{x} \in \left(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k\right)$

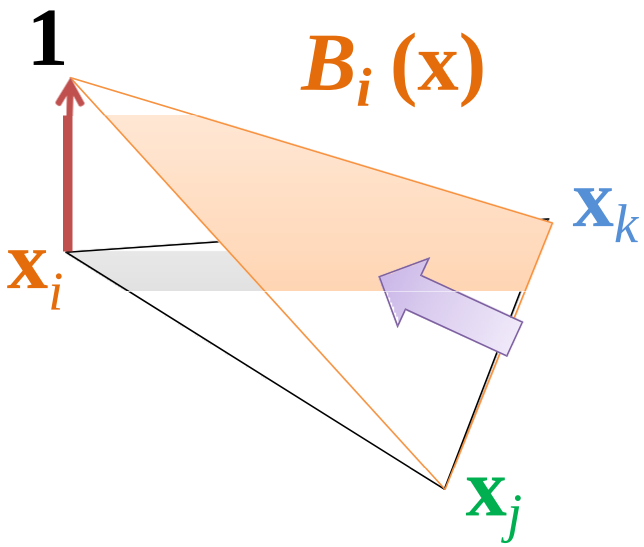
$$f(\mathbf{x}) = f_i B_i(\mathbf{x}) + f_j B_j(\mathbf{x}) + f_k B_k(\mathbf{x})$$



Gradient of a Function

$$f(\mathbf{x}) = f_i B_i(\mathbf{x}) + f_j B_j(\mathbf{x}) + f_k B_k(\mathbf{x})$$

$$\nabla f(\mathbf{x}) = f_i \nabla B_i(\mathbf{x}) + f_j \nabla B_j(\mathbf{x}) + f_k \nabla B_k(\mathbf{x})$$



Steepest ascent direction
perpendicular to opposite edge

$$\nabla B_i(\mathbf{x}) = \nabla B_i = \frac{(\mathbf{x}_k - \mathbf{x}_j)^{\perp}}{2A_T}$$

Constant in the triangle

Gradient of a Function

$$B_i(\mathbf{x}) + B_j(\mathbf{x}) + B_k(\mathbf{x}) = 1$$

$$\nabla B_i + \nabla B_j + \nabla B_k = 0$$

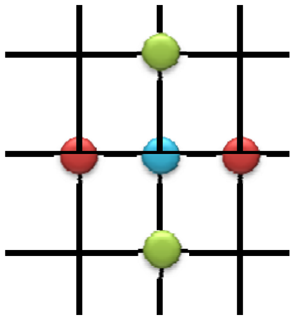
$$\nabla f(\mathbf{x}) = \left(f_j - f_i\right)\nabla B_j(\mathbf{x}) + \left(f_k - f_i\right)\nabla B_k(\mathbf{x})$$

$$\nabla f(\mathbf{x}) = \left(f_j - f_i\right)\frac{\left(\mathbf{x}_i - \mathbf{x}_k\right)^{\perp}}{2A_T} + \left(f_k - f_i\right)\frac{\left(\mathbf{x}_j - \mathbf{x}_i\right)^{\perp}}{2A_T}$$

Discrete Laplace-Beltrami
First Approach

- Laplace operator: $\Delta f = \operatorname{div} \nabla f = \sum_i \frac{\partial^2 f}{\partial x_i^2}$

- In 2D: $\Delta f = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$



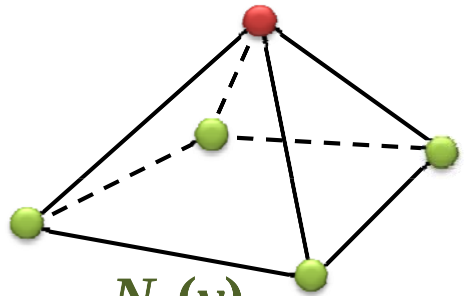
- On a grid – finite differences discretization:

$$\Delta f(x_i, y_i) = \frac{f(x_{i+1}, y_i) - f(x_i, y_i)}{h^2} + \frac{f(x_{i-1}, y_i) - f(x_i, y_i)}{h^2} + \frac{f(x_i, y_{i+1}) - f(x_i, y_i)}{h^2} + \frac{f(x_i, y_{i-1}) - f(x_i, y_i)}{h^2}$$

Discrete Laplace-Beltrami
Uniform Discretization

$$\Delta f = \sum_{v_i \in N_1(v)} (f(v) - f(v_i)) =$$

$$= |N_1(v)| f(v) - \sum_{v_i \in N_1(v)} f(v_i)$$



Normalized: $$\Delta f = \frac{1}{|N_1(v)|} \sum_{v_i \in N_1(v)} (f(v) - f(v_i)) =$$

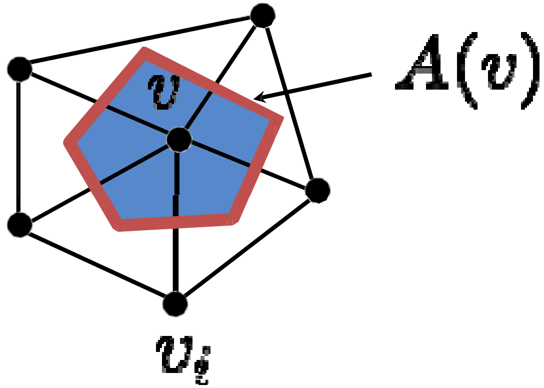
$$= \boxed{f(v) - \frac{1}{|N_1(v)|} \sum_{v_i \in N_1(v)} f(v_i)}$$

Discrete Laplace-Beltrami
Second Approach

- Laplace-Beltrami operator: $\Delta_{\mathcal{S}} f = \mathrm{div}_{\mathcal{S}}\, \nabla_{\mathcal{S}} f$
- Compute integral around vertex

Divergence theorem

$$\int_{A(v)} \Delta f(\mathbf{u})\, dA = \int_{A(v)} \left(\mathrm{div}\, \nabla f(\mathbf{u})\right) dA = \int_{\partial A(v)} \left(\nabla f(\mathbf{u}) \cdot \mathbf{n}(\mathbf{u})\right) ds$$

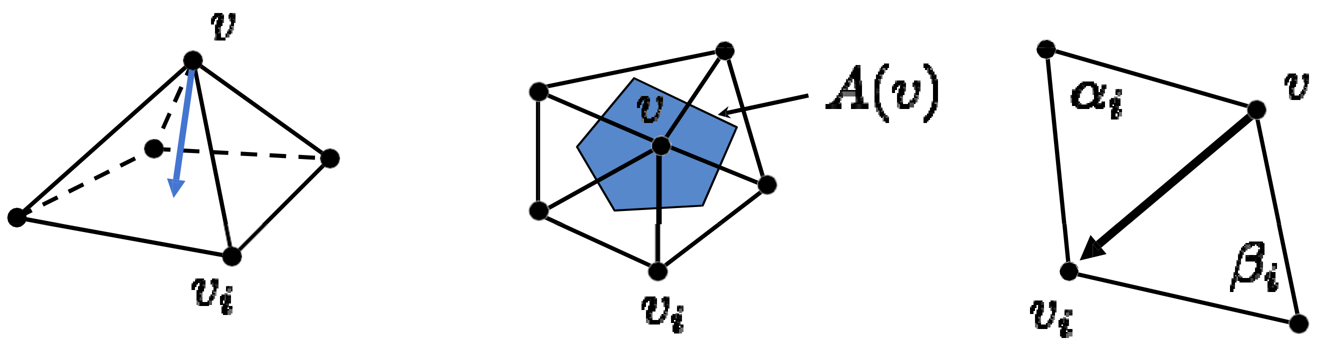


Discrete Laplace-Beltrami
Cotangent Formula

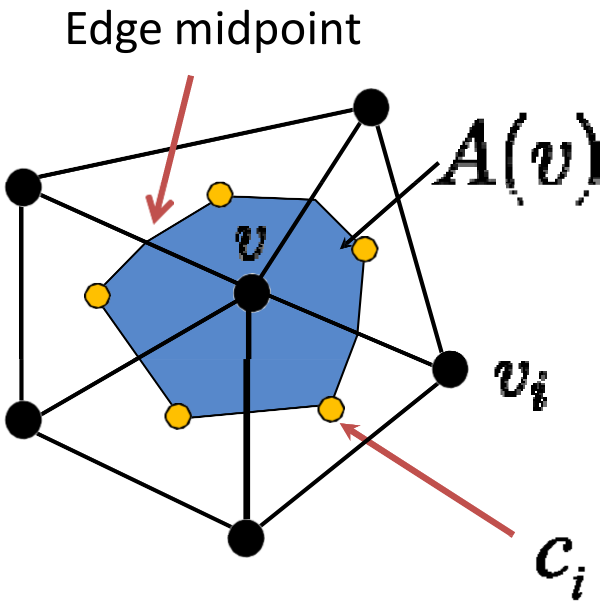
Plugging in expression for gradients gives:

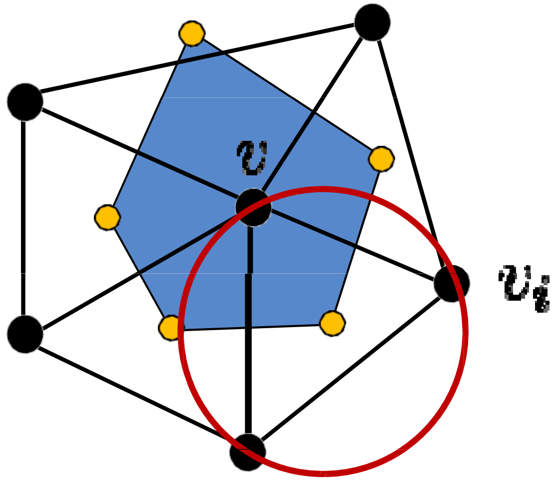
$$\Delta f(v) = \sum_{v_i \in N_1(v)} w_i \left(f(v_i) - f(v) \right)$$

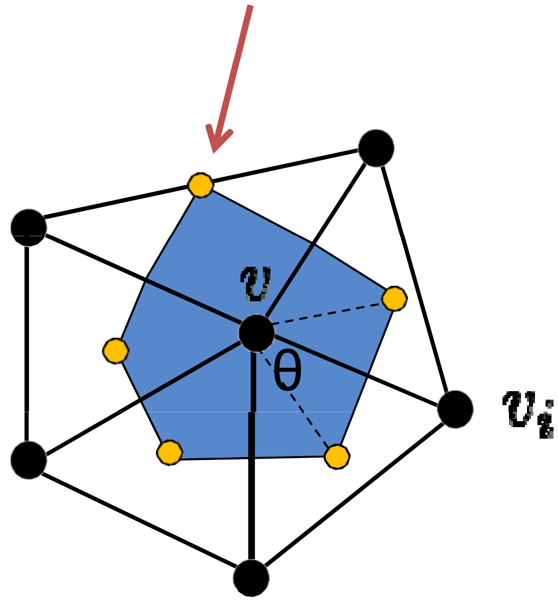
$$= \boxed{\frac{1}{2A(v)}} \sum_{v_i \in N_1(v)} \boxed{(\cot \alpha_i + \cot \beta_i)} \left(f(v_i) - f(v) \right)$$



The Averaging Region







Barycentric cell

c_i = barycenter of triangle

Voronoi cell

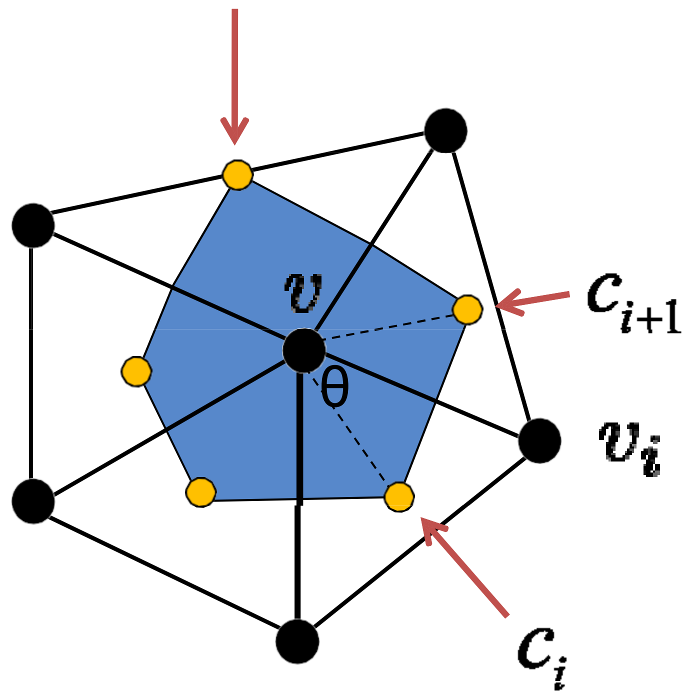
c_i = circumcenter of triangle

Mixed cell

The Averaging Region

Mixed Cell

If $\theta < \pi/2$, c_i is the circumcenter of the triangle $(v_i\,, v, v_{i+1})$



If $\theta \geq \pi/2$, c_i is the midpoint of the edge $(v_i\,, v_{i+1})$

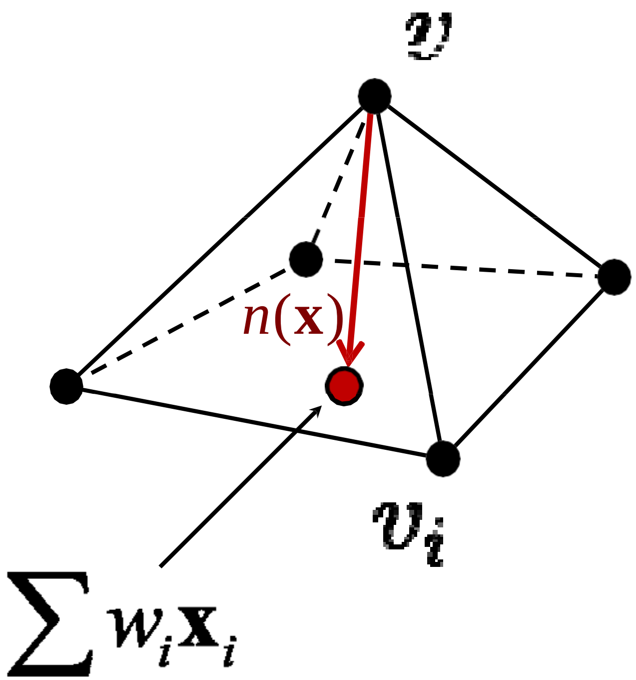
$$A(v) = \sum_{v_i \in \mathcal{N}(v)} \Big(Area(c_i, v, (v + v_i)/2) + Area(c_{i+1}, v, (v + v_i)/2) \Big)$$

Discrete Normal

$$\Delta_{\mathcal{S}}\mathbf{x} = \mathrm{div}_{\mathcal{S}}\, \nabla_{\mathcal{S}}\mathbf{x} = -2H\mathbf{n}$$

$$n(\mathbf{x}) = \sum_{v_i \in N_1(v)} w_i(\mathbf{x}_i - \mathbf{x}) \qquad \sum_i w_i = 1$$

$$= \left(\sum_{\mathbf{x}_i \in N_1(\mathbf{x})} w_i \mathbf{x}_i \right) - \mathbf{x}$$



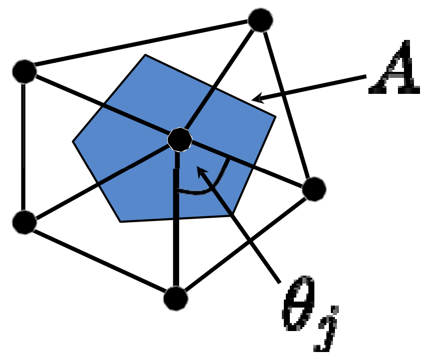
Discrete Curvatures

- Mean curvature

$$H = \|\Delta_S \mathbf{x}\|$$

- Gaussian curvature

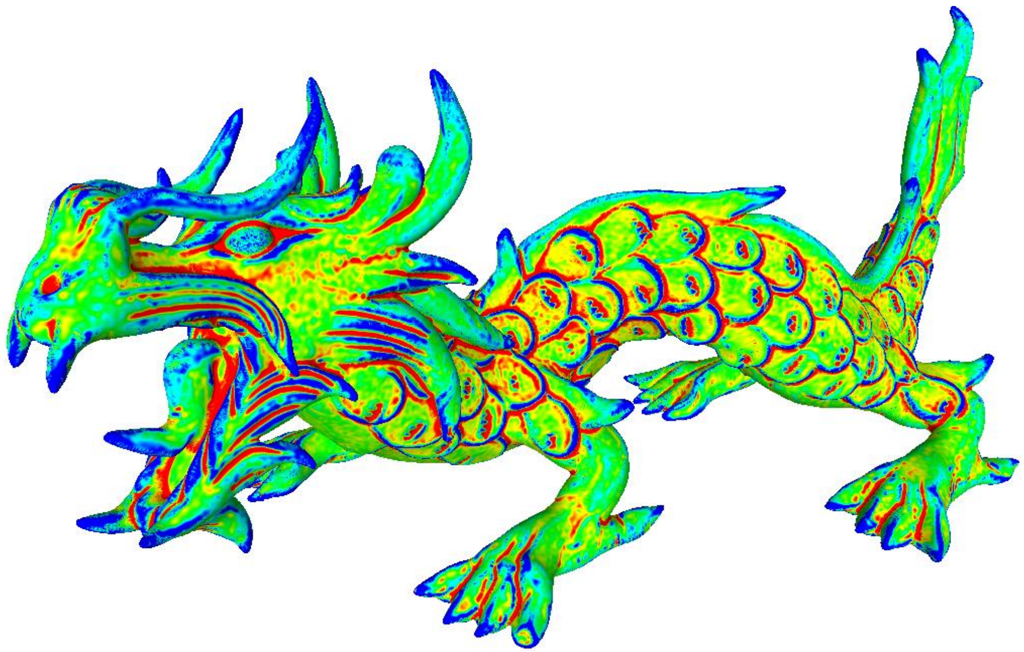
$$G = (2\pi - \sum_j \theta_j)/A$$



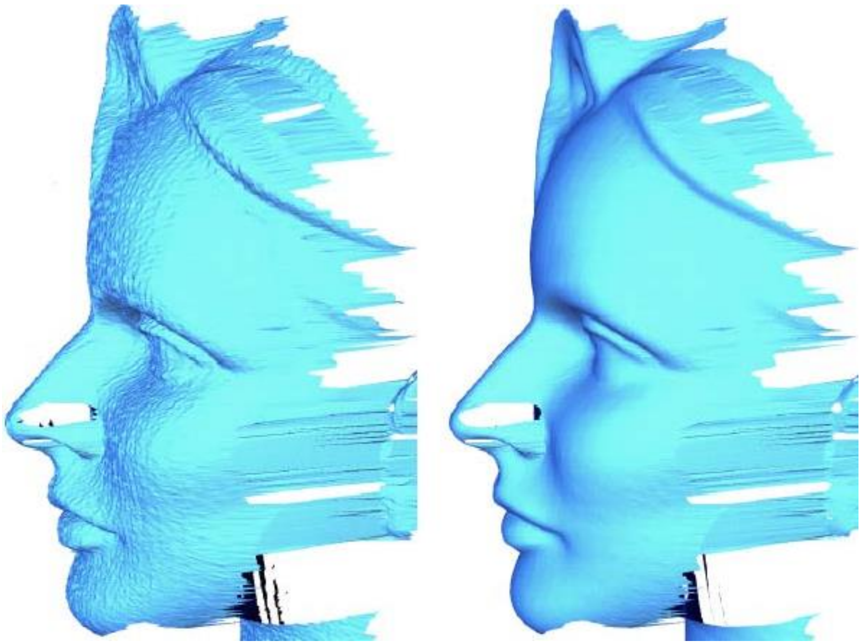
- Principal curvatures

$$\kappa_1 = H + \sqrt{H^2 - G} \qquad \kappa_2 = H - \sqrt{H^2 - G}$$

Example: Mean Curvature



Mesh Smoothing

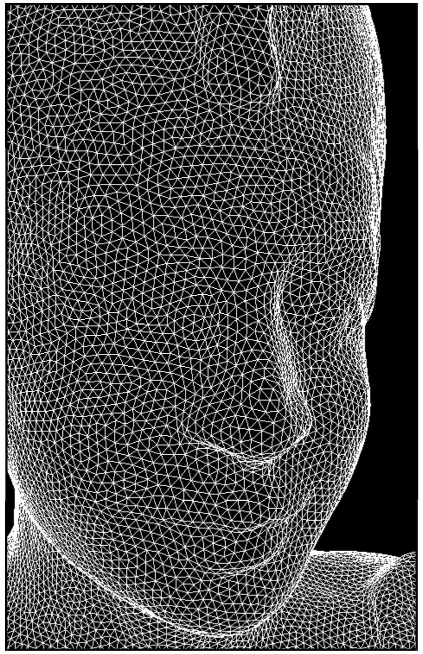


Mesh Processing Pipeline

Scan

Reconstruct

Clean

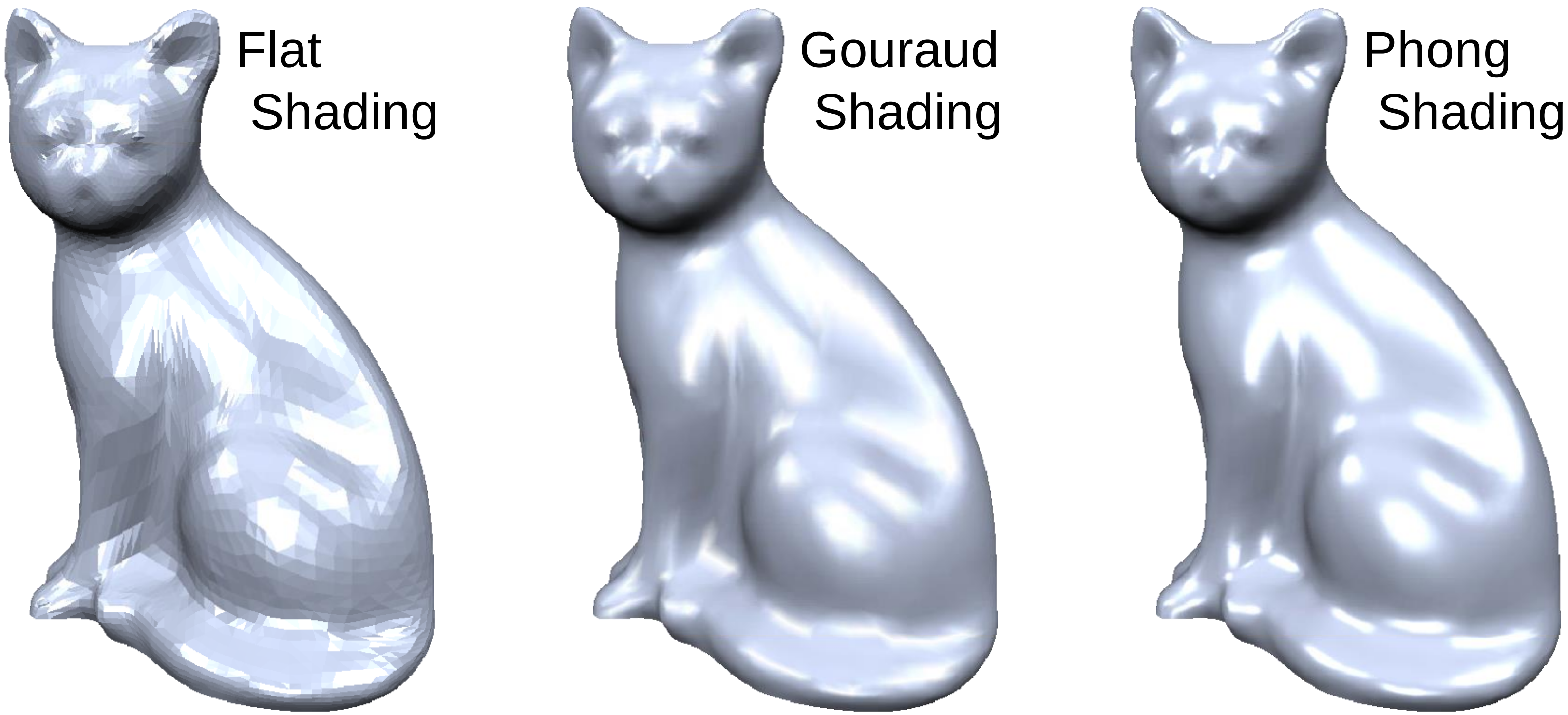


Remesh

…

Mesh Quality

- Visual inspection of “sensitive” attributes
  - Specular shading

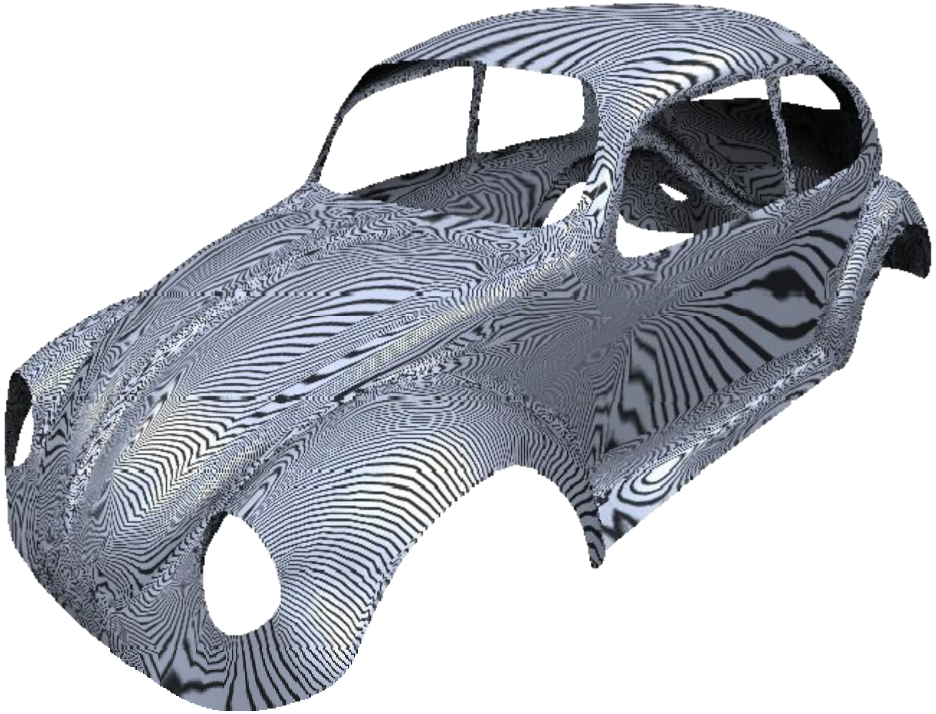


Mesh Quality

- Visual inspection of “sensitive” attributes
  - Specular shading

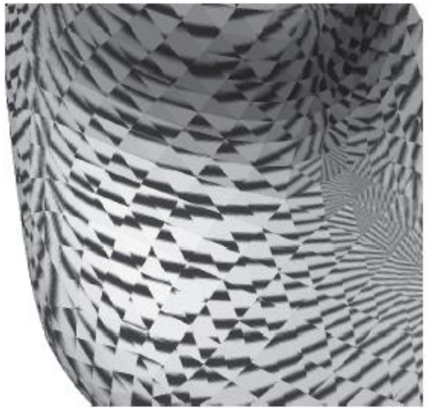
Mesh Quality

- Visual inspection of “sensitive” attributes
  - Specular shading
  - Reflection lines



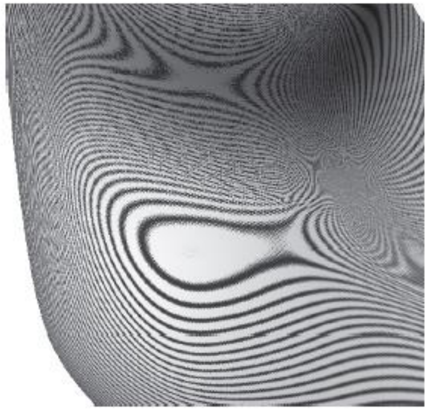
Mesh Quality

- **Visual inspection of “sensitive” attributes**
  - Specular shading
  - Reflection lines
    - differentiability one order lower than surface
    - can be efficiently computed using graphics hardware



C^0

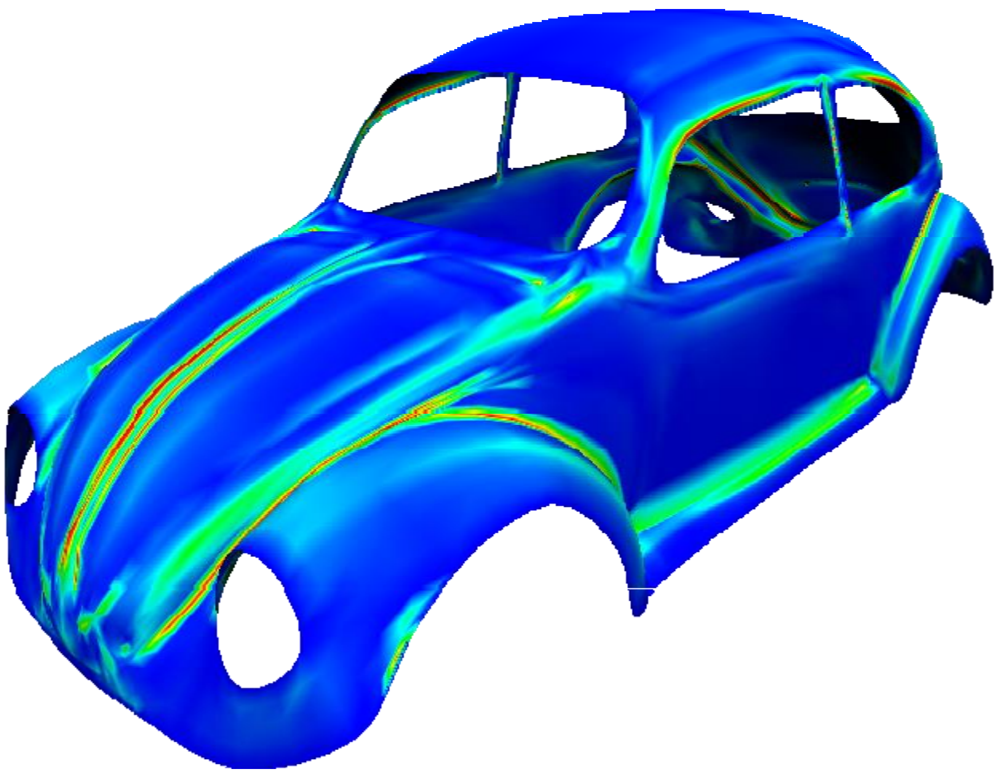
C^1



C^2

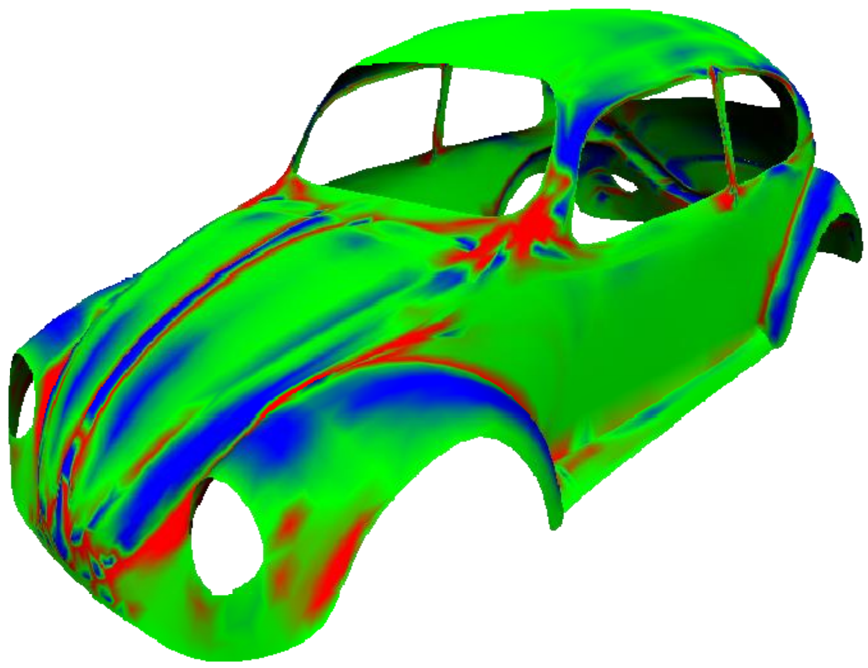
Mesh Quality

- Visual inspection of "sensitive" attributes
  - Specular shading
  - Reflection lines
  - Curvature
    - Mean curvature



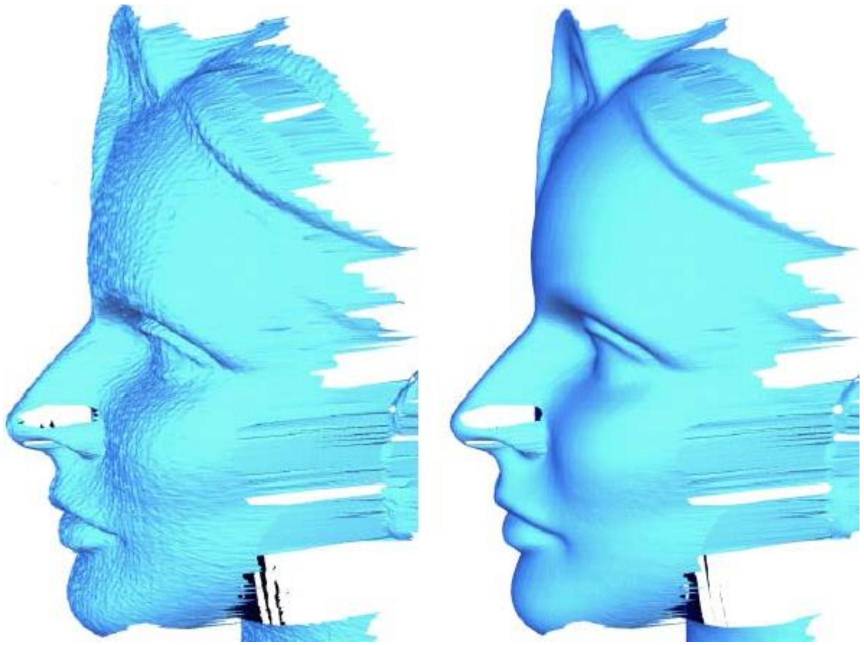
Mesh Quality

- Visual inspection of "sensitive" attributes
  - Specular shading
  - Reflection lines
  - Curvature
    - Mean curvature
    - Gaussian curvature



Motivation

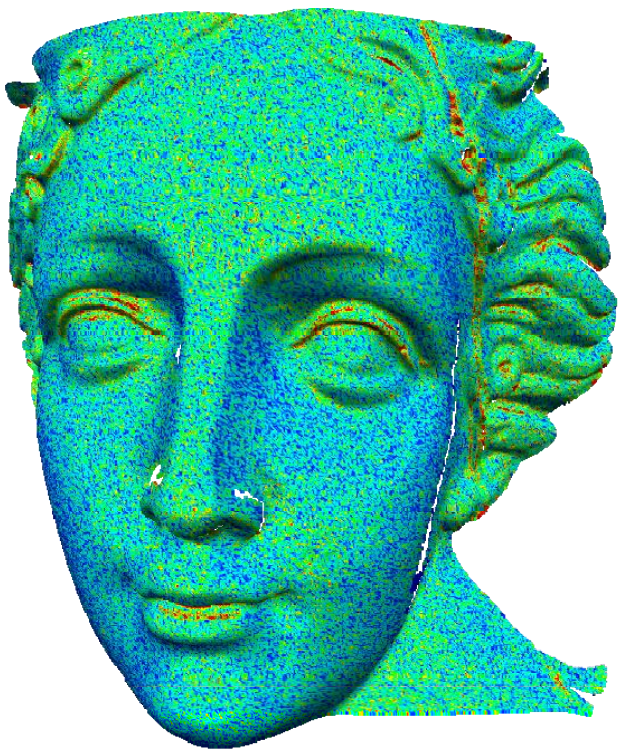
- Filter out high frequency noise



Mesh Smoothing

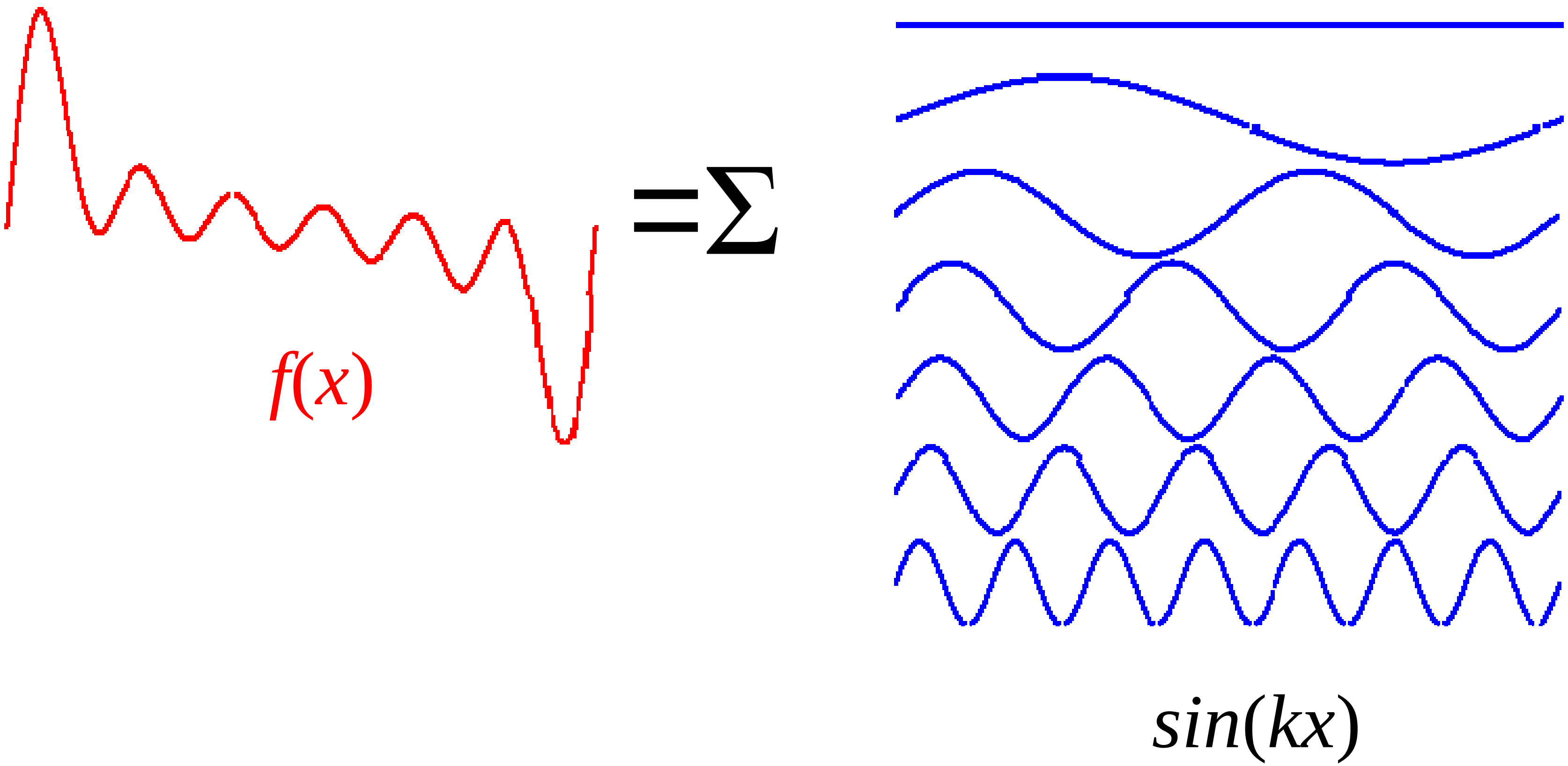
(aka Denoising, Filtering, Fairing)

Input: Noisy mesh (scanned or other)
Output: Smooth mesh
How: Filter out high frequency noise



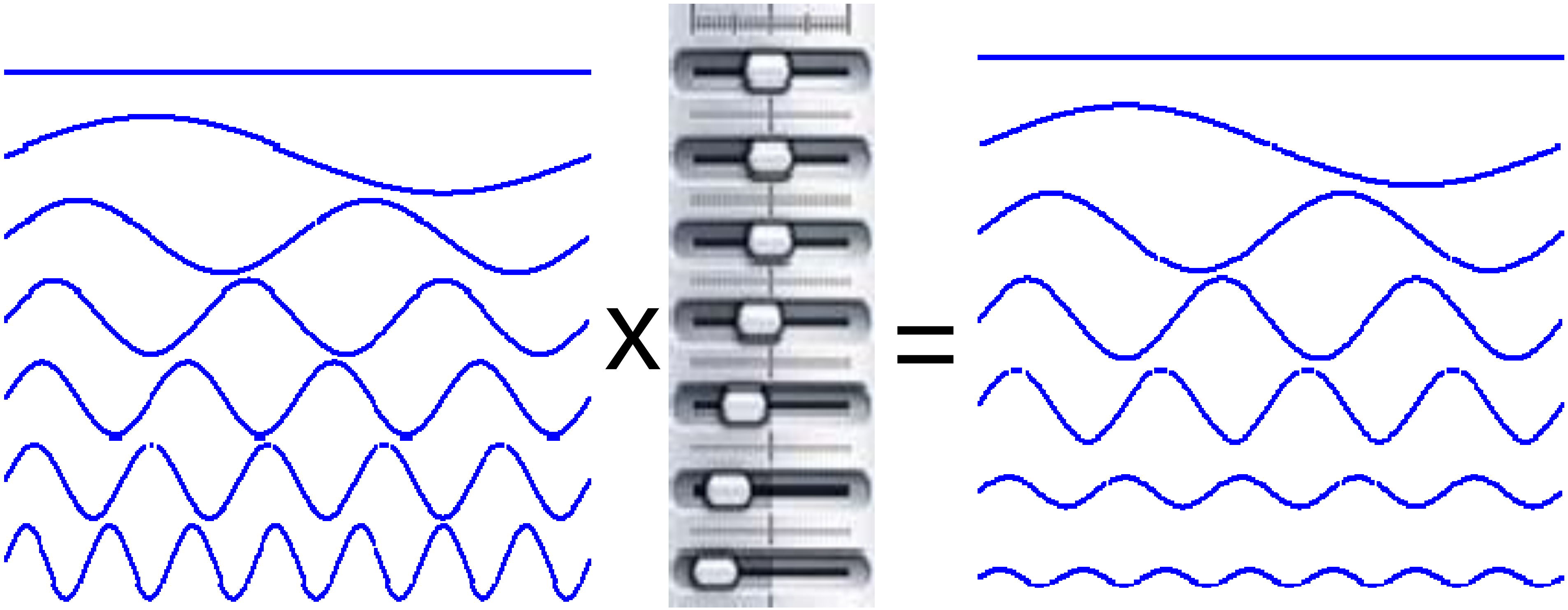
Smoothing by Filtering

Fourier Transform



Smoothing by Filtering

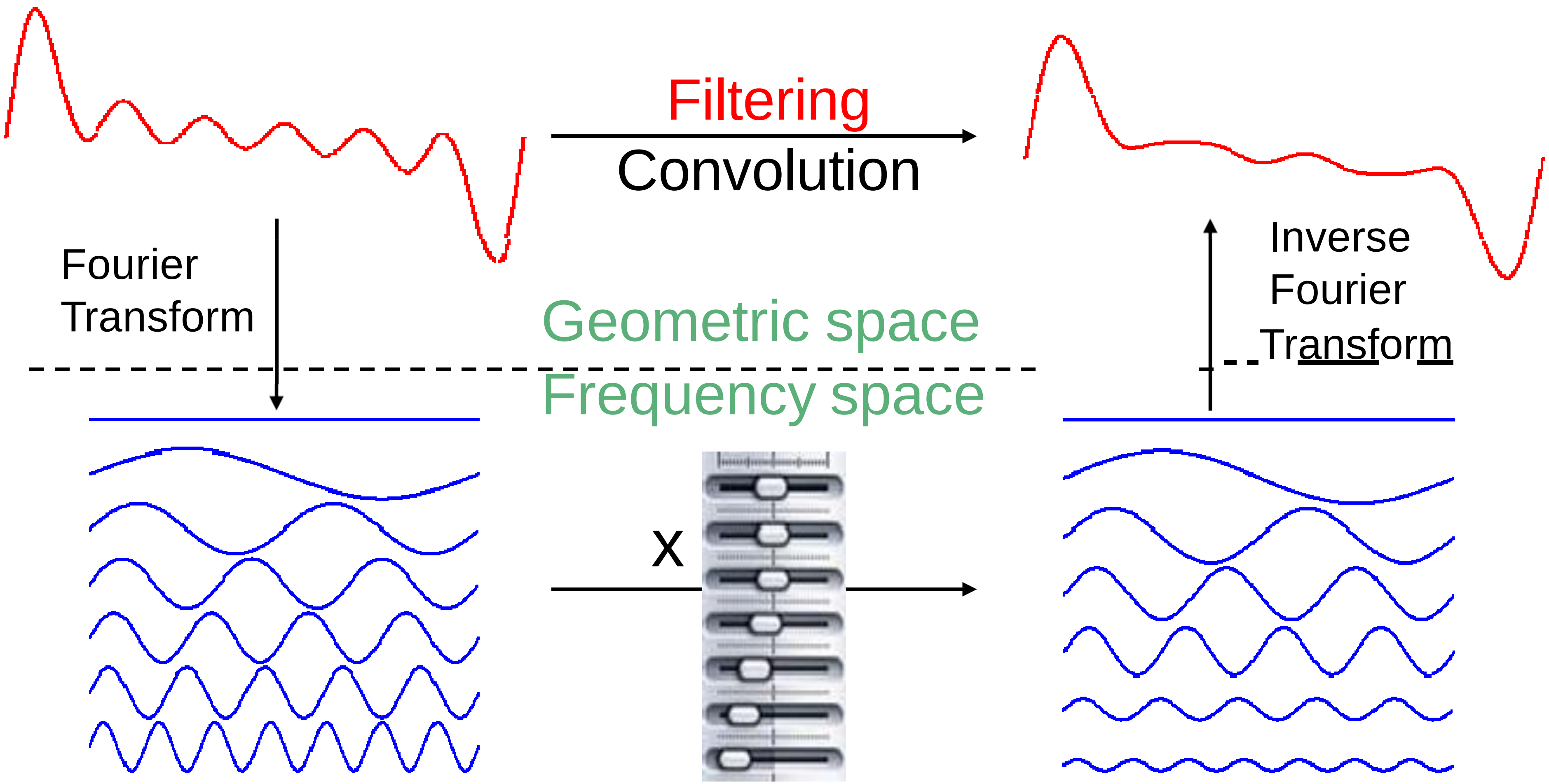
Fourier Transform



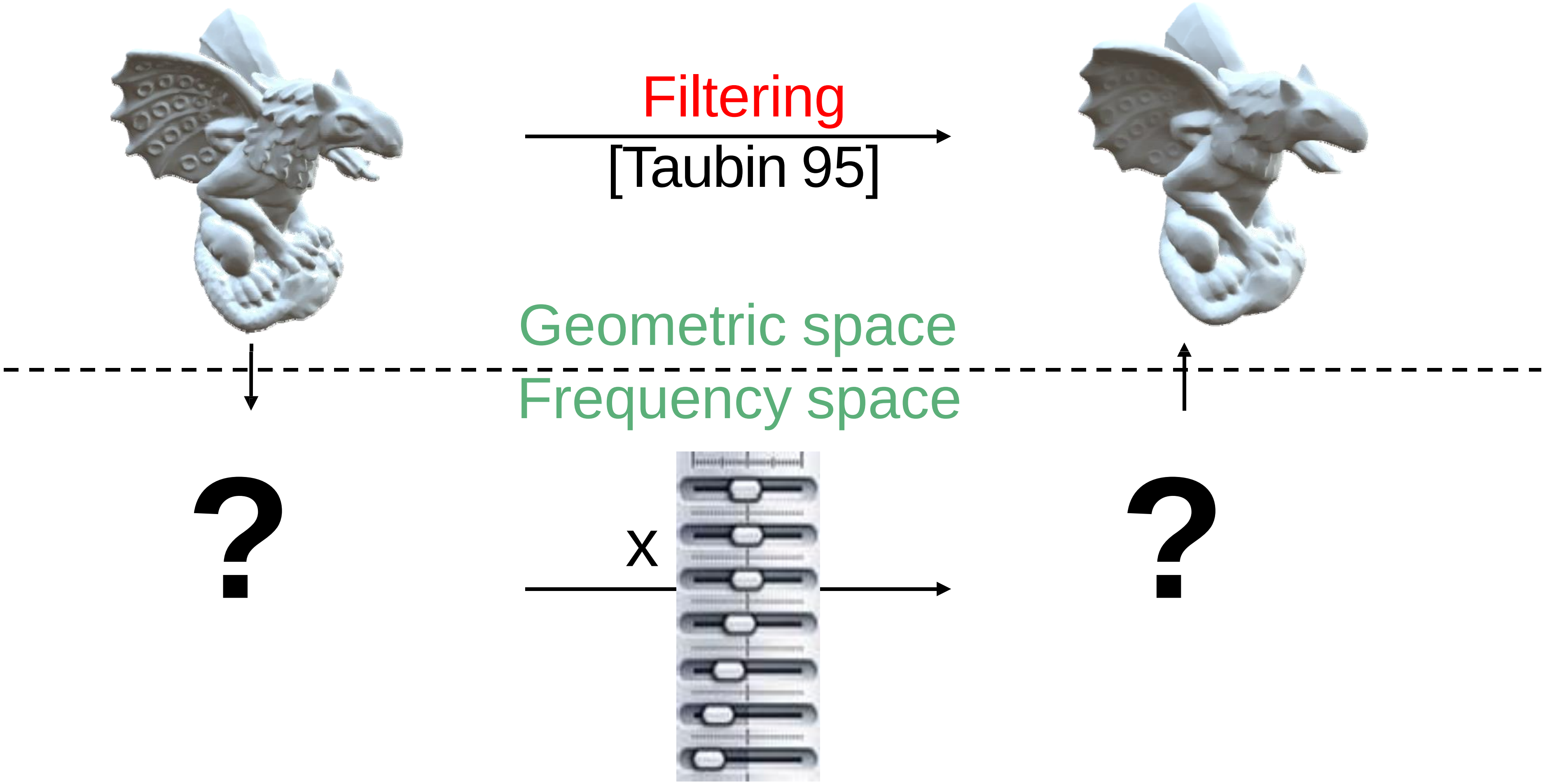
Slides by Levy et al., SigAsia Course 2009

Smoothing by Filtering

Fourier Transform

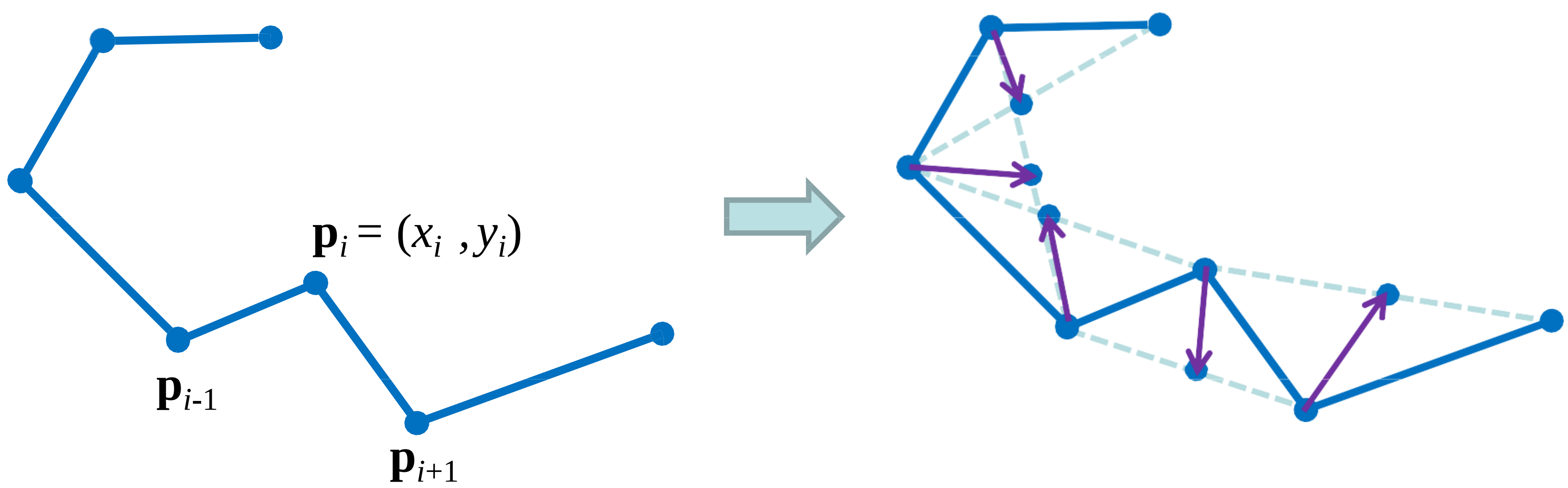


Filtering on a Mesh



Laplacian Smoothing

An easier problem: How to smooth a curve?

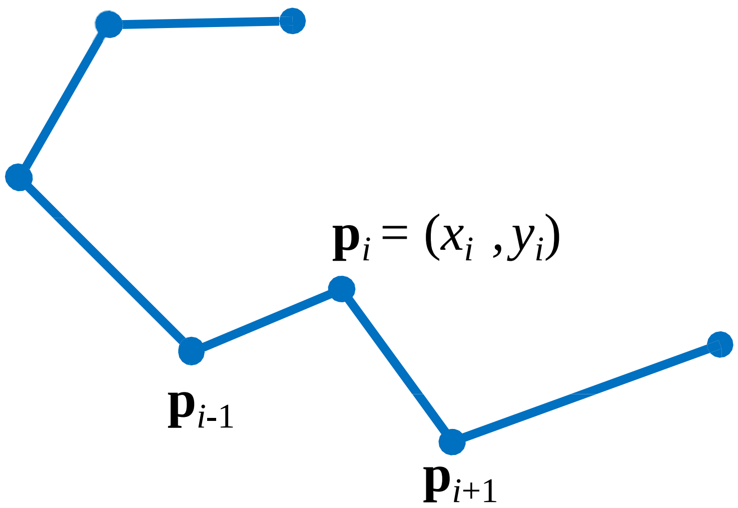


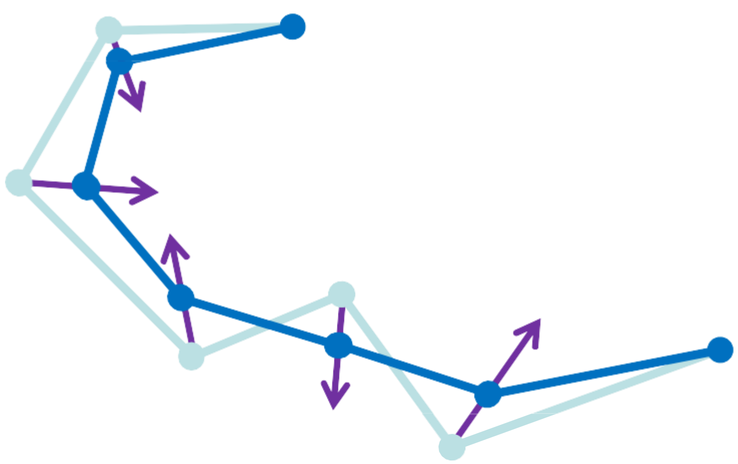
$(\mathbf{p}_{i\text{-}1} + \mathbf{p}_{i+1})/2 - \mathbf{p}_i$

$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

Laplacian Smoothing

An easier problem: How to smooth a curve?





$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \frac{1}{2} L(\mathbf{p}_i)$$

Finite difference discretization of second derivative
= Laplace operator in one dimension

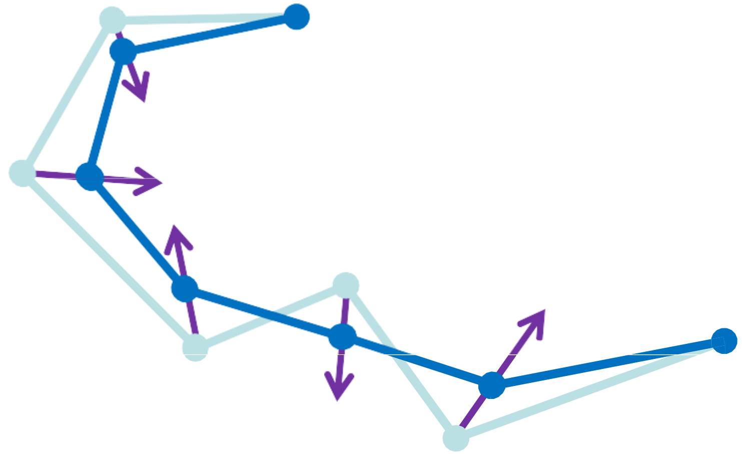
$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

Laplacian Smoothing

<u>Algorithm:</u>

Repeat for *m* iterations (for non boundary points):

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$



For which λ?

$0 < \lambda < 1$

Closed curve converges to?

Single point

Laplacian Smoothing on Meshes

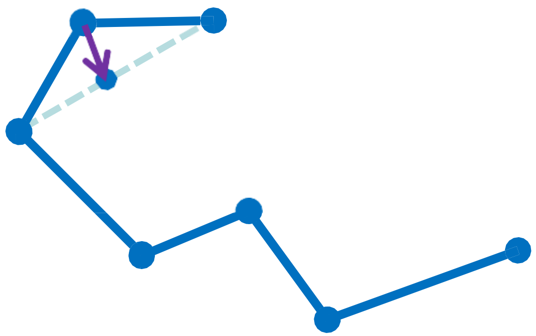
Same as for curves:

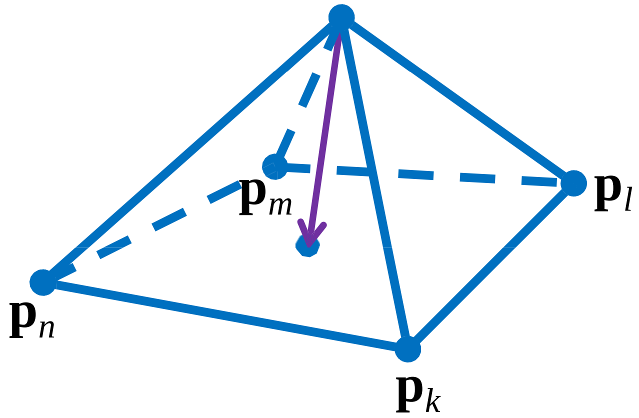
$$\mathbf{p}_i^{(t+1)} = \mathbf{p}_i^{(t)} + \lambda \Delta \mathbf{p}_i^{(t)}$$

$N_i = \{k, l, m, n\}$

$\mathbf{p}_i = (x_i, y_i, z_i)$

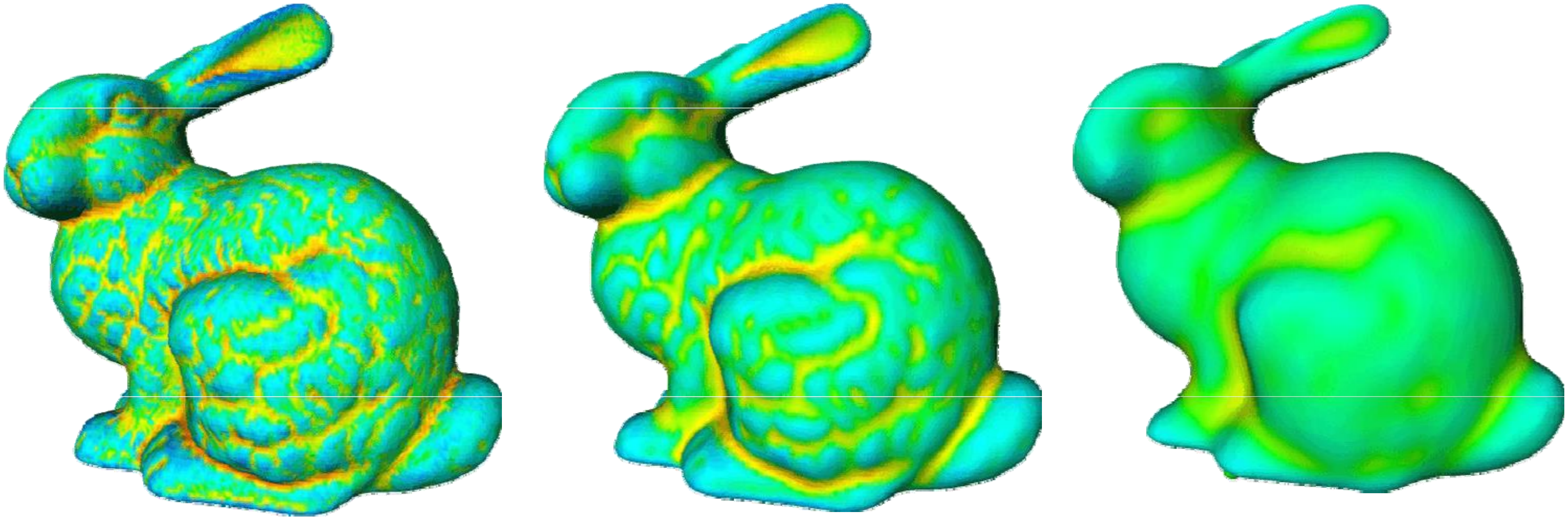
What is $\Delta \mathbf{p}_i$?





$$\frac{1}{2}\left(\mathbf{p}_{i+1} + \mathbf{p}_{i-1}\right) - \mathbf{p}_i \qquad \frac{1}{|N_i|}\left(\sum_{j \in N_i} \mathbf{p}_j\right) - \mathbf{p}_i$$

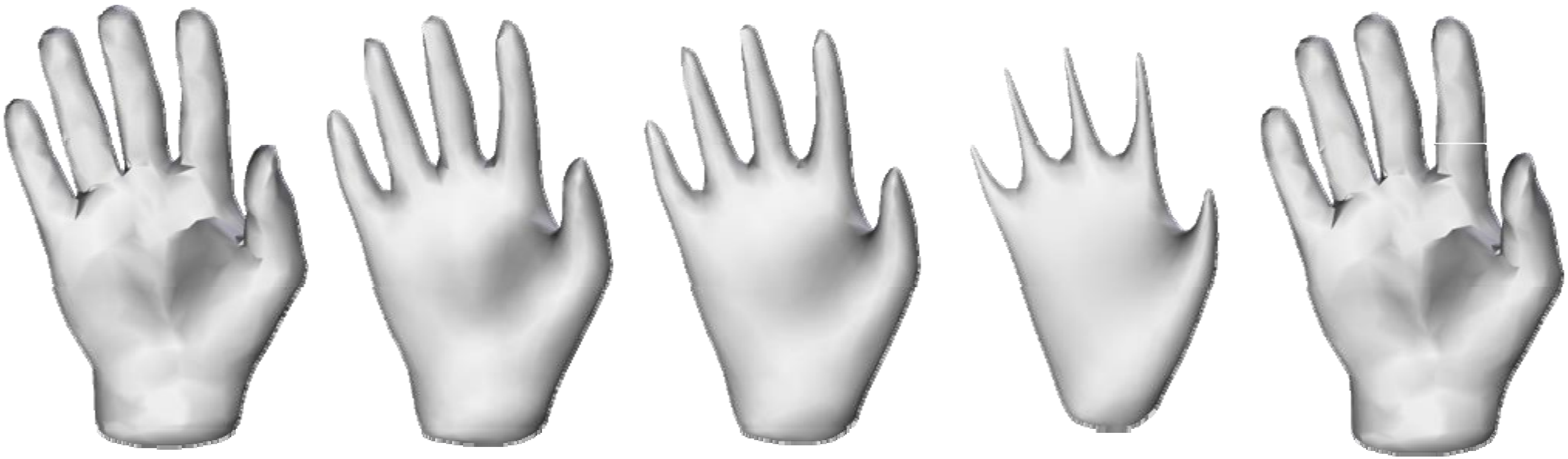
Laplacian Smoothing on Meshes



0 Iterations 5 Iterations 20 Iterations

Problem - Shrinkage

Repeated iterations of Laplacian smoothing shrinks the mesh



original 3 steps 6 steps 18 steps original

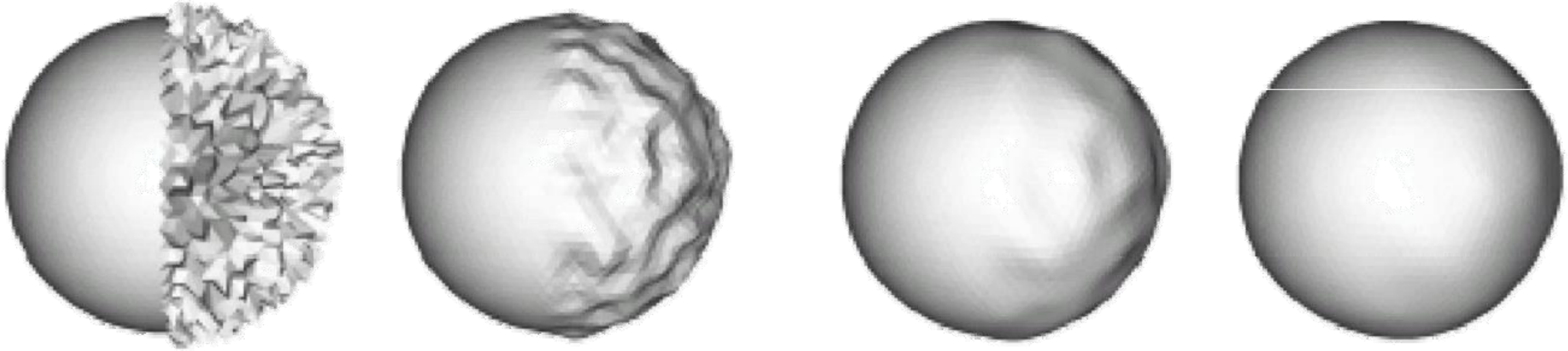
Taubin Smoothing

Iterate:

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda \Delta \mathbf{p}_i$$ Shrink

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \mu \Delta \mathbf{p}_i$$ Inflate

with $\lambda > 0$ and $\mu < 0$



original 10 steps 50 steps 200 steps

From Taubin, Siggraph 1995

Laplacian Smoothing

$$\mathbf{p}_i^{(t+1)} = \mathbf{p}_i^{(t)} + \lambda \Delta \mathbf{p}_i^{(t)}$$

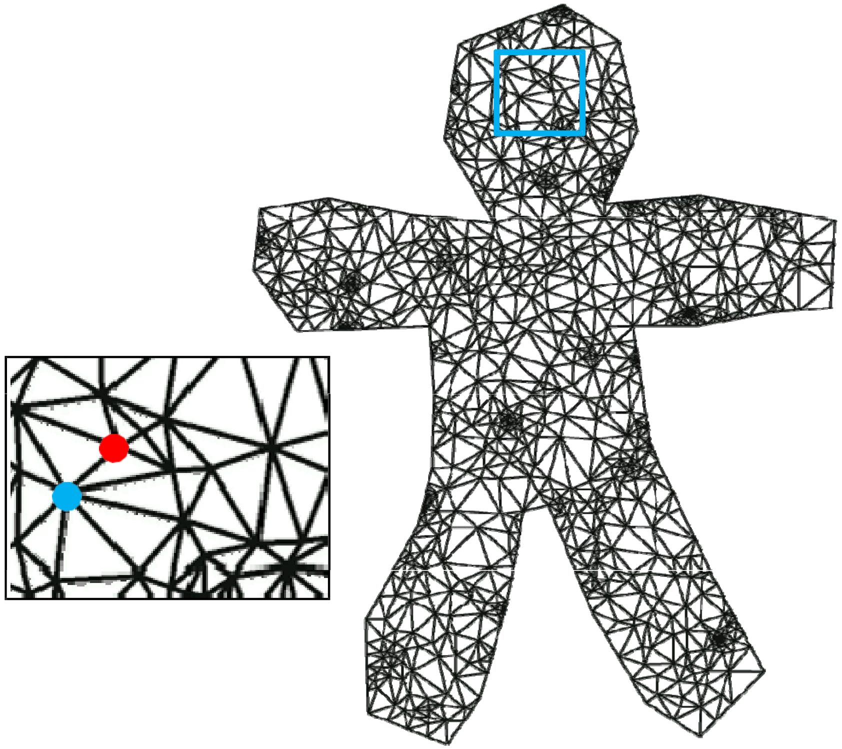
$\Delta\, \mathbf{p}_i$ = mean curvature normal

⇨ mean curvature flow

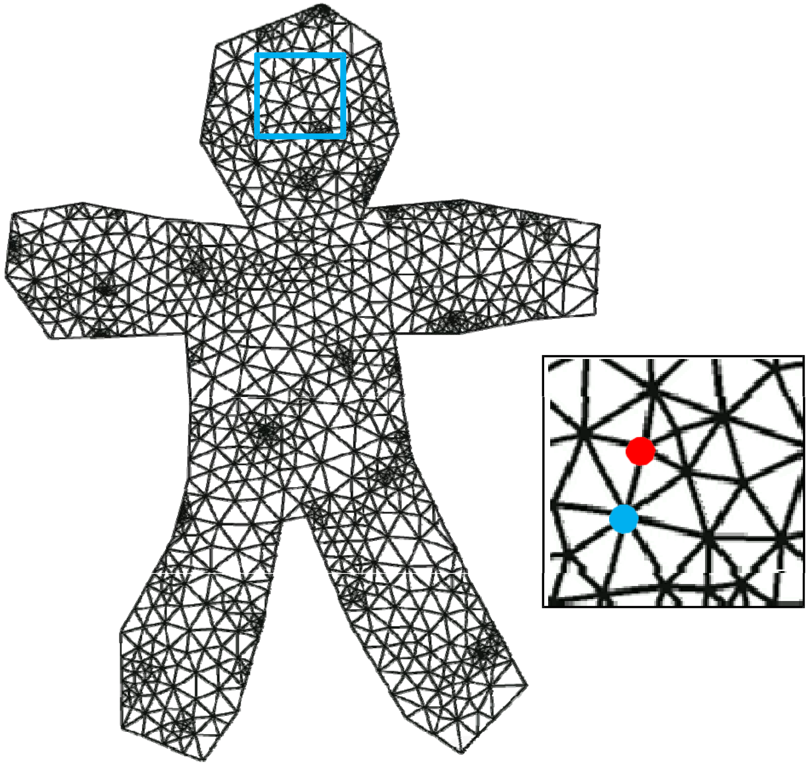
Laplace Operator Discretization

The Problem

Laplace Operator Discretization



0 Iterations

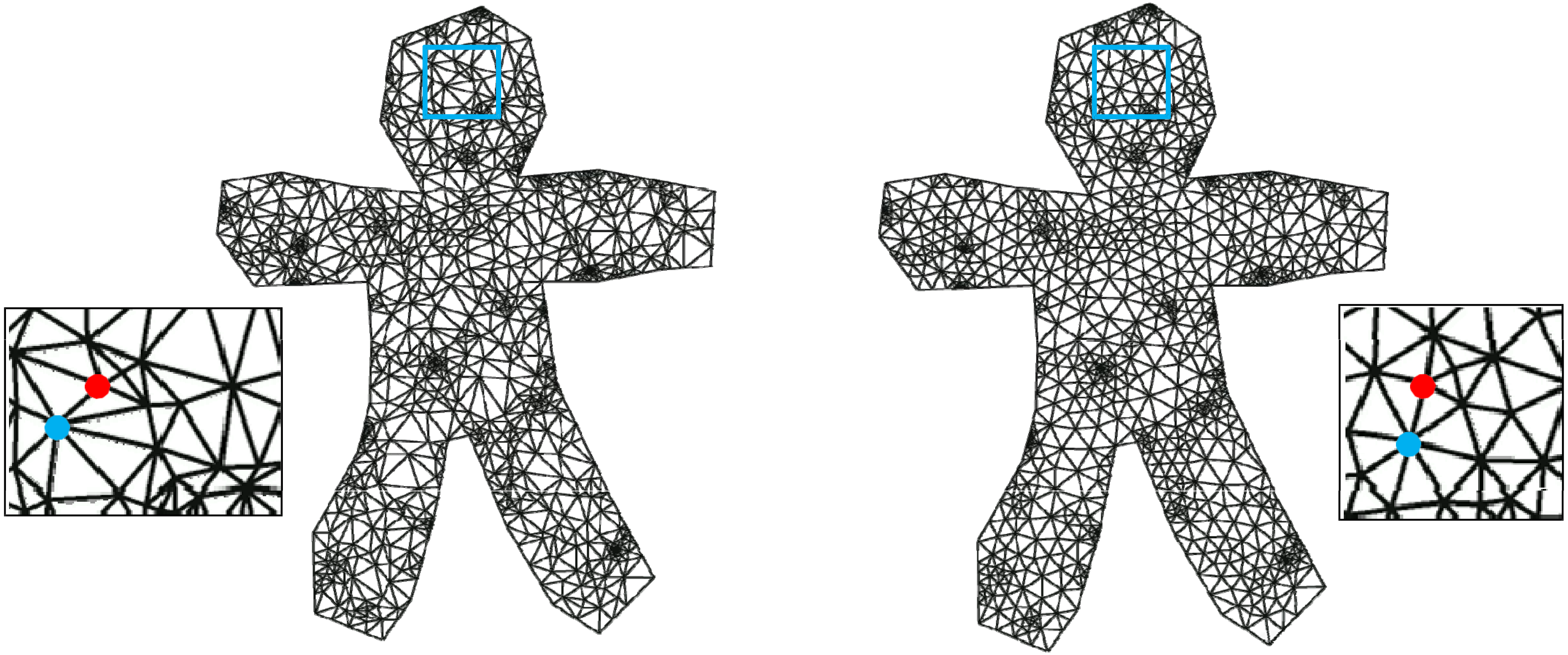


5 Iterations

Laplace Operator Discretization

The Problem

Laplace Operator Discretization

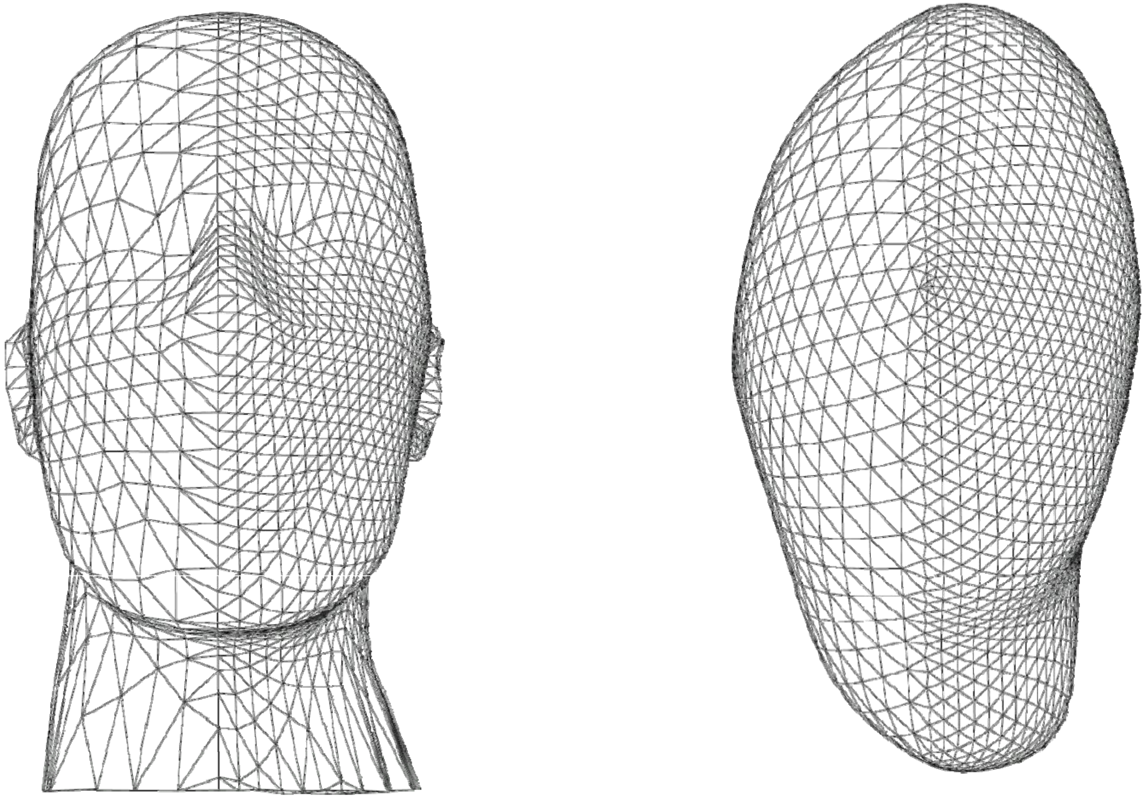


0 Iterations 5 Iterations

Laplace Operator Discretization

The Problem

Not good – The result should not depend on triangle sizes



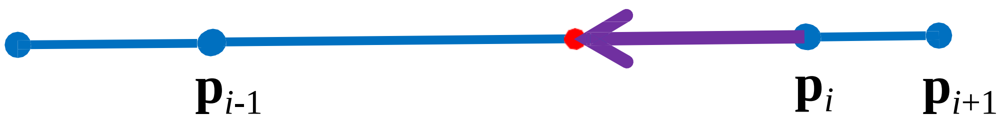
From Desbrun et al., Siggraph 1999

Laplace Operator Discretization

What Went Wrong?

Back to curves:

$$\boxed{\frac{1}{2}}\left(\mathbf{p}_{i+1}+\mathbf{p}_{i-1}\right)-\mathbf{p}_i$$



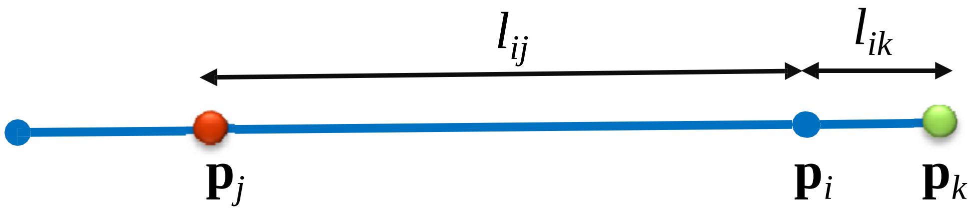
Same weight for both neighbors,
although one is closer

Laplace Operator Discretization

The Solution

Use a weighted average to define Δ

Which weights?



$$w_{ij} = \frac{1}{l_{ij}} \qquad w_{ik} = \frac{1}{l_{ik}}$$

$$L(\mathbf{p}_i) = \frac{w_{ij}\mathbf{p}_j + w_{ik}\mathbf{p}_k}{w_{ij} + w_{ik}} - \mathbf{p}_i$$

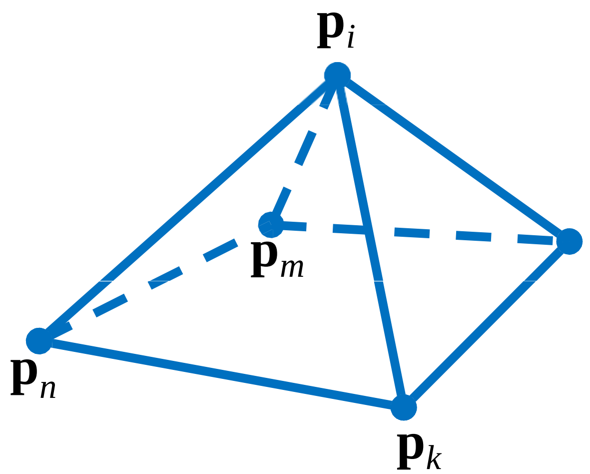
Straight curves will be invariant to smoothing

Laplace Operator Discretiztion

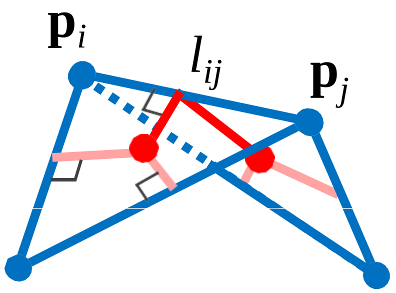
Cotangent Weights

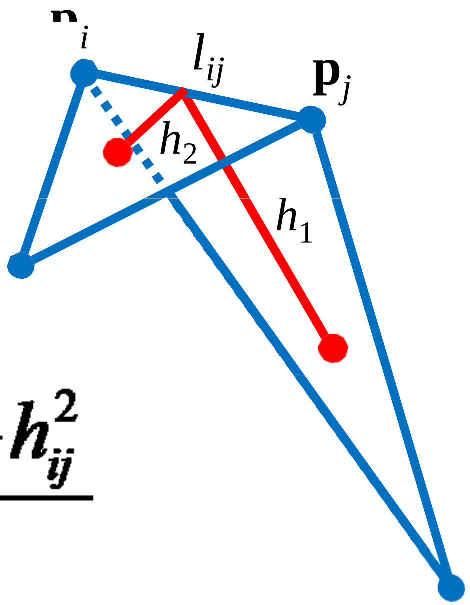
Use a weighted average to define Δ

$N_i = \{k, l, m, n\}$



Which weights? $w_{ij} = \dfrac{1}{l_{ij}}$?





$$L(\mathbf{p}_i) = \frac{1}{\sum_{j \in N_i} w_{ij}} \left(\sum_{j \in N_i} w_{ij} \mathbf{p}_j \right) - \mathbf{p}_i$$

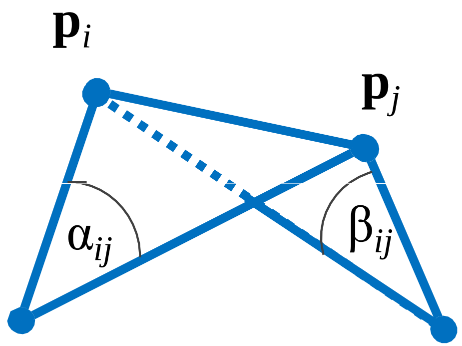
$$w_{ij} = \frac{h_{ij}^1 + h_{ij}^2}{l_{ij}}$$

Laplace Operator Discretiztion

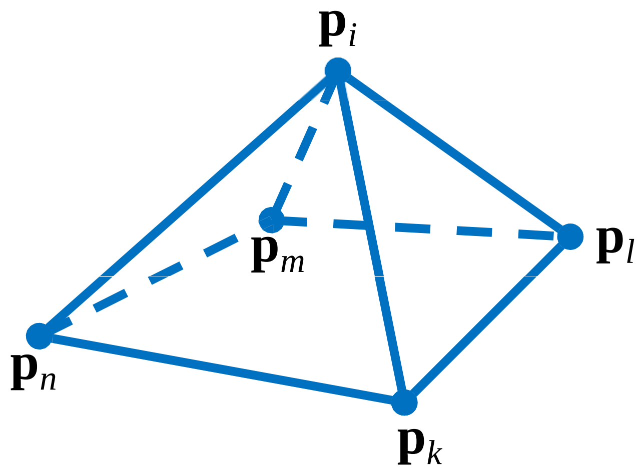
Cotangent Weights

Use a weighted average to define Δ

Which weights?



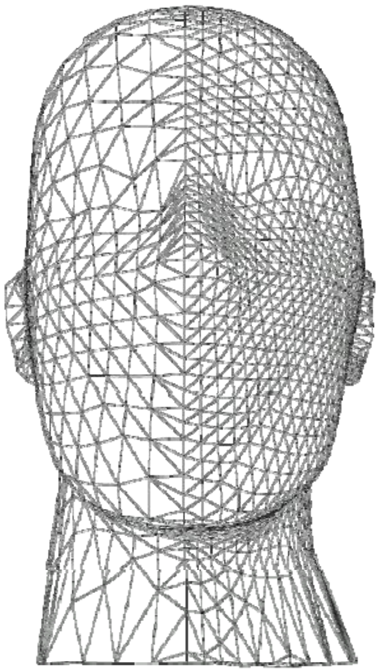
$N_i = \{k, l, m, n\}$



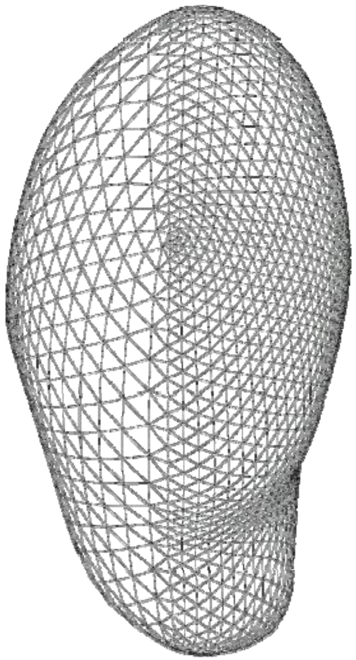
$$w_{ij} = \frac{h_{ij}^1 + h_{ij}^2}{l_{ij}} = \frac{1}{2}\left(\cot\alpha_{ij} + \cot\beta_{ij}\right) \qquad L(\mathbf{p}_i) = \frac{1}{\sum w_{ij}} \sum_{j \in N_i} w_{ij}\left(\mathbf{p}_j - \mathbf{p}_i\right)$$

Planar meshes will be invariant to smoothing

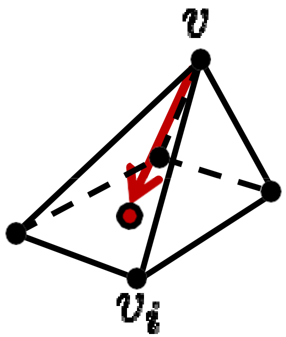
Smoothing with the Cotangent Laplacian



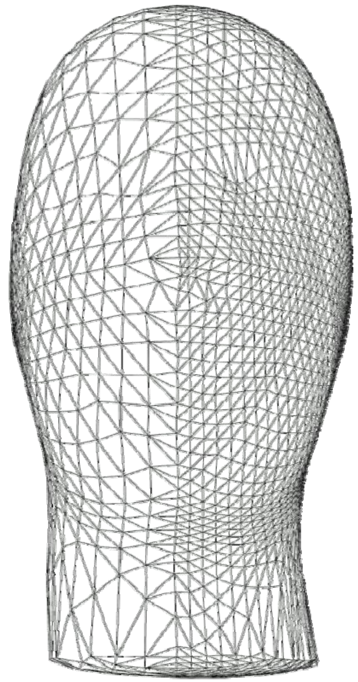
original



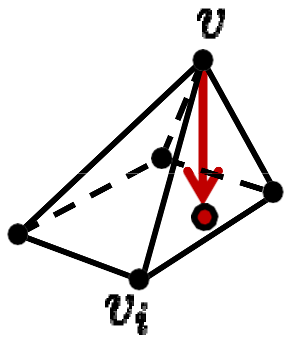
Uniform weights



normal and tangential movement



Cotangent weights



normal movement

From Desbrun et al., Siggraph 1999

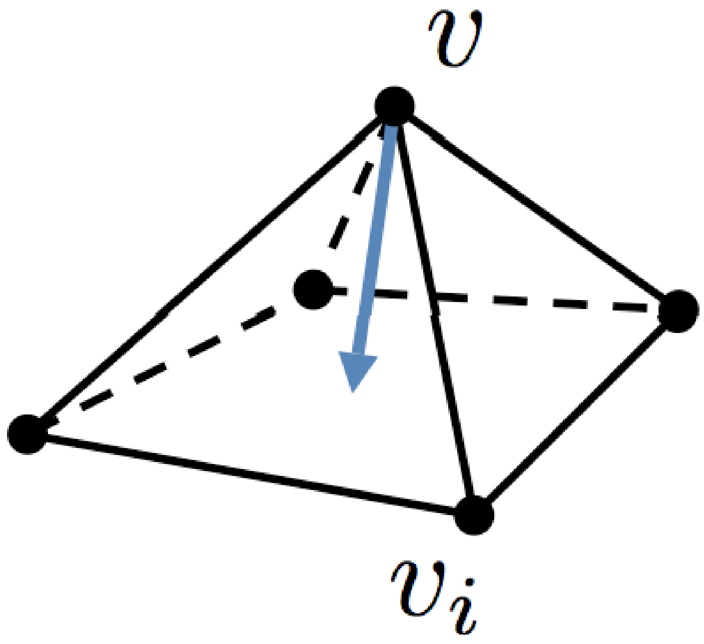
Exercise

- Smoothing
  - Uniform
  - Cotangent formula

- Smoothness visualization
  - Mean curvature (uniform, weighted)
  - Gaussian curvature
  - Triangle shape

Exercise

- Smoothing
  - Uniform Laplace-Beltrami



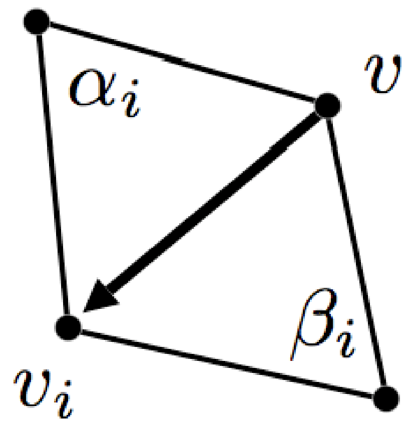
$$L_U(v) = \left(\frac{1}{n}\sum_i v_i\right) - v$$

Exercise

- Smoothing
  - Cotangent Formula (simplified)

$$L_B(v) = \frac{1}{\sum_i w_i} \sum_i w_i (v_i - v)$$

$$w_i = \frac{1}{2}(\cot \alpha_i + \cot \beta_i)$$



$$\cot(\theta) = \frac{\cos\theta}{\sin\theta} = \frac{\vec{u}.\vec{v}}{||\vec{u}\times\vec{v}||} = \frac{||\vec{u}||.||\vec{v}||cos(\theta)}{||\vec{u}||.||\vec{v}||sin(\theta)}$$

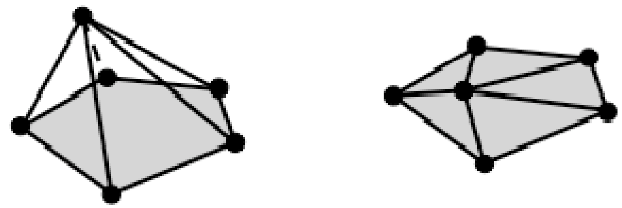
Exercise

- Smoothing
  - Uniform Laplace-Beltrami

$$v' = v + \frac{1}{2} \cdot L_U(v)$$

  - Cotangent Formula

$$v' = v + \frac{1}{2} \cdot L_B(v)$$



  - Move vertices in parallel!

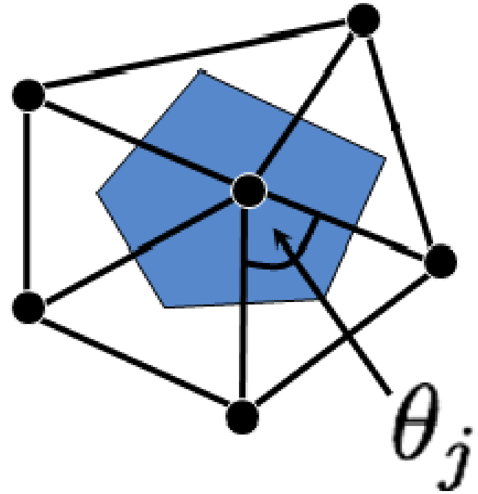
Exercise

- Curvature
  - Mean Curvature

$$H = \|\Delta_{\mathcal{S}} \mathbf{x}\|$$

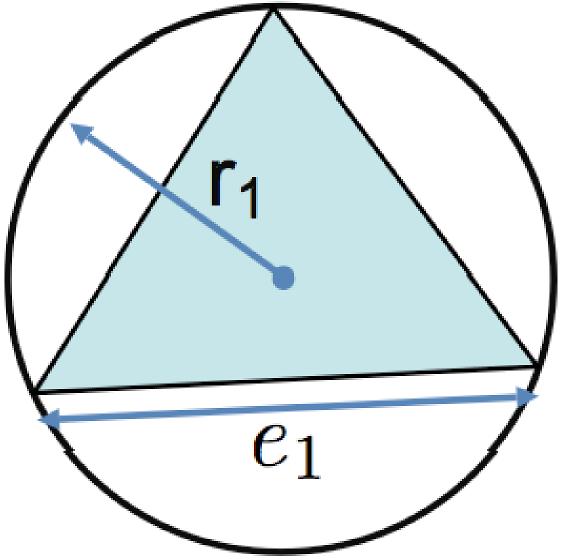
  - Gaussian Curvature (simplified)

$$G = (2\pi - \sum_j \theta_j)$$

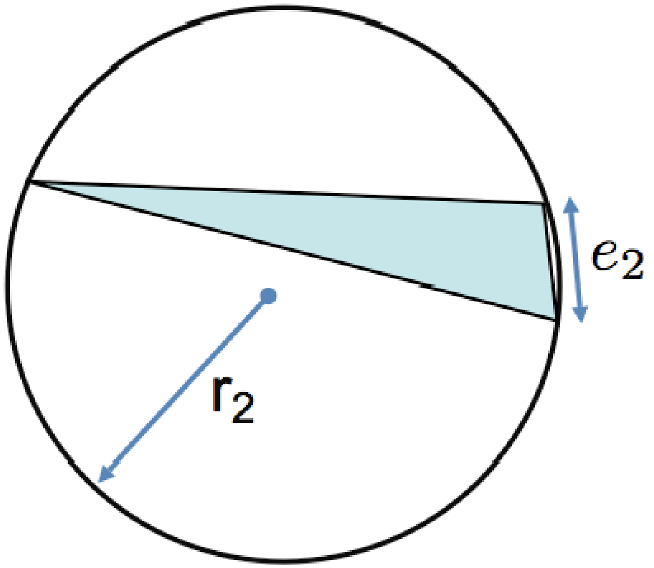


Exercise

- Triangle Shape – circumradius vs. minimal edge length

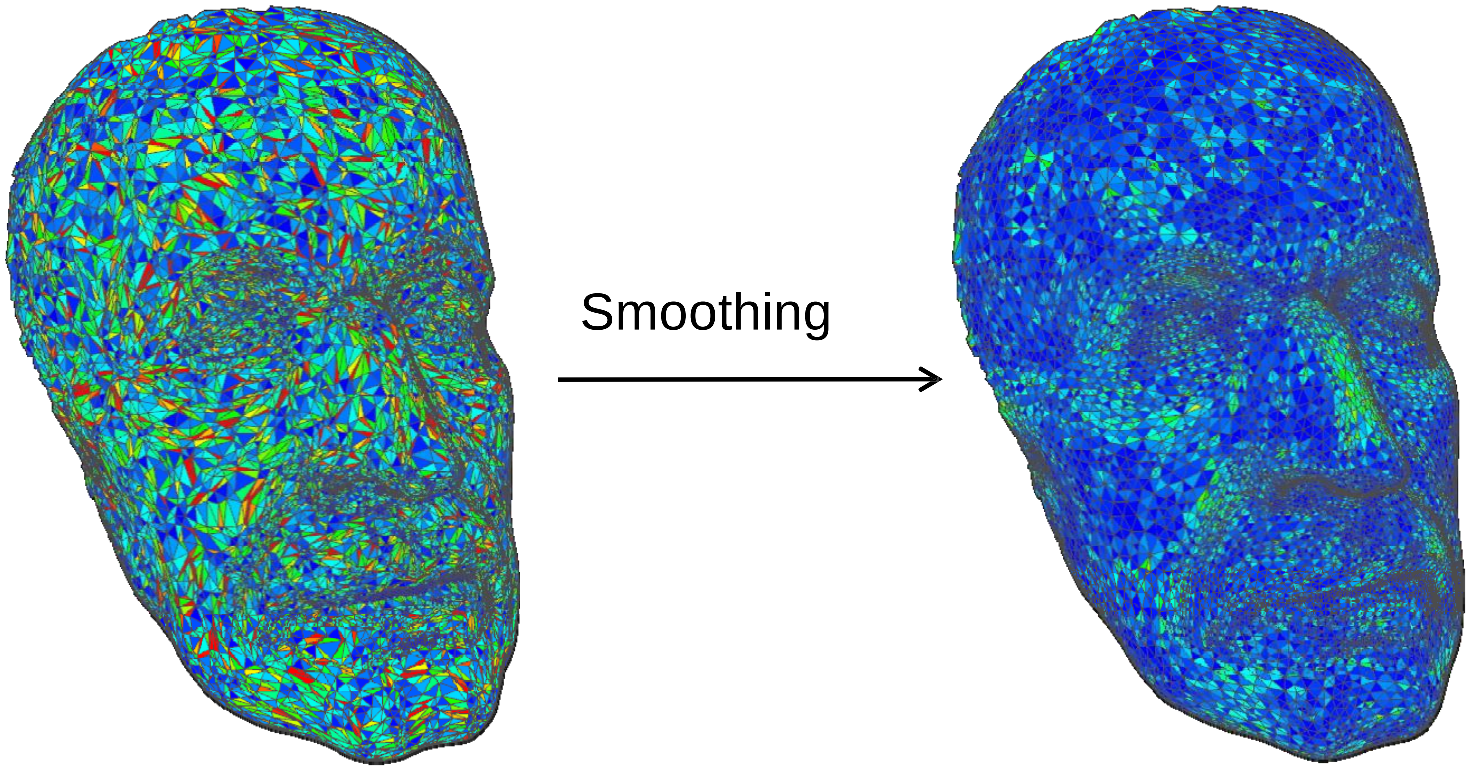


$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$



Exercise

- Triangle Shape



References

- "Discrete Differential-Geometry Operators for Triangulated 2-Manifolds", Meyer et al., '02
- "Restricted Delaunay triangulations and normal cycle", Cohen-Steiner et al., SoCG '03
- "On the convergence of metric and geometric properties of polyhedral surfaces , Hildebrandt et al., 06
- "Discrete Laplace operators: No free lunch", Wardetzky et al., SGP '07
- "A Signal Processing Approach to Fair Surface Design", Taubin, Siggraph '95
- "Implicit Fairing of Irregular Meshes using Diffusion and Curvature Flow", Desbrun et al., Siggraph '99
- "An Intuitive Framework for Real-Time Freeform Modeling", Botsch et al., Siggraph '04
- "Spectral Geometry Processing with Manifold Harmonics", Vallet et al., Eurographics '08

References

- “Discrete Differential-Geometry Operators for Triangulated 2-Manifolds”, Meyer et al., ’02
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