

Gromatica regulară

\Rightarrow produsele sunt unele din formele următoare

$A \rightarrow aB$ sau $A \rightarrow b$, unde $A, B \in N$ și $a, b \in \Sigma$

Produsul $A \rightarrow E$ poate să fie o clasa deoarece

A nu apare în membrul drept al niciunei produse

1) a) $L = \{a\}$ $\Sigma = \{a\}$

$\Rightarrow A \xrightarrow{\textcircled{A}} \textcircled{B} \quad | \quad A \rightarrow a$
 grom. regulară

b) $L = \{a^m \mid m \in N\}$

$\Rightarrow \textcircled{A} \xrightarrow{S_a} \begin{array}{l} A \rightarrow aA \\ A \rightarrow a \\ \underline{A \rightarrow \epsilon} \end{array}$ g. nu e regulară

\Rightarrow g. regulară.

$S \rightarrow \epsilon \quad A \rightarrow aA$

$S \rightarrow aA \quad A \rightarrow a$

$S \rightarrow a$

c) $L = \{a^m b \mid m \in N\}$

$\Rightarrow \textcircled{A} \xrightarrow{\substack{a \\ b}} \textcircled{B} \quad \begin{array}{l} A \rightarrow aA \\ A \rightarrow b \end{array}$

d) $L = \{\epsilon\} \cup \{a^m b \mid m \in N\}$

$S \rightarrow \epsilon$

$S \rightarrow aA \Rightarrow$ e regulară

$A \rightarrow aA$

$A \rightarrow b$

e) $\Rightarrow \textcircled{A} \xrightarrow{\substack{a, b \\ a}} \textcircled{B} \quad L = \{a^m b^n \mid m, n \in N, m+n > 0\}$

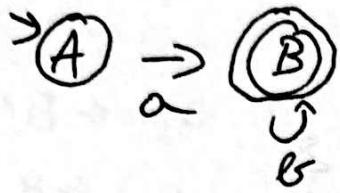
$A \rightarrow a$
 $A \rightarrow b$

$A \rightarrow aA$
 $A \rightarrow ab$

$A \rightarrow GB$
 $B \rightarrow GB$

$B \rightarrow b$

f) $L = \{ab^m \mid m \in \mathbb{N}\}$



Gramatica: $A \rightarrow a$
 $A \rightarrow aB$
 $B \rightarrow BB$
 $B \rightarrow b$

? $abb\epsilon L(G)$

$A \rightarrow aB \rightarrow abB \rightarrow abbB \rightarrow abbb \in L(G)$

? $abb\epsilon L(A)$

$(A, abbb) \vdash (B, bb) \vdash (B, b) \vdash (B, \epsilon) = B$

2) Descrieți construcția generând o nouă gram. regulată echivalentă cu un AF dat. $\Rightarrow abbb \in L(A)$

Teorema:

$M = (Q, \Sigma, \delta, q_0, F)$ automat finit, \exists o gram. regulată

$G = (N, \Sigma, \tau, S)$ a.t. $L(G) = L(M)$

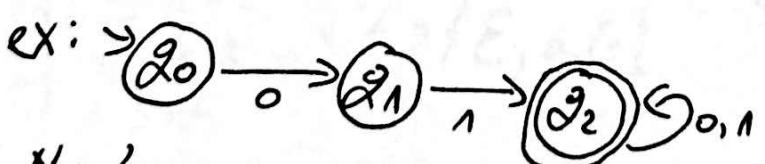
Construcție:

$N = Q$

$S = q_0$

$\Sigma \rightarrow e$ oclosi

$\tau = \{ A \rightarrow aB \mid B \in \delta(A, a) \} \cup \{ A \rightarrow a \mid B \in F, B \in \delta(A, a) \}$



$N = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$S = q_0$

$\tau: q_0 \rightarrow 0q_1$

$q_1 \rightarrow 1q_2$

$q_1 \rightarrow 1$

$q_2 \rightarrow 0q_2$

$q_2 \rightarrow 1q_2$

$q_2 \rightarrow 1$

$q_2 \rightarrow 0$

3) Pt. nemotociale limbice, doti de gram. regulatoare care le genereaza:

a) $L = \{a^{3m} \mid m \in \mathbb{N}\}^*$

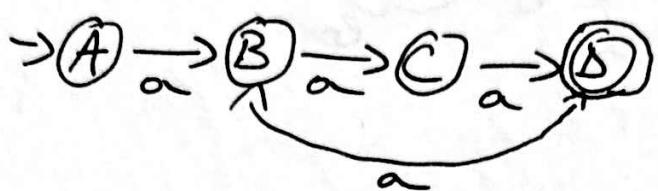
$A \rightarrow aB$

$B \rightarrow aC$

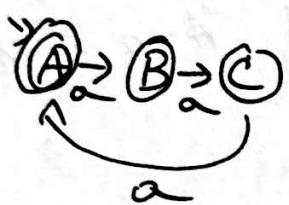
$C \rightarrow aD$

$C \rightarrow a$

$D \rightarrow aB$



b) $L = \{a^{3m} \mid m \in \mathbb{N}\}$



$A \rightarrow \epsilon$
 $A \rightarrow aB$
 $B \rightarrow aC$
 $C \rightarrow aA$
 $C \rightarrow a$

\Rightarrow doceză că este

$\rightarrow \alpha$ odoare ca în S' în N , unde simbolul de start

$\rightarrow \alpha$ odoare în P :

$S' \rightarrow \epsilon$

$S' \rightarrow \alpha$, și $\alpha : \exists S \rightarrow \alpha$ în P , unde S e vectorul simbolurilor de start

$S \rightarrow \epsilon$

$S \rightarrow aB$

$A \rightarrow aB$

$B \rightarrow aC$

$C \rightarrow aA$

$C \rightarrow a$

$$c) \mathcal{L} = \{a^m b^n / m, n \in \mathbb{N}^*\}$$

$$A \rightarrow aB$$

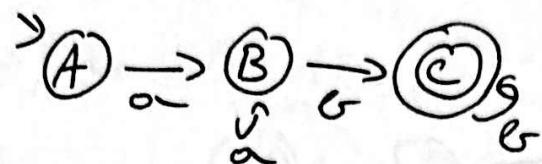
$$B \rightarrow aB$$

$$B \rightarrow b$$

$$B \rightarrow bC$$

$$C \rightarrow bC$$

$$C \rightarrow b$$



d) limbajul constantelor numerice fără semn reprezentate în baza 2

$$S \rightarrow 0S$$

$$S \rightarrow 0$$

$$S \rightarrow 1$$

$$S \rightarrow 1A$$

$$A \rightarrow 0A$$

$$A \rightarrow 1A$$

$$A \rightarrow 0$$

$$A \rightarrow 1$$

$$S \rightarrow AS$$

$$S \rightarrow 0$$

$$S \rightarrow 1$$

e) limbajul identificatorilor

$$S \rightarrow a | b | \dots | z$$

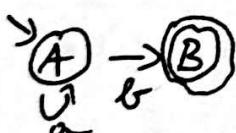
$$C \rightarrow a | f | 0 | .. | 9$$

$$S \rightarrow a | b | \dots | z$$

C

4) Pt. următoarele gram. regulare, descrieți limbajul generat. Dati AF echivalent

a) $A \rightarrow aA$ $\mathcal{L} = \{a^n b / n \in \mathbb{N}\}$
 $A \rightarrow b$



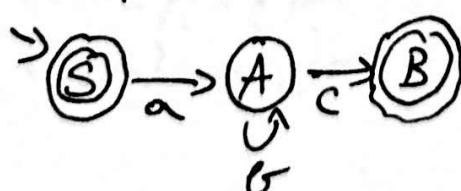
b) $S \rightarrow E$
 $S \rightarrow aA$
 ~~$A \rightarrow b$~~

$$\mathcal{L} = \{E, ab\}$$



c) $S \rightarrow E$
 $S \rightarrow aA$
 $A \rightarrow bA$
 $A \rightarrow C$

$$\mathcal{L} = \{E\} \cup \{a b^n c / n \in \mathbb{N}\}$$



5) Descrieți construcția generală a unui AF echivalent cu și
fără regulile date.

$G \rightarrow AF?$

$$G = (N, \Sigma, S, P)$$

$$AF = (Q, \Sigma, \delta, q_0, F)$$

$$Q = N \cup \{K\}$$

$$q_0 = S$$

$$F = \begin{cases} \{K\}, \text{ dacă } S \rightarrow \epsilon \notin P \\ \{S, K\}, \text{ dacă } S \rightarrow \epsilon \in P \end{cases}$$

$$\delta(A, a) = \{B \mid (A \rightarrow_a B \in P)\} \cup K, \text{ unde}$$

$$K = \begin{cases} \{K\}, \text{ dacă } A \rightarrow a \in P \\ \emptyset, \text{ altfel, } \exists A \rightarrow a \end{cases}$$

ex: $G = (N, \Sigma, S, P)$

$$P: S \rightarrow aB \mid bA$$

$$A \rightarrow c B$$

$$C \rightarrow aS \mid c$$

$$B \rightarrow \epsilon$$

Automatul finit echivalent

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$S \rightarrow aB \Leftrightarrow \delta(S, a) = \{B\}$$

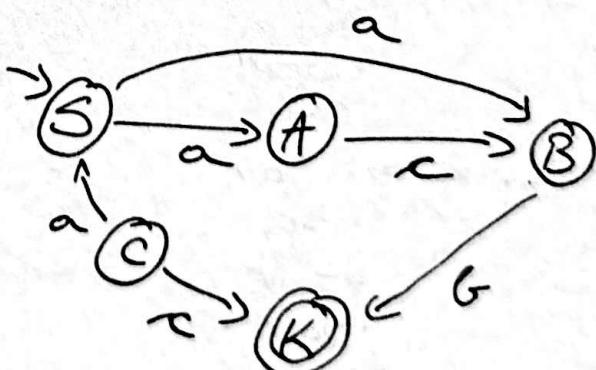
$$S \rightarrow bA \Rightarrow \delta(S, b) = \{A\}$$

$$A \rightarrow cB \Rightarrow \delta(A, c) = \{B\}$$

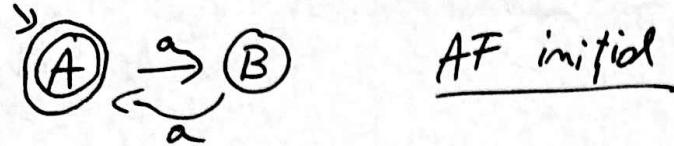
$$C \rightarrow aS \Rightarrow \delta(C, a) = \{S\}$$

$$C \rightarrow c \Rightarrow \delta(C, c) = \{K\}$$

$$B \rightarrow \epsilon \Rightarrow \delta(B, \epsilon) = \{K\}$$



$$6) a) L = \{a^{2^n} \mid n \in \mathbb{N}\}$$



$$G = (\{A, B\}, \{a\}, P, A)$$

$$\begin{array}{l} A \rightarrow \epsilon \\ A \rightarrow aB \\ B \rightarrow aA \\ B \rightarrow a \end{array} \left. \begin{array}{l} \text{nu e regulat} \end{array} \right\} \Rightarrow$$

$$\begin{array}{l} S \rightarrow \epsilon \\ S \rightarrow aB \\ A \rightarrow aB \\ B \rightarrow aA \\ B \rightarrow a \end{array}$$

Grom regulat generate din AF

AF constraint pe fozo gramatici.

$$M = (\{S, A, B, K\}, \{a\}, \delta, S, \{K, S\})$$

$$S \rightarrow aB \Rightarrow \delta(S, a) = \{B\}$$

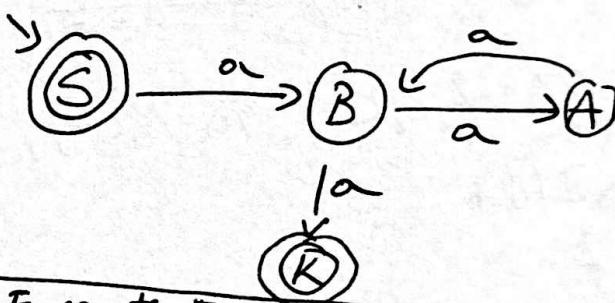
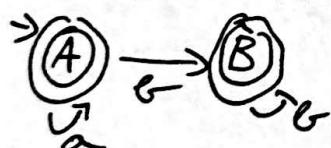
$$A \rightarrow aB \Rightarrow \delta(A, a) = \{B\}$$

$$B \rightarrow aA \Rightarrow \delta(B, a) = \{A\}$$

$$B \rightarrow a \Rightarrow \delta(B, a) = \{K\}$$

$$g) L = \{a^m b^n \mid m, n \in \mathbb{N}\}$$

AF initial



Grom constrainte echivalente.

$$G = (\{A, B\}, \{a, b\}, P, A)$$

$$\begin{array}{l} A \rightarrow \epsilon \\ A \rightarrow aA \\ A \rightarrow a \\ A \rightarrow b \\ A \rightarrow bB \\ B \rightarrow bB \\ B \rightarrow b \end{array} \left. \begin{array}{l} \text{nu e reg} \end{array} \right\} \Rightarrow$$

AF constraint

$$\begin{array}{ll} M = (\{S, A, B, K\}, \{a, b\}, \delta, S, \{S, K\}) & \\ \delta(S, \epsilon) = \{S\} & \delta(S, a) = \{A, K\} \\ \delta(S, b) = \{B\} & \delta(A, a) = \{A, K\} \\ \delta(A, b) = \{K, B\} & \delta(B, b) = \{K, B\} \end{array}$$

