

Sem 4 - LFTC

Gramatici regulare

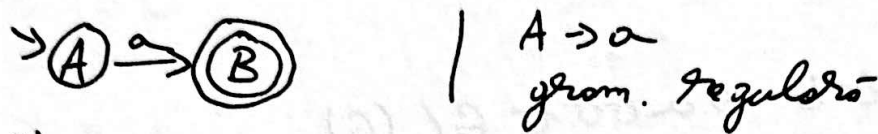
\Rightarrow \forall producție de una din formele următoare

$A \rightarrow aB$ sau $A \rightarrow b$, unde $A, B \in N$ și $a, b \in \Sigma$

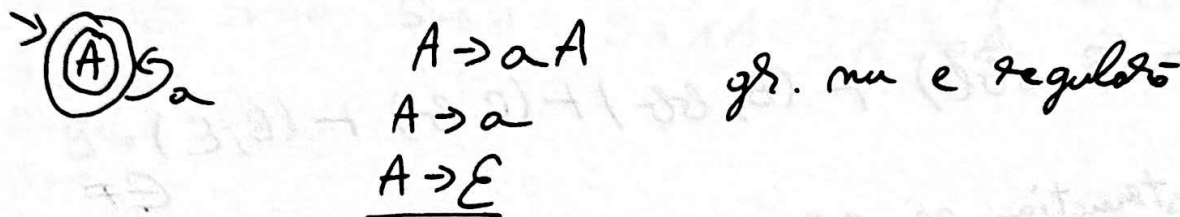
Producția $A \rightarrow \epsilon$ poate să apară doar dacă

A nu apare în membrul drept al niciunei producții

1) a) $L = \{a\}$ $\Sigma = \{a\}$



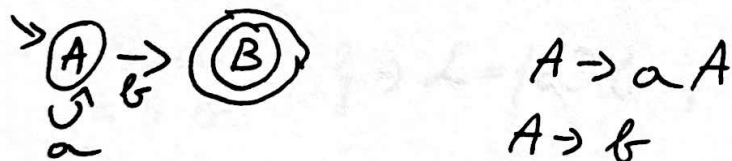
b) $L = \{a^m \mid m \in \mathbb{N}\}$



\Rightarrow gr. regulare

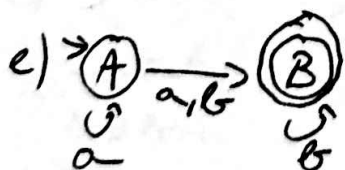
$S \rightarrow \epsilon$ $A \rightarrow aA$
 $S \rightarrow aA$ $A \rightarrow a$
 $S \rightarrow a$

c) $L = \{a^m b \mid m \in \mathbb{N}\}$



d) $L = \{\epsilon\} \cup \{a^m b \mid m \in \mathbb{N}\}$

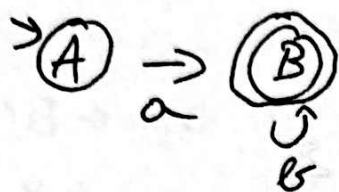
$S \rightarrow \epsilon$
 $S \rightarrow aA$ \Rightarrow e regulare
 $A \rightarrow aA$
 $A \rightarrow b$



$L = \{a^m b^m \mid m, n \in \mathbb{N}, m+n > 0\}$

$A \rightarrow a$ $A \rightarrow aA$ $A \rightarrow bB$ $B \rightarrow b$
 $A \rightarrow b$ $A \rightarrow aB$ $B \rightarrow bB$

f) $L = \{a b^m \mid m \in \mathbb{N}\}$



Gramatica: $A \rightarrow a$
 $A \rightarrow aB$
 $B \rightarrow bB$
 $B \rightarrow b$

? $abbb \in L(G)$

$A \Rightarrow aB \Rightarrow abB \Rightarrow abbB \Rightarrow abbb \in L(G)$

? $abbb \in L(A)$

$(A, abbb) \vdash (B, bbb) \vdash (B, bb) \vdash (B, b) \vdash (B, \epsilon) = B \in F$

$\Rightarrow abbb \in L(A)$

2) Descrieti constructia generala a unei gram. regulare echivalente cu un AF dat.

Teorema:

$\forall M = (Q, \Sigma, \delta, q_0, F)$ automat finit, \exists o gram. regulare $G = (N, \Sigma, P, S)$ a.i. $L(G) = L(M)$

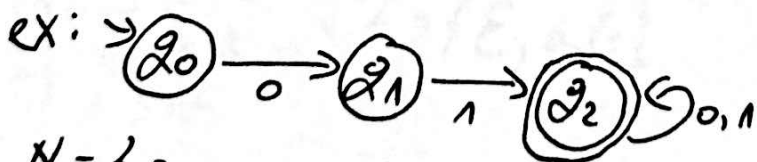
Constructia:

$N = Q$

$S = q_0$

$\Sigma \rightarrow e$ oclapi

$P = \{ A \rightarrow aB \mid B \in \delta(A, a) \} \cup \{ A \rightarrow a \mid B \in F, B \in \delta(A, a) \}$



$N = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$S = q_0$

P : $q_0 \rightarrow 0q_1$

$q_2 \rightarrow 0q_2$

$q_2 \rightarrow 1$

$q_1 \rightarrow 1q_2$

$q_2 \rightarrow 1q_2$

$q_2 \rightarrow 0$

$q_1 \rightarrow 1$

3) Pt. nemotoarele limbaj, doti o gram. regulare care le genereaza.

a) $L = \{a^{3m} \mid m \in \mathbb{N}^*\}$

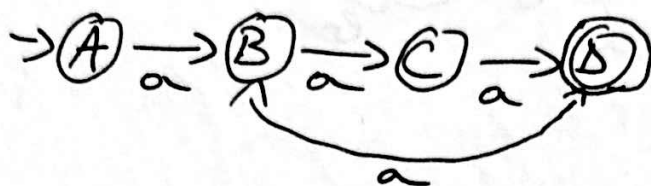
$A \rightarrow aB$

$B \rightarrow aC$

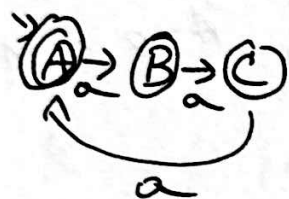
$C \rightarrow aD$

$C \rightarrow a$

$D \rightarrow aB$



b) $L = \{a^{3m} \mid m \in \mathbb{N}\}$



$A \rightarrow \epsilon$

$A \rightarrow aB$

$B \rightarrow aC$

$C \rightarrow aA$

$C \rightarrow a$

} m e regulare

\Rightarrow Deci $20 \in F$

$\rightarrow \alpha$ odango un S' in N , modul simbol de start

$\rightarrow \alpha$ odango in P :

$S' \rightarrow \epsilon$

$S' \rightarrow \alpha$, $\forall \alpha: \exists S \rightarrow \alpha$ in P , unde S e vechiul simbol de start

$S \rightarrow \epsilon$

$S \rightarrow aB$

$A \rightarrow aB$

$B \rightarrow aC$

$C \rightarrow aA$

$C \rightarrow a$

$$c) L = \{ a^m b^m \mid m, m \in \mathbb{N}^* \}$$

$$A \rightarrow aB$$

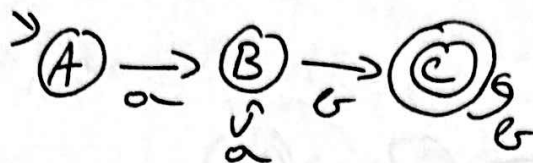
$$B \rightarrow aB$$

$$B \rightarrow b$$

$$B \rightarrow bC$$

$$C \rightarrow bC$$

$$C \rightarrow b$$



d) limbojul constantelor numerice ~~foarte~~ ~~num~~ reprezentate în baza-2

$$S \rightarrow 0S \quad S \rightarrow 0 \quad S \rightarrow 1 \quad S \rightarrow 1A \quad A \rightarrow 0A \quad A \rightarrow 1A \quad A \rightarrow 0 \quad A \rightarrow 1$$

$$S \rightarrow AS$$

$$S \rightarrow 0$$

$$S \rightarrow 1$$

e) limbojul identificatorilor

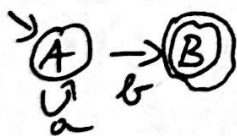
$$S \rightarrow a | b | \dots | z$$

$$C \rightarrow a | \dots | z | 0 | \dots | 9$$

$$S \rightarrow a | b | \dots | z C$$

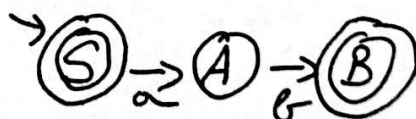
4) Pt. următoarele gram. regulate, descrieți limbojul generat. Dați AF echivalent

a) $A \rightarrow aA$
 $A \rightarrow b$ $\Rightarrow L = \{ a^m b \mid m \in \mathbb{N} \}$



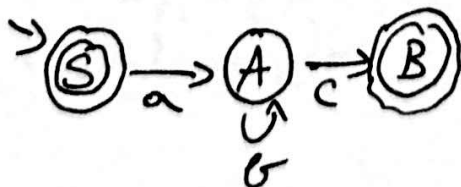
b) $S \rightarrow \epsilon$
 $S \rightarrow aA$
 $A \rightarrow b$

$$L = \{ \epsilon, ab \}$$



c) $S \rightarrow \epsilon$
 $S \rightarrow aA$
 $A \rightarrow bA$
 $A \rightarrow c$

$$L = \{ \epsilon \} \cup \{ a b^m c \mid m \in \mathbb{N} \}$$



5) Descrieti constructia generală a unui AF echivalent cu 2 gram. regulor date

$$G \rightarrow AF?$$

$$G = (N, \Sigma, S, P)$$

$$AF = (Q, \Sigma, \delta, q_0, F)$$

$$Q = N \cup \{K\}$$

$$q_0 = S$$

$$F = \begin{cases} \{K\}, & \text{dacă } S \rightarrow \epsilon \notin P \\ \{S, K\}, & \text{dacă } S \rightarrow \epsilon \in P \end{cases}$$

$$\delta(A, a) = \{B \mid (A \rightarrow aB \in P)\} \cup K, \text{ unde}$$

$$K = \begin{cases} \{K\}, & \text{dacă } A \rightarrow a \in P \\ \emptyset, & \text{altfel, } \exists \overline{A \rightarrow a} \end{cases}$$

ex: $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$

$$P: S \rightarrow aB \mid bA$$

$$A \rightarrow cB$$

$$C \rightarrow aS \mid c$$

$$B \rightarrow b$$

Automatul finit echivalent

$$M = (\{S, A, B, C, K\}, \{a, b, c\}, \delta, S, \{K\})$$

$$S \rightarrow aB \Rightarrow \delta(S, a) = \{B\}$$

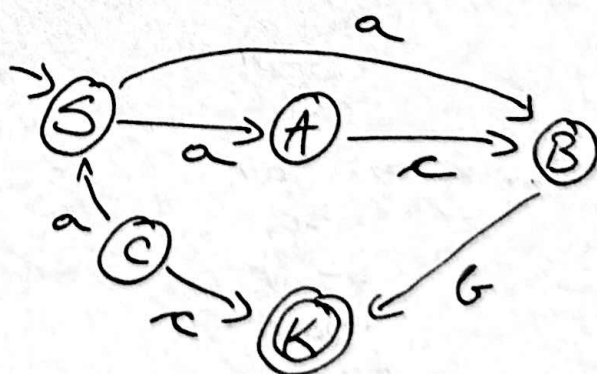
$$S \rightarrow bA \Rightarrow \delta(S, b) = \{A\}$$

$$A \rightarrow cB \Rightarrow \delta(A, c) = \{B\}$$

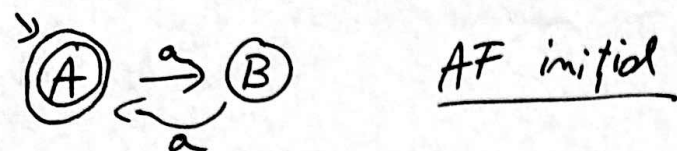
$$C \rightarrow aS \Rightarrow \delta(C, a) = \{S\}$$

$$C \rightarrow c \Rightarrow \delta(C, c) = \{K\}$$

$$B \rightarrow b \Rightarrow \delta(B, b) = \{K\}$$



$$6) a) L = \{a^{2n} \mid n \in \mathbb{N}\}$$



$$G = (\{A, B\}, \{a\}, P, A)$$

$$\left. \begin{array}{l} A \rightarrow \varepsilon \\ A \rightarrow aB \\ B \rightarrow aA \\ B \rightarrow a \end{array} \right\} \text{nu e regulată} \Rightarrow \left. \begin{array}{l} S \rightarrow \varepsilon \\ S \rightarrow aB \\ A \rightarrow aB \\ B \rightarrow aA \\ B \rightarrow a \end{array} \right\}$$

Gram regulată generată din AF

AF construit pe baza gramaticii:

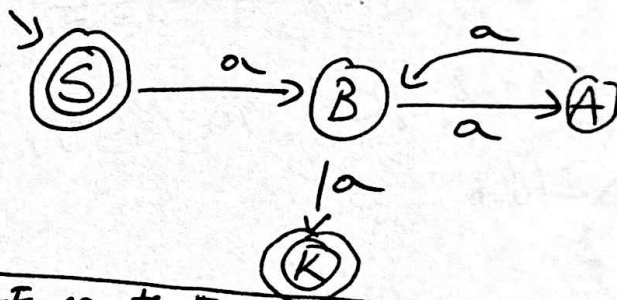
$$M = (\{S, A, B, K\}, \{a\}, \delta, S, \{K, S\})$$

$$S \rightarrow aB \Rightarrow \delta(S, a) = \{B\}$$

$$A \rightarrow aB \Rightarrow \delta(A, a) = \{B\}$$

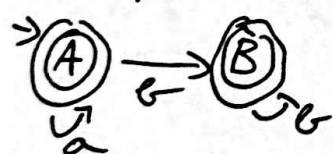
$$B \rightarrow aA \Rightarrow \delta(B, a) = \{A\}$$

$$B \rightarrow a \Rightarrow \delta(B, a) = \{K\}$$



$$b) L = \{a^m b^n \mid m, n \in \mathbb{N}\}$$

AF. inițial



Gram construită echivalentă.

$$G = (\{A, B\}, \{a, b\}, P, A)$$

$$\left. \begin{array}{l} A \rightarrow \varepsilon \\ A \rightarrow aA \\ A \rightarrow a \\ A \rightarrow bB \\ B \rightarrow bB \\ B \rightarrow b \end{array} \right\} \text{nu e reg} \Rightarrow \left. \begin{array}{l} S \rightarrow A \\ S \rightarrow \varepsilon \\ S \rightarrow aA \\ S \rightarrow a \\ S \rightarrow bB \\ A \rightarrow a \\ A \rightarrow bB \\ B \rightarrow bB \\ B \rightarrow b \end{array} \right\}$$

AF construit

$$M = (\{S, A, B, K\}, \{a, b\}, \delta, S, \{S, K\})$$

$$\begin{array}{ll} \delta(S, \varepsilon) = \{S\} & \delta(S, a) = \{A, K\} \\ \delta(S, b) = \{B\} & \delta(A, a) = \{A, K\} \\ \delta(A, b) = \{K, B\} & \delta(B, b) = \{K, B\} \end{array}$$

