# Curs – sapt.2

- Automate finite

# Automat finit (AF)

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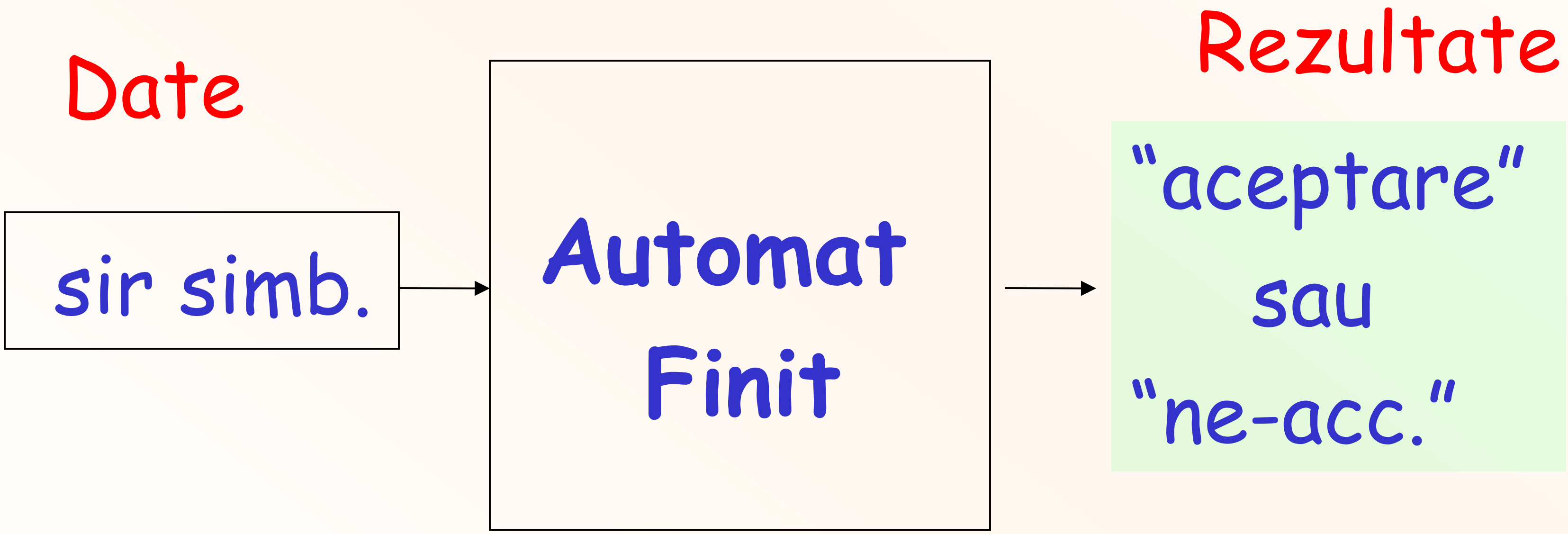
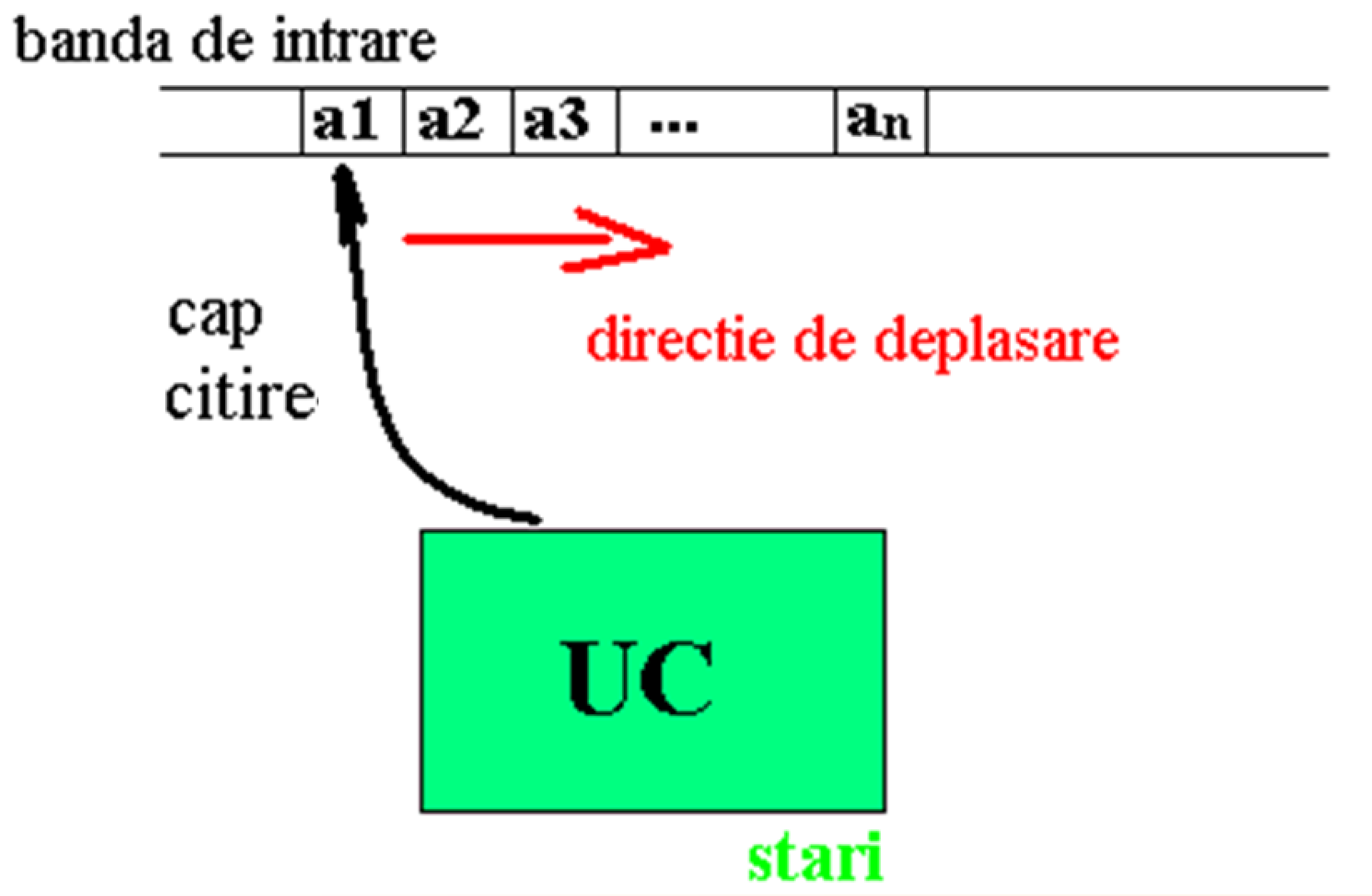


# Automat finit: model fizic



# Automat finit: model matematic

- Un ***automat finit*** este un ansamblu

  $M = (Q, \Sigma, \delta, q_0, F)$ :
- Q – alfabetul starilor
- $\Sigma$ – alfabet de intrare
- $\delta : Q x \Sigma \to \mathcal{P}(Q)$ functie de tranzitie
- $q_0 \in Q$ - stare initialã
- $F \subseteq Q$ multimea stãrilor finale

# AF – reprezentare tabelara

$\Sigma$

| $\delta$ | | $a_j$ | | |
|---|---|---|---|---|
| | | | | |
| $q_i$ | | | | $z_i$ |
| | | | | |
| | | | | |

**Q**

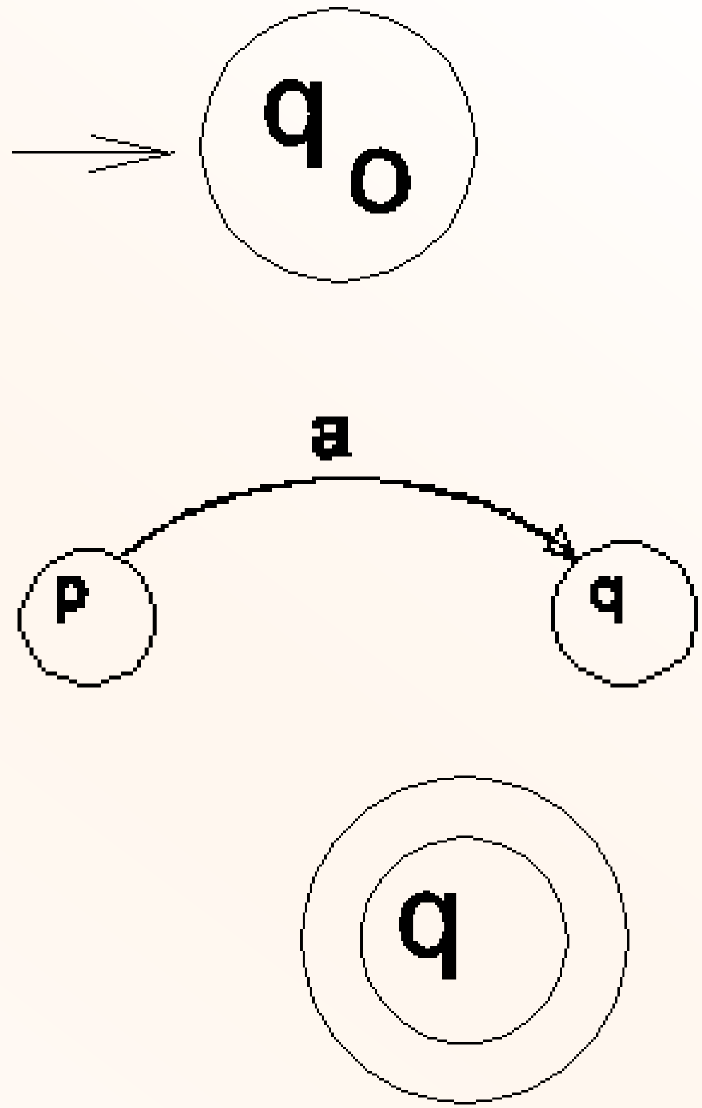
$$z_i = \begin{cases} 0 \text{ daca } q_i \text{ nu e stare finala} \\ 1 \text{ daca } q_i \text{ este stare finala} \end{cases}$$

# AF reprezentat tabelar; exemplu

| δ | 0 | 1 | |
|---|---|---|---|
| **p** | q | p | 0 |
| **q** | r | p | 0 |
| **r** | r | r | 1 |

# AF – reprezentare sub forma de graf

- graf orientat
- cu noduri si arce etichetate
- (graf de tranzitii)



# Configuratii si relatii de tranzitie

$M = (Q, \Sigma, \delta, q_0, F)$.

configuratie: $(q,x) \in \mathbf{Q x \Sigma^*}$

tranzitie: element din $\mathbf{(Q x \Sigma^*) \, x \, (Q x \Sigma^*)}$

$(p,aw) \vdash (q,w) \iff \delta(p,a) \ni q;$
$p,q \in Q,\ a \in \Sigma,\ w \in \Sigma^*$

- $\vdash$ tranzitie directa
- $\vdash^k$ k-tranzitie
- $\vdash^+$ +-tranzitie
- $\vdash^*$ *-tranzitie

# Limbaj acceptat; autom. echivalente

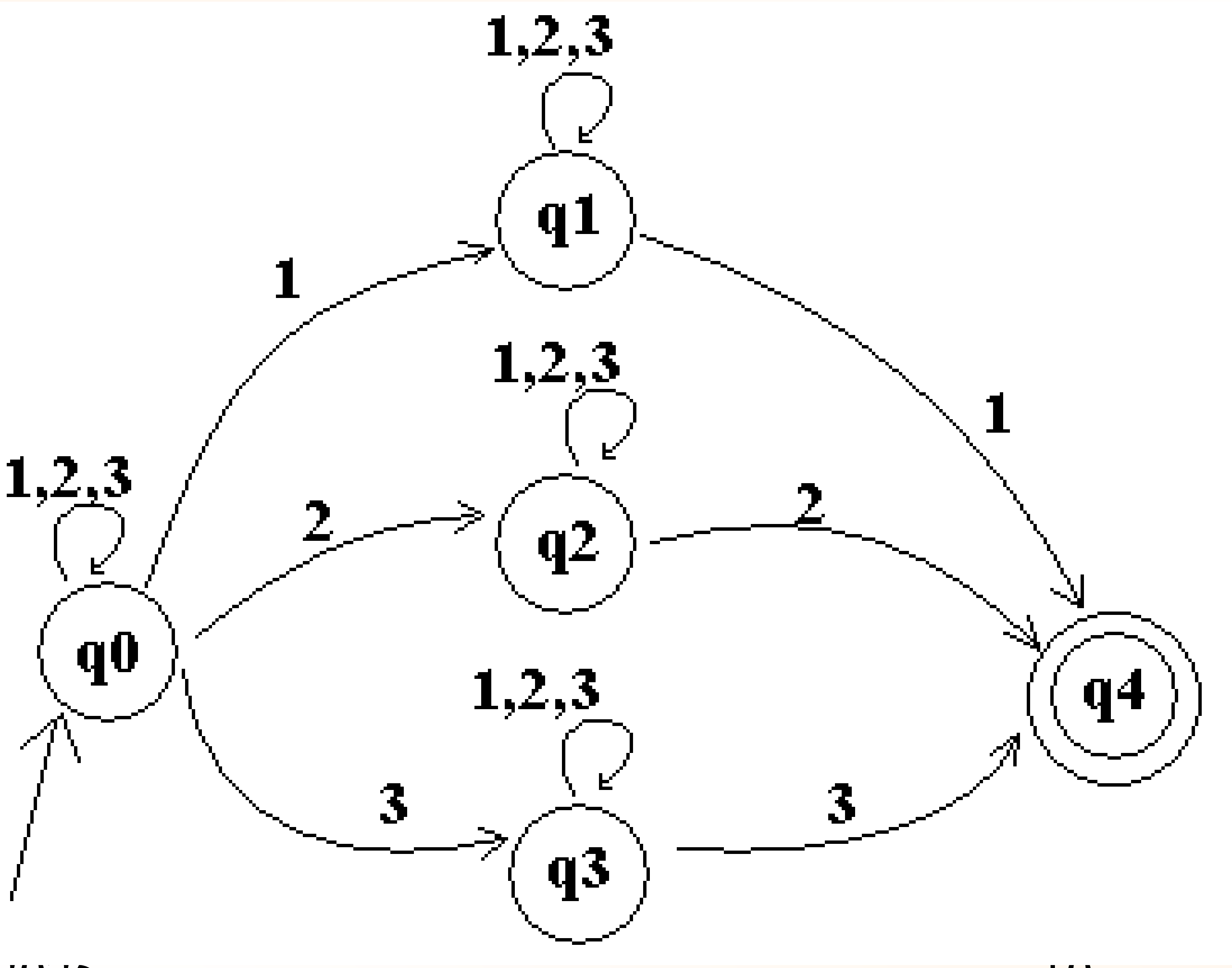
- Limbaj acceptat de automat

  $\mathbf{L(M) = \{w \mid w \in \Sigma^*, (q_0,w) \vdash^* (q_f,\varepsilon), q_f \in F\}}$

- Automate echivalente

  $M_1$ echivalent cu $M_2$ daca: $L(M_1) = L(M_2)$

# Automat finit - exemplu



# Determinism

- Automat finit determinist (AFD)

  $|\delta(q,a)| <= 1 \quad \forall\, q \in Q,\ a \in \Sigma$

- Automat finit nedeterminist (AFN)

  $\exists\, q \in Q,\ a \in \Sigma$ astfel incat $|\delta(q,a)| > 1$

- Automat finit determinist complet definit

  $|\delta(q,a)| = 1 \quad \forall\, q \in Q,\ a \in \Sigma$

# Echivalenta dintre AFD si AFN

Teorema:

- $\forall$ $M_1$ – AFN $\exists$ $M_2$ – AFD echivalent

Constructie (nu demonstratie!):

- Pornim cu: $M_1=(Q_1,\Sigma_1,\delta_1,q_{01},F_1)$ – AFN oarecare
- Construim: $M_2=(Q_2,\Sigma_2,\delta_2,q_{02},F_2)$ – AFD

  pe baza lui $M_1$

  a.i. $L(M_1) = L(M_2)$

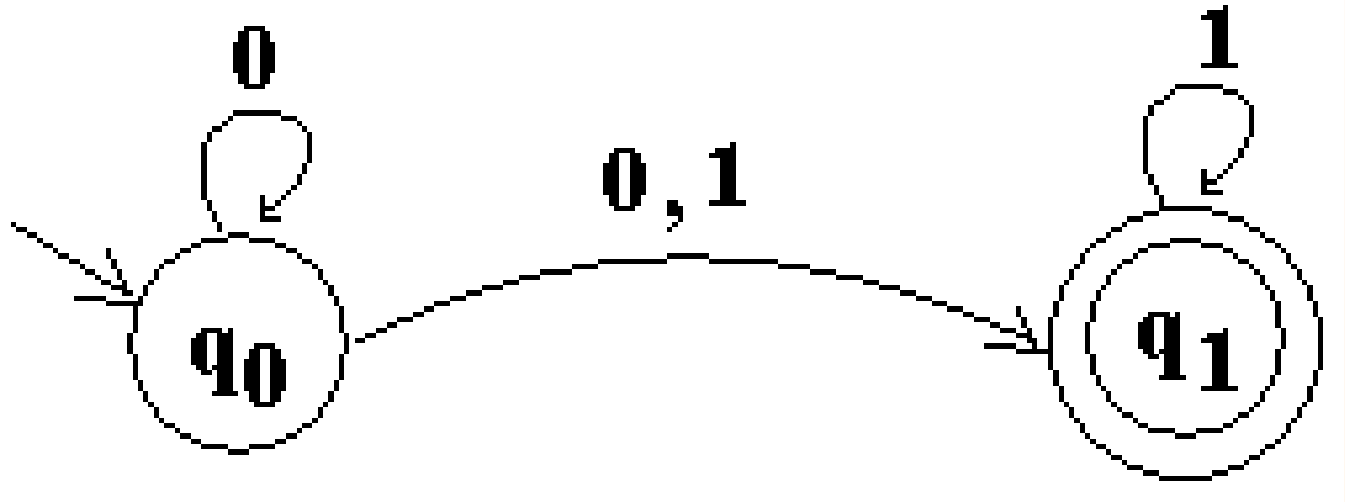
# Teor: $\forall$ $M_1$ – AFN $\exists$ $M_2$ – AFD echivalent

- $\Sigma_2 = \Sigma_1$
- $Q_2 = \mathcal{P}(Q_1)$
- $q_{02} = \{q_{01}\}$
- $F_2 = \{S \in \mathcal{P}(Q_1) \mid S \cap F_1 <> \Phi\}$
- $\delta_2(q,a) = \{r \in Q_1 \mid \exists\, q_1 \in q \text{ a.i. } r \in \delta_1(q_1,a)\}$

$$= \bigcup_{q1 \in q} \delta(q_1,a)$$

**$M_2$ – determinist (?)**

# Problema: determinati AFD echiv. pt.



# AF – stari care nu contribuie la acceptarea unui cuvant

- stare neproductiva – (nu e stare productiva)
- stare inaccesibila – (nu e stare accesibila)

- stare productiva: $q \in Q$ a.i.

  $$\exists\, w \in \Sigma^{*} \text{ si } q_f \in F \text{ a.i. } (q,w) \vdash^{*} (q_f,\varepsilon)$$

- stare accesibila: $q \in Q$ a.i.

  $$\exists\, w \in \Sigma^{*} \text{ a.i. } (q_0,w) \vdash^{*} (q,\varepsilon)$$

# Algoritm determin. stari accesibile

1. i:=0

   $A_0:=\{q_0\}$

2. **Repeta**

   i:=i+1

   $A_{i+1} := A_i \cup \{\, q \in Q \mid \exists p \in A_i, \exists a \in \Sigma \text{ a.i. } q \in \delta(p,a)\}$

   **pana cand** $A_i = A_{i+1}$

$\{A_i$ – multimea starilor accesibile$\}$

# Algoritm determin. stari productive

1. $i:=0$

   $A_0:=F$

2. **Repeta**

   $i:=i+1$

   $A_{i+1}=A_i \cup \{ q \in Q \mid \exists p \in A_i, \exists a \in \Sigma \text{ a.i. } p \in \delta(q,a)\}$

 **pana cand** $A_i=A_{i+1}$

$\{A_i$ – multimea starilor productive$\}$

# Teorema:

## $\forall$ $M_1$ – AF exista $M_2$ – AF fara st. neproductive **echiv**.

Constructie (nu demonstratie!):

- Pornim cu: $M_1=(Q_1,\Sigma_1,\delta_1,q_{01},F_1)$ – AF oarecare
- determinam A – multimea starilor productive (algoritmul anterior)
- Construim: $M_2$ pe baza lui $M_1$ (a.i. $L(M_1) = L(M_2)$)

$$M_2=(A,\Sigma_1,\delta_{1/A},q_{01},F_1)$$

$$L(M_1) = L(M_2) \quad !$$

Teorema:

$\forall$ $M_1$ – AF exista $M_2$ – AF fara st. inaccesibile **echiv**.

Constructie (nu demonstratie!):

- … analog …

# ? alta metoda de determinare a AFD echivalent pentru un AFN dat

- $M_1 \Rightarrow ?\ M_2$

Ideea:

1. $\{q_{01}\} \in Q_2$

   pornim cu $Q_2=\{q_{01}\}$
2. adaugam la $Q_2$ toate submultimile lui $Q_1$ la care se poate ajunge prin functia de tranzitie, atunci cand se aplica unei stari $q \in Q_2$ deja adaugata

# AFD complet definit

$M = (Q, \Sigma, \delta, q_0, F)$ **(AFD)**

- $\delta : Qx\Sigma \rightarrow \mathcal{P}(Q)$ functie de tranzitie ; $|\,\delta(q,a)\,| <= 1$
- ...

<u>Teor</u>: $\forall$ AFD $\exists$ AFD complet definit echivalent

Constructie:

AFD => AFD complet definit:

- adaugam o stare (neproductiva) *r* si extindem $\delta$ astfel:
- $\forall\ (q,a) \in Qx\Sigma$ a.i. $\delta(q,a)=\phi$ devine: $\delta(q,a)= \{\mathbf{r}\}$
- $\forall\ a \in \Sigma$ $\delta(r,a)= \{\mathbf{r}\}$

# Minimizarea automatelor finite

**Ce vrem:**

**Automat determinist cu numar minim de stari !**

**Automat redus**

- AFD
- nu contine stari inaccesibile si neproductive
- nu contine perechi de **stari echivalente**

# Minimizarea AFD

- automat cu numar minim de stari

  - fara stari –inaccesibile, neproductive

  - mai putine stari ?

  ideea: relatie de echivalenta; clase de echivalenta
- stari diferentiate
- stari k diferentiate
- stari echivalente
- stari k-echivalente

# Automatul redus

Fie M1 – un automat finit oarecare

- Determinam AFD echivalent
- Eliminam starile inaccesibile si neproductive
- Determinam AFD echivalent complet definit

Fie $M = (Q, \Sigma, \delta, q_0, F)$ automatul rezultat.

- Determinam relatia $\equiv$ (stari echivalente, clase de echivalenta)
- Pe baza relatiei $\equiv$ determinam automatul:

  $M_{\equiv} = (Q/\equiv, \Sigma, \delta_{\equiv}, [q_0], F_{\equiv})$

  $Q/\equiv$ - multimea claselor de echivalenta

  $\delta_{\equiv}([q],a) = [\delta(q,a)]$

  $F_{\equiv} = \{[q] \mid q \in F\}$

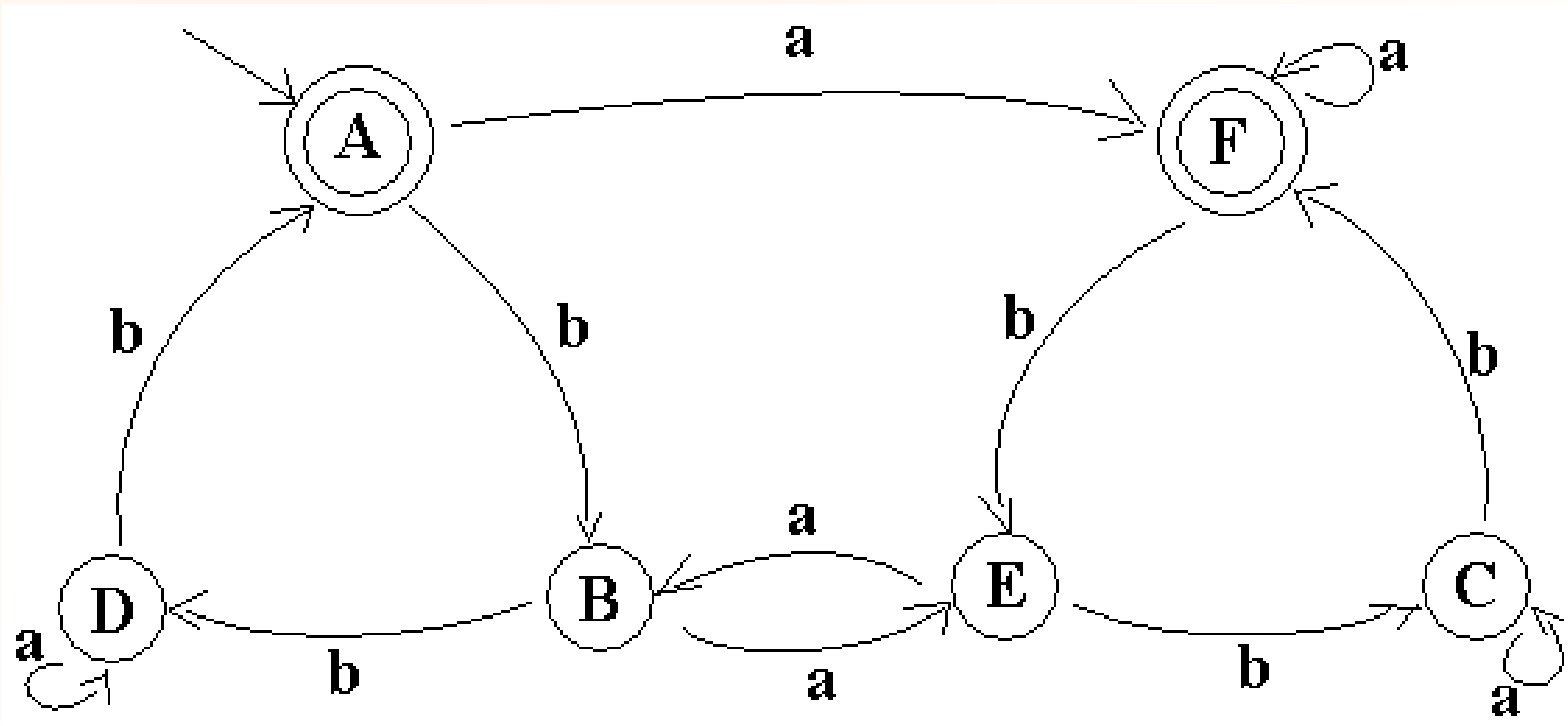
# Automat redus

- **Teorema**

  Automatul redus are numar minim de stari dintre toate AFD echivalente
- **Teorema**

  $\forall$ $M_1$ – AF $\exists$ M2 – automat redus echivalent

# Determinati clasele de echivalenta ale starilor automatului de mai jos




determinati automatul redus !

# Stari diferentiate

- o alta exprimare

$q_1,q_2$ sunt *stari diferentiate* de $x \in \Sigma^*$

daca $\exists\ q_f \in F$ a.i. $(q_1,x) \vdash^* (q_f,\varepsilon)$

si nu exista nici un $q \in F$ a.i. $(q_2,x) \vdash^* (q,\varepsilon)$

sau

daca $\exists\ q_f \in F$ a.i. $(q_2,x) \vdash^* (q_f,\varepsilon)$

si nu exista nici un $q \in F$ a.i. $(q_1,x) \vdash^* (q,\varepsilon)$

- x (de mai sus) *diferentiaza* pe $q_1$ si $q_2$

# Stari diferentiate

- o alta exprimare

PP. AFD complet definit

$q_1, q_2$ sunt *stari diferentiate* de $x \in \Sigma^*$

daca $\exists$ $r_1$, $r_2$ astfel incat: $(q_1,x) \vdash^* (r_1,\varepsilon)$

si $(q_2,x) \vdash^* (r_2,\varepsilon)$

are loc una dintre:

1. $r_1 \in F$ si $r_2 \in Q\text{-}F$
2. $r_1 \in Q\text{-}F$ si $r_2 \in F$

# Relatii intre stari

- $q_1,q_2$ - stari diferentiate
  - cf. def. de mai sus
  - ( $\exists$ x $\in \Sigma^*$ care sa le diferentieze)
- stari k diferentiate
  - daca $\exists$ x $\in \Sigma^*$, $|x| \leq k$ care sa le diferentieze
- stari echivalente ($\equiv$)
  - daca nu exista x $\in \Sigma^*$ care sa le diferentieze
- stari k-echivalente ($\equiv^k$)
  - daca nu exista x $\in \Sigma^*$, $|x| \leq k$, care sa le diferentieze

# Proprietati ale rel. de k-echivalenta ($\equiv^k$)

- $q_1 \equiv^0 q_2$ ddaca ($q_1, q_2 \in F$) sau ($q_1, q_2 \in Q\text{-}F$)

- $\equiv^0 \supseteq \equiv^1 \supseteq \equiv^2 \supseteq \cdots \supseteq \equiv^n \supseteq \cdots$
- Daca ($\equiv^k$) = ($\equiv^{k+1}$) atunci ($\equiv^k$) = $\equiv$

Lema:

Pt. orice M exista $n \in \mathbf{N}$ a.i. $q_1 \equiv^n q_2 \Rightarrow q_1 \equiv q_2$

- ideea: pot avea un nr. finit de relatii distincte (max. |Q|)