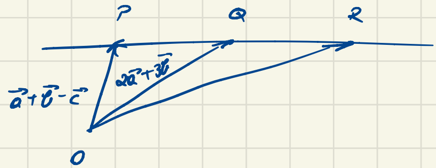
Modele parțial 1

Model 1

1. ip. $\begin{cases} \vec{a}, \vec{b}, \vec{c} \text{ coplanari} \\ \vec{OP} = \vec{a} + \vec{b} - \vec{c} \\ \vec{OQ} = 2\vec{a} + 3\vec{b} \\ \vec{OR} = -\vec{b} - t\vec{c}, \ t \in \mathbb{R} \\ P, Q, R \text{ coliniare} \end{cases}$

c) $t = ?$



R: Considerăm baza de coordonate $(\vec{a}, \vec{b}, \vec{c})$

$\Rightarrow \vec{OP}(1,1,-1), \ \vec{OQ}(2,3,0), \ \vec{OR}(0,-1,-t)$

P, Q, R coliniare $\Rightarrow \vec{PQ}, \vec{QR}$ coliniari $\Rightarrow$ au coordonate proporționale $\Rightarrow$

$\vec{PQ} = \vec{PO} + \vec{OQ} = (-1,-1,1) + (2,3,0) = (1,2,1)$

$\vec{QR} = \vec{QO} + \vec{OR} = (-2,-3,0) + (0,-1,-t) = (-2,-4,-t)$

$\Rightarrow -\frac{1}{2} = -\frac{1}{t} \Rightarrow \underline{t = 2}$

2. ip. $\begin{cases} ABC\Delta \text{ tetraedru} \\ A(1,-6,10), \ B(-1,-3,7), \ C(5,-1,a), \ \Delta(7,-4,7), \ a \in \mathbb{R} \\ V_{ABC\Delta} = 11 \end{cases}$

c) $a = ?$

R: $\vec{AB}(-2, 3, -3)$

$\vec{AC}(4, 5, a-10)$

$\vec{A\Delta}(6, 2, -3)$

$A_{ABC\Delta} = \frac{1}{6} |(\vec{AB}, \vec{AC}, \vec{A\Delta})| = \frac{1}{6} \left| \begin{vmatrix} -2 & 3 & -3 \\ 4 & 5 & a-10 \\ 6 & 2 & -3 \end{vmatrix} \right| = \frac{1}{6}|30 + 18a - 180 - 24 + 90 + 4a - 40 + 36|$

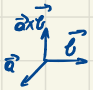
$= \frac{1}{6}|22a - 88|$

$\Rightarrow \frac{1}{6}|22a - 88| = 11 \Rightarrow |22a - 88| = 66$

1) $22a - 88 = 66 \Rightarrow 22a = 154 \Rightarrow a = 7$

2) $22a - 88 = -66 \Rightarrow 22a = 22 \Rightarrow a = 1$

3. $\vec{a}, \vec{b}$ vectori necoliniari



$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) \perp \vec{a}$

4. $d_1: x + 2y + 3 = 0$

$d_2: x + 2y - 7 = 0$

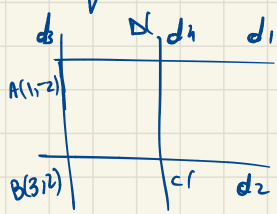
$d_3: 2x - y - 4 = 0$

Fie $d: ax + by + c = 0$ cea de-a patra latură

$$\left.\begin{aligned} \frac{a_1}{a_2} &= \frac{1}{1} = 1 \\ \frac{b_1}{b_2} &= \frac{2}{2} = 2 \\ \frac{c_1}{c_2} &= \frac{-3}{-7} \end{aligned}\right\} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow d_1 \parallel d_2 \Rightarrow \begin{cases} d \perp d_1 \\ d \perp d_2 \\ d \parallel d_3 \end{cases}$$

$$d \parallel d_3 \Rightarrow \frac{a}{a_3} = \frac{b}{b_3} \neq \frac{c}{c_3} \Rightarrow \frac{a}{2} = \frac{b}{-1} \neq \frac{c}{-4}$$

$$\frac{a}{2} = \frac{b}{-1} \Rightarrow -a = 2b \Rightarrow a = -2b \Rightarrow d: -2bx + by + c = 0$$



Fie $A = d_1 \cap d_3$

$$\Rightarrow \left.\begin{aligned} &x_A + 2y_A + 3 = 0 \,|\cdot 2 \Rightarrow 2x_A + 4y_A + 6 = 0 \\ &2x_A - y_A - 4 = 0 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow 5y_A + 10 = 0 \Rightarrow y_A = -2 \Rightarrow x_A - 4 + 3 = 0 \Rightarrow x_A = 1$$

$B = d_2 \cap d_3$

$$\Rightarrow \left.\begin{aligned} &x_B + 2y_B - 7 = 0 \,|\cdot 2 \Leftrightarrow 2x_B + 4y_B - 14 = 0 \\ &2x_B - y_A - 4 = 0 \end{aligned}\right\} \Rightarrow 5y_B - 10 = 0 \Rightarrow y_B = 2 \Rightarrow x_B + 4 - 7 = 0$$

$$\Rightarrow x_B = 3$$

$$AB = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = 2\sqrt{5}$$

Var1: $2x-y+6=0 \Rightarrow \begin{cases} 2x_\Delta - y_\Delta + 6 = 0 \\ x_\Delta + 2y_\Delta + 3 = 0 \mid \cdot 2 \Leftrightarrow 2x_\Delta + 4y_\Delta + 6 = 0 \end{cases} \Rightarrow$

$\Rightarrow 5y_\Delta = 0 \Rightarrow y_\Delta = 0 \Rightarrow x_\Delta = -3$ $\qquad A(1,-2)$

$\Rightarrow A\Delta = \sqrt{4^2+2^2} = 2\sqrt{20} \Rightarrow$ r. corect.

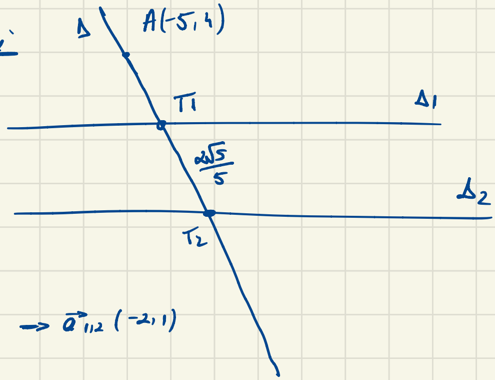
Var2: $2x-y+8=0 \Rightarrow \begin{cases} x_\Delta + 2y_\Delta + 3 = 0 \rightarrow 2x_\Delta + 4y_\Delta + 6 = 0 \\ 2x_\Delta - y_\Delta + 8 = 0 \end{cases} \Rightarrow 5y_\Delta - 14 = 0$

$\Rightarrow y_\Delta = \frac{14}{5}$

Var 3: $2x - y - 10 = 0 \rightarrow \begin{cases} 2x_\Delta + 4y_\Delta + 6 = 0 \\ 2x_\Delta - y_\Delta - 10 = 0 \end{cases}$ ...

<u>Cu încercări</u>

5.



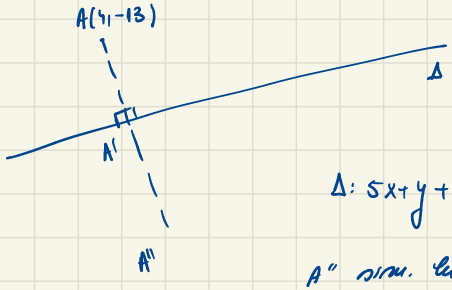
$\Delta_1: x+2y+1=0$
$\Delta_2: x+2y-1=0$ $\Rightarrow \vec{a}_{1,2}(-2,1)$

$\left.\begin{array}{l} y=0 \rightarrow B(-1,0) \in \Delta_1 \\ d(B,\Delta_2) = \frac{|-1-1|}{\sqrt{1^2+2^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \end{array}\right\} \Rightarrow \Delta \perp \Delta_1$ și $\Delta \perp \Delta_2 \Rightarrow$

$\Rightarrow \Delta \perp \vec{a}(-2,1) \Rightarrow \left.\begin{array}{l} -2x + y + c = 0 \\ A(-5,4) \in \Delta \end{array}\right\} \Rightarrow 10 + 4 + c = 0 \Rightarrow c = -14$

$\Rightarrow \Delta: -2x + y - 14 = 0$

6. Simetricul pct. $A(4,-13)$ relativ la dreapta $5x+y+6=0$ este:



$\Delta: 5x+y+6=0 \rightarrow \vec{n}(5,1)$

A'' sim. lui A față de $\Delta \Rightarrow AA'' \perp \Delta \Rightarrow AA'' \parallel \vec{n}(5,1)$

$\Rightarrow \left.\begin{array}{l} AA'': x-5y+c=0 \\ A(4,-13)\in AA'' \end{array}\right\} \Rightarrow 4+65+c=0 \Rightarrow c=-69$

$\Rightarrow AA'': x-5y-69=0$

Fie $A' = AA'' \cap \Delta \rightarrow A'$ - mijl. $[AA''] \Rightarrow \begin{cases} x_{A'} = \dfrac{x_A + x_{A''}}{2} \\ y_{A'} = \dfrac{y_A + y_{A''}}{2} \end{cases}$

$\left.\begin{array}{l} 5x_{A'} + y_{A'} + 6 = 0 \mid \cdot 5 \Rightarrow 25x_{A'} + 5y_{A'} + 30 = 0 \\ x_{A'} - 5y_{A'} - 69 = 0 \end{array}\right\} \overset{\oplus}{\Rightarrow}$

$26x_{A'} - 39 = 0 \Rightarrow x_{A'} = \dfrac{39}{26} = \dfrac{3}{2} \Rightarrow \dfrac{15}{2} + y_{A'} + 6 = 0$

$\Rightarrow y_{A'} = -6 - \dfrac{15}{2} = -\dfrac{27}{2}$

$\Rightarrow \dfrac{3}{2} = \dfrac{x_A + x_{A''}}{2} \Rightarrow 6 = 2\cdot 4 + 2x_{A''} \Rightarrow x_{A''} = -1$

$-\dfrac{27}{2} = \dfrac{y_A + y_{A''}}{2} \Rightarrow -54 = -26 + 2y_{A''} \Rightarrow y_{A''} = -14$

$\Rightarrow A''(-1,-14)$

7. $P(-3,5,2) \in \Delta$

$\sphericalangle(\Delta, Ox) = \sphericalangle(\Delta, Oy) = \sphericalangle(\Delta, Oz) = \varphi$

Fie $\vec{a_\Delta}(x_0, y_0, z_0)$

$\cos\varphi = \dfrac{\vec{a}\cdot\vec{i}}{\|\vec{a}\|\,\|\vec{i}\|} = \dfrac{x_0}{\sqrt{x_0^2+y_0^2+z_0^2}} = \dfrac{y_0}{\sqrt{x_0^2+y_0^2+z_0^2}} = \dfrac{z_0}{\sqrt{x_0^2+y_0^2+z_0^2}} \Rightarrow x_0 = y_0 = z_0$

$\Rightarrow$ Fie $\vec{a_\Delta}(1,1,1) \Rightarrow \Delta: \begin{cases} x = -3+t \\ y = 5+t \\ z = 2+t \end{cases} \Rightarrow d = \dfrac{\|(1,-2,-1)\times(1,1,1)\|}{\sqrt{3}}$

$A(-2,3,1)$

$$\begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = -\vec{i} + 2\vec{j} + 3\vec{k}$$

$$\Rightarrow d = \sqrt{\frac{14}{3}}$$

8. $\Delta_1: \frac{x-5}{0} = \frac{y}{3-a} = \frac{z}{-2}$

coplanare

$\Delta_2: \frac{x-a}{0} = \frac{y}{-1} = \frac{z}{2-a}$

$\vec{a}_{\Delta_1} (0, 3-a, -2)$

$a_{\Delta_2} (0, -1, 2-a)$

$M_1(5,0,0)$ $\Rightarrow \overrightarrow{M_1M_2}(a-5, 0, 0)$

$M_2(a,0,0)$

$$(\overrightarrow{M_1M_2}, \vec{a}_{\Delta_1}, \vec{a}_{\Delta_2}) = \begin{vmatrix} a-5 & 0 & 0 \\ 0 & 3-a & -2 \\ 0 & -1 & 2-a \end{vmatrix} = a^3 - 10a^2 + 29a - 20$$

$$a^3 - 10a^2 + 29a - 20 = 0$$

$$a^3 - a^2 - 9a^2 + 9a + 20a - 20 = 0$$

$$\Leftrightarrow a^2(a-1) - 9a(a-1) + 20(a-1) = 0$$

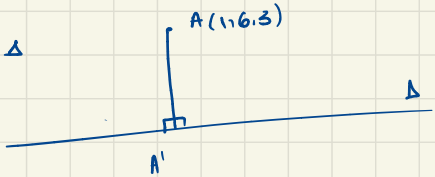
$$\Leftrightarrow (a-1)(a^2 - 9a + 20) = 0$$

$$\Delta = 81 - 4 \cdot 1 \cdot 20 = 1 \Rightarrow a_2 = \frac{9-1}{2} = 4 \qquad \Rightarrow a \in \{1, 4, 5\}$$

$$a_3 = \frac{9+1}{2} = 5$$

9. Proiecția punctului $A(1,6,3)$ pe dreapta $\Delta: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ este punctul de coordonate?

R: Fie $A'(x_{A'}, y_{A'}, z_{A'})$ proiecția $\Rightarrow AA' \perp \Delta$



$$\Delta: \begin{cases} x = t \\ y = 1 + 2t \\ z = 2 + 3t \end{cases} \qquad \vec{a}_\Delta (1,2,3)$$

$\Rightarrow AA' \perp \vec{a}_\Delta$

$\Rightarrow \overrightarrow{AA'} \cdot \vec{a}_\Delta = 0$

$AA' = (x_{A'} - 1, y_{A'} - 6, z_{A'} - 3)$

$\Rightarrow x_{A'} - 1 + 2y_{A'} - 12 + 3z_{A'} - 9 = 0$

$\Rightarrow x_{A'} + 2y_{A'} + 3z_{A'} - 22 = 0$

$$A' \in \Delta \Rightarrow \frac{x_{A'}}{1} = \frac{y_{A'}-1}{2} = \frac{z_{A'}-2}{3}$$

$$\Rightarrow \begin{cases} x_{A'} = \frac{y_{A'}-1}{2} \\ x_{A'} = \frac{z_{A'}-2}{3} \end{cases} \Rightarrow \frac{y_{A'}-1}{2} = \frac{z_{A'}-2}{3} \Rightarrow 2z_{A'} - 4 = 3y_{A'} - 3$$

$$\Rightarrow z_{A'} = \frac{3y_{A'}+1}{2}$$

$$\Rightarrow \frac{y_{A'}-1}{2} + 2y_{A'} + \frac{9y_{A'}+3}{2} - 22 = 0 \, / \cdot 2$$

$$\Leftrightarrow y_{A'} - 1 + 4y_{A'} + 9y_{A'} + 3 - 44 = 0$$

$$\Leftrightarrow 14y_{A'} - 42 = 0 \Rightarrow y_{A'} = 3 \Rightarrow x_{A'} = \frac{3-1}{2} = 1$$

$$z_{A'} = \frac{3 \cdot 3 + 1}{2} = 5$$

Model 3

1. $A(2,2,3)$, $B(1,0,4)$, $C(2,3,5)$

$\vec{AB}(-1,-2,1)$

$\vec{AC}(0,1,2)$

$\vec{BC}(1,3,1)$

$$\vec{BC} \times \vec{AB} = \begin{vmatrix} i & j & k \\ 1 & 3 & 1 \\ -1 & -2 & 1 \end{vmatrix} = 5\vec{i} - 2\vec{j} + \vec{k}$$

$$\vec{AB} + \vec{AC} = (-1,-1,3)$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ -1 & -1 & 3 \\ 5 & -2 & 1 \end{vmatrix} = 5\vec{i} + 16\vec{j} + 7\vec{k}$$

3. $\Delta_1 : \frac{x-6}{3} = \frac{y-3}{-2} = \frac{z+3}{4} \Rightarrow M_1(6,3,-3)$ $\quad \vec{a_1}(3,-2,4)$

$\Delta_2 : \frac{x+1}{3} = \frac{y+7}{-3} = \frac{z-4}{8} \Rightarrow M_2(-1,-7,4)$ $\quad \vec{a_2}(3,-3,8)$

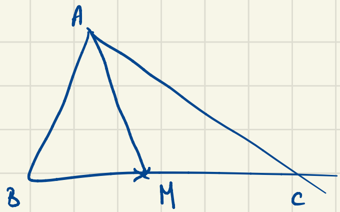
$\vec{M_1M_2}(-7,-10,7)$

$$(\vec{M_1M_2}, \vec{a_1}, \vec{a_2}) = \begin{vmatrix} -7 & -10 & 7 \\ 3 & -2 & 4 \\ 3 & -3 & 8 \end{vmatrix} = 127 \neq 0 \Rightarrow \text{drepte strâmbe}$$

$$\Rightarrow d(\Delta_1, \Delta_2) = \frac{|127|}{13} = \frac{127}{13}$$

$$\vec{a_1} \times \vec{a_2} = \begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ 3 & -3 & 8 \end{vmatrix} = -4\vec{i} - 12\vec{j} - 3\vec{k} \Rightarrow \sqrt{16+144+9} = \sqrt{144+25} = \sqrt{169} = 13$$

4.



$\vec{BC} = \vec{BA} + \vec{AC} = -\vec{AB} + \vec{AC} =$
$= (3,0,-4) + (5,-2,4)$
$= (8,-2,0)$

$\vec{AM} = \vec{AB} + \vec{BM}$
$\vec{AM} = \vec{AC} + \vec{CM}$ (+)

$2\vec{AM} = \vec{AB} + \vec{AC} + \underbrace{\vec{BM} + \vec{CM}}_{0} = \vec{AB} + \vec{AC} = (-3,0,4) + (5,-2,4) = (2,-2,8) \Rightarrow$
$\Rightarrow \vec{AM} = (1,-1,4)$

$\|AM\|^2 = \vec{AM} \cdot \vec{AM} = 1 + 1 + 16 = 18 \Rightarrow \vec{AM} = \sqrt{18} = 3\sqrt{2}$

5. $d_1 : x + 2y + 3 = 0$
$d_2 : x + 2y - 7 = 0$
$d_3 : 2x - y - 4 = 0$

$\frac{a_1}{a_2} = 1 \quad \frac{b_1}{b_2} = 1 \Rightarrow d_1 \parallel d_2$

Fie $A = d_1 \cap d_3 \Rightarrow \begin{cases} x_A + 2y_A + 3 = 0 \,|\cdot 2 \Leftrightarrow 2x_A + 4y_A + 6 = 0 \\ 2x_A - y_A - 4 = 0 \end{cases} \overset{\ominus}{\Rightarrow}$

$\Rightarrow 5y_A + 10 = 0 \Rightarrow \underline{y_A = -2} \Rightarrow x_A = -3 + 4 = 1 \Rightarrow A(1,-2)$

Fie $B = d_2 \cap d_3 \Rightarrow \begin{cases} x_B + 2y_B - 7 = 0 \,|\cdot 2 \Leftrightarrow 2x_B + 4y_B - 14 = 0 \\ 2x_B - y_B - 4 = 0 \end{cases} \overset{\ominus}{\Rightarrow}$

$\Rightarrow 5y_B - 10 = 0 \Rightarrow y_B = 2 \Rightarrow x_B = 7 - 4 = 3 \Rightarrow B(3,2)$

$AB = \sqrt{(3-1)^2 + (2-(-2))^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ ... încercări

6. $r = (1 + u - v) \cdot \vec{i} + (2 - u) \cdot \vec{j} + (3 - 2u + 2v) \cdot \vec{k}$

$\begin{cases} x = 1 + u - v \\ y = 2 - u \\ z = 3 - 2u + 2v \end{cases}$

$u = 2 - y \Rightarrow v = 1 + u - x = 1 + 2 - y - x = -x - y + 3$

$z = 3 - 2(2-y) + 2(x-y+3) \Leftrightarrow z = 3 - 4 + \cancel{2y} - 2x - \cancel{2y} + 6 \Leftrightarrow$
$\Leftrightarrow z = 5 - 2x \rightarrow \pi: \underline{2x + z - 5 = 0}$

7. $\Delta: \frac{x-1}{1} = \frac{y-2}{2} = \frac{z+1}{3}$

$\Rightarrow \Delta: r = (1,2,-1) + t(1,2,3)$

$4 \cdot 7 = 28 \cdot$
$\frac{8}{4}$

$OA = \sqrt{6} \rightarrow x_A^2 + y_A^2 + z_A^2 = 6$
$x_A - 1 = \frac{y_A - 2}{2} / \cdot 2$

$\Leftrightarrow 2x_A - 2 = y_A - 2 \Leftrightarrow y_A = 2x_A$
$x_A - 1 = \frac{z_A + 1}{3} \Leftrightarrow 3x_A - 3 = z_A + 1 \Rightarrow z_A = 3x_A - 4$

$\Rightarrow x_A^2 + 4x_A^2 + 9x_A^2 - 24x_A + 16 = 6$
$\Leftrightarrow 14x_A^2 - 24x_A + 10 = 0 \Leftrightarrow 7x_A^2 - 12x_A + 5 = 0$
$\Delta = 144 - 4 \cdot 7 \cdot 5 = 4$

$\Rightarrow \begin{cases} x_{A_1} = \frac{12-2}{14} = \frac{10}{14} = \frac{5}{7} \Rightarrow y_{A_1} = \frac{10}{7} \Rightarrow a) \\ x_{A_2} = \frac{12+2}{14} = 1 \Rightarrow y_{A_2} = 2 \rightarrow c) \end{cases}$

Modelul 4

1. $\pi_1: 2x - y + 2z - 2 = 0$
$\pi_2: x - 2y + z + 4 = 0$
$\pi_3: x + \lambda y + z - 4 = 0$

Planele sunt fețele laterale ale unei prisme $\Rightarrow$ unghiul dintre oricare două este egal

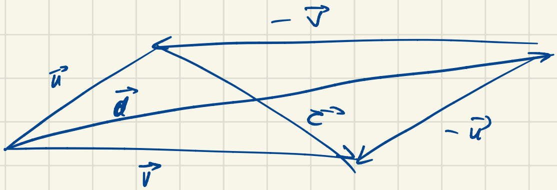
$\cos(\widehat{\pi_1, \pi_2}) = \frac{2+2+2}{\sqrt{4+1+4} \cdot \sqrt{1+4+1}} = \frac{6}{\sqrt{9} \cdot \sqrt{6}} = \frac{6}{3 \cdot \sqrt{6}} = \frac{6\sqrt{6}}{3 \cdot 6} = \frac{\sqrt{6}}{3}$ $\Rightarrow$

$\cos(\widehat{\vec{n_2},\vec{n_3}}) = \frac{1-2\lambda+1}{\sqrt{1+4+1}\cdot\sqrt{1+\lambda^2+1}} = \frac{2-2\lambda}{\sqrt6\cdot\sqrt{\lambda^2+2}} = \frac{\cancel{2}\sqrt6(1-\lambda)}{\cancel{6}_3\sqrt{\lambda^2+2}} = \frac{(1-\lambda)\sqrt6}{3\sqrt{\lambda^2+2}}$

$\Rightarrow \frac{(1-\lambda)\cancel{\sqrt6}}{\cancel{3}\sqrt{\lambda^2+2}} = \frac{\cancel{\sqrt6}}{\cancel{3}}$

$\Rightarrow (1-\lambda) = \sqrt{\lambda^2+2}\ \big|^2 \Leftrightarrow 1-2\lambda+\cancel{\lambda^2} = \cancel{\lambda^2}+2 \Rightarrow -2\lambda = 1 \Rightarrow \lambda = -\frac{1}{2}$

2.



Fie $\vec d$ diagonală $\vec d = \vec u + \vec v \Rightarrow \vec d(3,6,-2)$

$\vec t(t_1,t_2,t_3)$ vectorul căutat $t_1^2+t_2^2+t_3^2=1$

$\vec t \parallel \vec d \Rightarrow (\exists)\lambda\in\mathbb{R}$ a.î. $\vec t=\lambda\vec d \Rightarrow \frac{t_1}{3}=\frac{t_2}{6}=\frac{t_3}{-2}$

$\to \begin{cases} t_2 = 2t_1 \\ t_3 = \frac{-2t_1}{3}\end{cases} \Rightarrow t_1^2+4t_1^2+\frac{4t_1^2}{9} = 1 \ \big|\cdot 9$

$\Leftrightarrow 9t_1^2+36t_1^2+4t_1^2=9$

$\Leftrightarrow 49t_1^2 = 9 \Rightarrow t_1^2=\frac{9}{49} \Rightarrow t_1=\frac{3}{7} \Rightarrow t_2=\frac{6}{7} \Rightarrow t_3 = -\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{7} = -\frac{2}{7}$

$\Rightarrow t = \frac{1}{7}(3\vec i+6\vec j-2\vec k) \Rightarrow$ b)

Fie $\vec c$ diagonala $\Rightarrow \vec c = -\vec u + -\vec v = (-3,-6,2)$

Fie $\vec s(s_1,s_2,s_3)$ vectorul căutat $\Rightarrow s_1^2+s_2^2+s_3^2=1$

$\frac{s_1}{-3}=\frac{s_2}{-6}=\frac{s_3}{2}$

$\Rightarrow \begin{cases} s_2=2s_1\\ s_3=-\frac{2}{3} s_3\end{cases} \to$ același rez.

3. $\begin{cases} 4x-4y-z+11=0 \\ x+2y-z-1=0 \end{cases}$ $\quad \vec{n_1}(4,-4,-1)$, $\vec{n_2}(1,2,-1)$

$\vec{a}=\vec{n_1}\times\vec{n_2}$

$$\vec{n_1}\times\vec{n_2}=\begin{vmatrix} i & j & k \\ 4 & -4 & -1 \\ 1 & 2 & -1 \end{vmatrix}=6\vec{i}+3\vec{j}+12\vec{j} \Rightarrow \vec{a}(2,1,4)$$

Fie $M(0,2,z_M)$ $\quad M\in\Delta \Rightarrow \begin{cases} -8-z_M+11=0 \Leftrightarrow -z_M=-3 \Rightarrow z_M=3 \\ 4-z_M-1=0 \Leftrightarrow -z_M=-3 \Leftrightarrow z_M=3 \end{cases} \Rightarrow$

$\Rightarrow M(0,2,3)\in\Delta \Rightarrow \Delta: \dfrac{x}{2}=\dfrac{y-2}{1}=\dfrac{z-3}{4}$

4. $A(-5,0)$, $B(3,0)$

$A_{\Delta ABC}=20$

$C\in\Delta: \begin{cases} x=1+t \\ y=-1+t, \ t\in\mathbb{R} \end{cases}$ $\quad C=?$

$C\in\Delta \Rightarrow C\in\Delta: \dfrac{x-1}{1}=\dfrac{y+1}{1} \Rightarrow x_C-1=y_C+1 \Rightarrow y_C=x_C-2$

$\Rightarrow x_C=y_C+2$

$$A_{\Delta ABC}=\frac{1}{2}\begin{vmatrix} -5 & 0 & 1 \\ 3 & 0 & 1 \\ y_C+2 & y_C & 1 \end{vmatrix}$$

$$\begin{vmatrix} -5 & 0 & 1 \\ 3 & 0 & 1 \\ y_C+2 & y_C & 1 \end{vmatrix}=3y_C+5y_C=8y_C$$

$\Rightarrow 20=\frac{1}{2}8y_C \Leftrightarrow 4y_C=20$

$\Rightarrow y_C=5$

$\Rightarrow x_C=7$

$\Rightarrow C(7,5)$

$\Rightarrow d)$

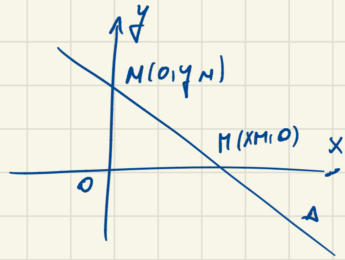
5. $\Delta: y-y_0=m(x-x_0)$, $M_0(8,6)$ $\Rightarrow \Delta: y-6=m(x-8) \Leftrightarrow \Delta: y-6=mx-8m \Leftrightarrow$

$\Leftrightarrow \Delta: mx-y-8m+6=0$

Fie $M(x_M,0)=\Delta\cap Ox$ şi $N(0,y_N)=\Delta\cap Oy$

$\Rightarrow x_M \cdot y_N = 24$

$$\left.\begin{array}{l} M\in\Delta \Rightarrow m x_M - 8m + 6 = 0 \Rightarrow x_M = \dfrac{8m-6}{m} \\ N\in\Delta \Rightarrow -y_N - 8m + 6 = 0 \Rightarrow y_N = 6-8m \end{array}\right\} \Rightarrow$$



$$\Rightarrow \frac{8m-6}{m}\cdot(6-8m) = 24 \;|\cdot m \Leftrightarrow \cancel{2}(4m-3)\cdot\cancel{2}(3-4m) = \cancel{24}^{\,6}\, m$$

$$\Leftrightarrow -(4m-3)^2 = 6m \Leftrightarrow -16m^2 + 24m - 9 = 6m \Leftrightarrow$$

$$\Leftrightarrow 16m^2 - 18m + 9 = 0 \qquad \Delta = 18^2 - 4\cdot 16\cdot 9 =$$

6. $b\times(a+b-a\times b)$ cu $a(1,-1,1)$, $b = 2i + j - 3k$

$$a\times b = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix} = 2\vec{i} + 5\vec{j} + 3\vec{k}$$

$$a+b = (3,0,-2)$$

$$a+b-a\times b = (3,0,-2) - (2,5,3) = (1,-5,-5)$$

$$b\times(a+b-a\times b) = \begin{vmatrix} i & j & k \\ 2 & 1 & -3 \\ 1 & -5 & -5 \end{vmatrix} = -20\vec{i} + 7\vec{j} - 9\vec{k}$$

7. $\Delta_1: \dfrac{x-1}{2} = \dfrac{y+2}{-3} = \dfrac{z-5}{4} \Rightarrow \begin{array}{l}\vec{a_1}(2,-3,4) \\ M_1(1,-2,5)\end{array}$

$\Delta_2: \dfrac{x-7}{3} = \dfrac{y-2}{2} = \dfrac{z-1}{-2} \Rightarrow \left\{\begin{array}{l}\vec{a}(3,2,-2) \\ M_2(7,2,1)\end{array}\right.$

$\overrightarrow{M_1M_2} = (6,4,-4)$

$$(\overrightarrow{M_1M_2}, \vec{a_1}, \vec{a_2}) = \begin{vmatrix} 6 & 4 & -4 \\ 2 & -3 & 4 \\ 3 & 2 & -2 \end{vmatrix} = \cancel{36} + \cancel{48} - \cancel{16} - \cancel{36} - \cancel{48} + \cancel{16} = 0 \Rightarrow$$ nu sunt coplanare

Modelul 3/ ex2

$$\Delta: \left.\begin{array}{l} y - y_0 = m(x-x_0) \\ M_0(2,4) \end{array}\right\} \Rightarrow y-4 = m(x-2) \Leftrightarrow y-4 = mx-2m \Leftrightarrow \Delta: mx - y - 2m + 4 = 0$$

$$d(A,\Delta) = \frac{|-3-2m+4|}{\sqrt{m^2+1^2}} = 1 \Leftrightarrow |-2m+1| = \sqrt{m^2+1} \;\Big|^2$$

c) $4m^2-4m+1 = m^2+1 \Leftrightarrow 3m^2-4m=0 \Leftrightarrow m(3m-4)=0 \Rightarrow m=0$ sau $m=\frac{4}{3}$

$m=0 \Rightarrow \Delta: -y+4=0 \Leftrightarrow \Delta: y=4$

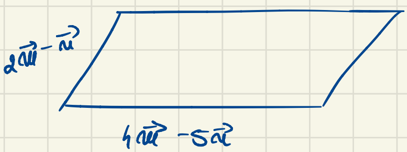
sau $\Delta: \frac{4}{3}x - y - \frac{8}{3}+4=0 /\cdot 3 \Leftrightarrow 4x-y-8+4=0 \Leftrightarrow 4x-y-4=0$

V: $d(A,\Delta_1)=\frac{|3-4|}{\sqrt{1^2}}=\frac{+1}{1}=1$

$d(A,\Delta_2)=\frac{|-1+4|}{\sqrt{1^2+1^2}} = \frac{3}{3}=1$

Subiect azi

1.



$A_{\square} = \|(2\vec{m}-\vec{n})\times(4\vec{m}-5\vec{n})\| = \|2\vec{m}-\vec{n}\|\cdot\|4\vec{m}-5\vec{n}\|\cdot \sin(\widehat{2\vec{m}-\vec{n}, 4\vec{m}-5\vec{n}})$

$\|2\vec m-\vec n\|^2 = (2\vec m-\vec n)\cdot(2\vec m-\vec n) = 4\underbrace{\vec m^2}_{1} - 4\vec m\cdot\vec n + \underbrace{\vec n^2}_{1} = 5-4\cdot\cos(\widehat{\vec m,\vec n}) = 5-4\cdot\frac{\sqrt2}{2} =$

$= 5-2\sqrt2 \Rightarrow \|2\vec m-\vec n\| = \sqrt{5-2\sqrt2}$

$\|4\vec m-5\vec n\| = (4\vec m-5\vec n)\cdot(4\vec m-5\vec n) = 16\vec m^2 - 40\vec m\cdot\vec n + 25\vec n^2 = 16-40\frac{\sqrt2}{2}+25 =$

$=41-40\frac{\sqrt2}{2} \Rightarrow \|4\vec m-5\vec n\| = \sqrt{41-40\frac{\sqrt2}{2}} = \sqrt{41-20\sqrt2}$

$\cos(2\vec m-\vec n, 4\vec m-5\vec n) = \frac{(2\vec m-\vec n)(4\vec m-5\vec n)}{\|2\vec m-\vec n\|\cdot\|4\vec m-5\vec n\|} = \frac{8\vec m^2-4\vec n\vec m-10\vec m\vec n+5\vec n^2}{\sqrt{5-2\sqrt2}\cdot\sqrt{41-20\sqrt2}} =$

$= \frac{13-7\sqrt2}{\sqrt{5-2\sqrt2}\cdot\sqrt{41-20\sqrt2}}$

2. $\Delta_1: 3x-2y-1=0 \quad \vec a(2,3)$

$\Delta_2: \frac{x-1}{2}=\frac{y+5}{3} \Leftrightarrow 3x-3=2y+10 \Leftrightarrow 3x-2y-13=0$

$\Delta: 3x-2y+c=0$

$$d(\Delta_1,\Delta) = \frac{|c+1|}{\sqrt{3^2+2^2}} = \frac{|c+1|}{\sqrt{13}}$$
$$d(\Delta_2,\Delta) = \frac{|c+13|}{\sqrt{3^2+2^2}} = \frac{|c+13|}{\sqrt{13}}$$
$$\Rightarrow |c+1| = |c+13| \Rightarrow c+1 = \pm(c+13)$$
$$c+1 = c+13 \Rightarrow 1 = 13\ F$$
$$\Rightarrow c+1 = -c-13 \Rightarrow 2c = -14 \Rightarrow c = -7$$

$$\Rightarrow \Delta: 3x-2y-7=0$$

3. $\begin{cases} 2x-2y+z+3=0 \\ 3x-2y+2z+17=0 \end{cases}$ $\quad \vec{n_1}(2,-2,1)$, $\vec{n_2}(3,-2,2)$

$$\vec{a} = \vec{n_1}\times\vec{n_2}$$
$$\vec{n_1}\times\vec{n_2} = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 3 & -2 & 2 \end{vmatrix} = -2\vec{i} - \vec{j} + 2\vec{k} \Rightarrow \vec{a}(-2,-1,2)$$

Fie punctul $M(1, y_M, z_M) \Rightarrow \begin{cases} 2-2y_M+z_M+3=0 \\ 3-2y_M+2z_M+17=0 \end{cases} \Leftrightarrow \begin{cases} -2y_M+z_M+5=0 \\ -2y_M+2z_M+20=0 \end{cases} \ominus$

$$\Leftrightarrow z_M+15=0 \Rightarrow z_M=-15$$
$$-2y_M-15+5=0 \Rightarrow -2y_M=10 \Rightarrow y_M=5 \Rightarrow M(1,5,-15) \qquad A(2,3,-1)$$

$$\Rightarrow \vec{MA}(1,8,14)$$

$$d(A,\Delta) = \frac{\|\vec{MA}\times\vec{a}\|}{\|\vec{a}\|}$$

$$\vec{MA}\times\vec{a} = \begin{vmatrix} i & j & k \\ 1 & 8 & 14 \\ -2 & -1 & 2 \end{vmatrix} = 15(2\vec{i} - 2\vec{j} + \vec{k}) \Rightarrow \begin{cases} \|\vec{MA}\times\vec{a}\| = 5\sqrt{9} = 5\cdot 3 = 15 \\ \|\vec{a}\| = \sqrt{9} = 3 \end{cases} \Rightarrow$$

$$\Rightarrow d(A,\Delta) = 3$$

4. $$(\vec{a},\vec{b},\vec{c}) = \begin{vmatrix} x & x-1 & x-2 \\ x-3 & x-4 & x-5 \\ x-6 & x-7 & x-8 \end{vmatrix} = 0$$

$$\Delta_1 : \begin{cases} 3x + y + z + 3 = 0 \\ x + y - z + 7 = 0 \end{cases}$$

$$\Delta_2 : \begin{cases} x = -3 - 4t \\ y = -4 + 4t \\ z = 3 + t \end{cases} \Rightarrow \vec{a_2}(-1, 1, 1) \text{ și } M_2(-3, -4, 3)$$

Δ_1: $\vec{u_1}(3, 1, 1)$, $\vec{u_2}(1, 1, -1)$

$$\vec{a_1} = \vec{u_1} \times \vec{u_2} = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -2\vec{i} + 4\vec{j} + 2\vec{k} \Rightarrow \vec{a_1}(-1, 2, 1)$$

Fie $M_1(1, y_1, z_1) \Rightarrow \begin{cases} 1 + y_1 + z_1 + 3 = 0 \\ 1 + y_1 - z_1 + 7 = 0 \end{cases} \oplus$

$$2y_1 + 12 = 0 \Rightarrow y_1 = -6 \Rightarrow z_1 = -4 + 6 = 2$$

$$\Rightarrow M_1(1, -6, 2) \in \Delta_1$$

$\vec{M_1M_2}(-4, 2, 1)$

$$(\vec{M_1M_2}, \vec{a_1}, \vec{a_2}) = \begin{vmatrix} -4 & 2 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} = -3 \Rightarrow \Delta_1, \Delta_2 \text{ necoplanare}$$

$$\vec{a} = \vec{a_1} \times \vec{a_2} = \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \vec{i} + \vec{k} \Rightarrow a(1, 0, 1)$$

$M_1(1, -6, 2) \in \pi_1$

$\pi_1 \parallel a_1(-1, 2, 1)$

$\pi_2 \parallel a(-1, 1, 1)$

$$\Rightarrow \begin{vmatrix} x-1 & y+6 & z-2 \\ -1 & 2 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0 \Leftrightarrow \cancel{x+z+3=0} \Leftrightarrow \ldots$$

Analog $(M_2, \vec{a_2}, \vec{a})$

$$\pi : (M_1, \vec{a_1}, \vec{a_2}) = \begin{vmatrix} x-1 & y+6 & z-2 \\ -1 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 0 \Leftrightarrow x + z - 3 = 0$$

$$d(\Delta_1, \Delta_2) = d(M_2, \pi) = \frac{|-3 + 3 - 3|}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$ ♡.

$$\Delta: \frac{x-2}{3} = \frac{y+1}{2} = \frac{z-3}{-2}$$

$$\Delta \in \pi: ax + y - 2z + d = 0 \Rightarrow \vec{n}(a, 1, -2)$$

$$\Rightarrow M(2,-1,3) \in \Delta$$

$$\vec{a}(3,2,-2)$$

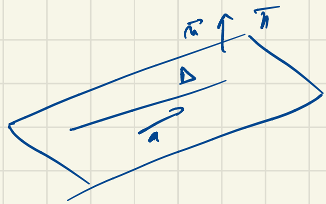
$$\Delta \in \pi \Rightarrow M \in \pi \Rightarrow 2a - 1 - 6 + d = 0 \Leftrightarrow 2a + d - 7 = 0$$

$$\Delta \in \pi \Rightarrow \pi \parallel \Delta \Rightarrow \pi \parallel (3,2,-2)$$

$$\vec{n} \perp \vec{a} \Rightarrow \vec{n} \cdot \vec{a} = 0$$

$$\Rightarrow 3a + 2 + 4 = 0$$

$$\Leftrightarrow a - 2$$



$\Rightarrow a$

$$d_1: \begin{cases} x - y + z + 1 = 0 \\ 2x - y - z + 2 = 0 \end{cases}$$

$$(\overrightarrow{M_1M_2}, \vec{a_1}, \vec{a_2})$$

$$d_2: \begin{cases} 3x + y + z = 0 \\ x + y - 2z - 1 = 0 \end{cases}$$

$$\Delta: \frac{x-1}{1} = \frac{y-2}{2} = \frac{z+1}{3} \Rightarrow M(1,2,-1) \quad \vec{a}(1,2,3)$$

$$d(O, M_0) = \sqrt{6}$$

$$\Rightarrow \frac{\|\overrightarrow{OM_0} \times \vec{a}\|}{\|\vec{a}\|}$$

$$\overrightarrow{OM_0}(x_0, y_0, z_0)$$

$$\begin{vmatrix} i & j & k \\ x_0 & y_0 & z_0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\pi_1: 4x - 4y - z + 11 = 0$$

$$\pi_2: x + 2y - z - 1 = 0$$

$$d: \begin{cases} 4x - 4y - z + 11 = 0 \\ x + 2y - z - 1 = 0 \end{cases}$$

$$\vec{n_1}(4,-4,-1)$$

$$\vec{n_2}(1,2,-1)$$