Complex Operations

Addition

Algebraic Interpretation

Let $s, t, x, y \in \mathbb{R}$ and let $z, w \in \mathbb{C}$ such that $z = (x, y)$ and $w = (s, t)$ then,

$$z + w = (x + s, y + t)$$

We could also write:

$$z + w = (x + yi) + (s + ti) = (x + s) + (y + t)i$$

The above forms are equivalent.

Geometric Interpretation

The geometric form of complex number addition gives us a better intuition for what these numbers really mean.

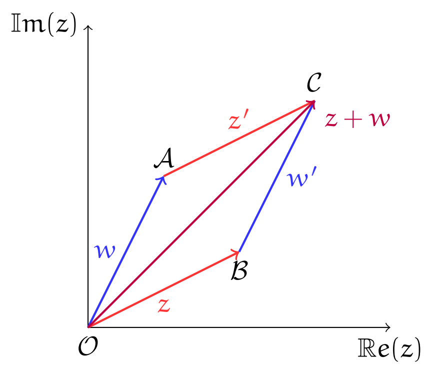

Figure 1: Addition over Complex Numbers

$$\begin{aligned} z &= (x, y) = x + yi \\ w &= (s, t) = s + ti \\ v = z + w &= (x + s) + (y + t)i \end{aligned}$$

Notice:
We have constructed a parallelogram $\mathcal{O}ACB$ with vertices O, A, B, C where C is the point created by adding points A and B component-wise.

We have also constructed two congruent triangles ΔOAC and ΔOBC which share a hypotenuse OC whose corresponding sides OA and BC as well as AC and OB are of equal length (respectively). The shared hypotenuse of these triangles (OC) has a length which is the sum of the lengths of z and w.

Subtraction

Subtraction of complex numbers is just a sign change for the number being subtracted.

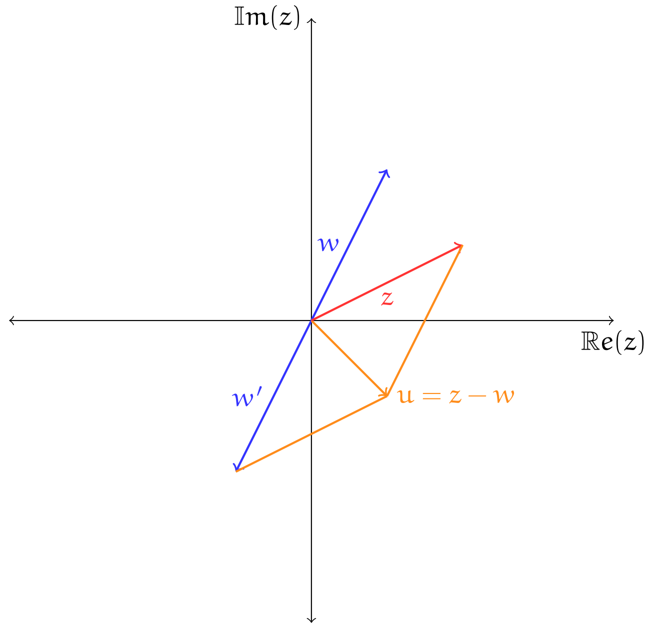

Figure 2: Subtraction over Complex Numbers

Algebraic Interpretation

Geometric Interpretation

Scalar Multiplication

Algebraic Interpretation

Geometric Interpretation

Multiplication

Algebraic Interpretation

Geometric Interpretation

Conjugation

Algebraic Interpretation

Geometric Interpretation

Modulus

Algebraic Interpretation

Geometric Interpretation

Forms

Cartesian

Polar

Euler

Proof Simplifications

Solutions

What does it mean to have an "imaginary" root?

What does an "imaginary" root look like?

Cubic Solutions

Rotation by i