Actividad 1.1

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1. Let X be a finite set. Give a recursive definition of the set of subsets of X (the Power set of X). Use union as the operator on the definition

$X = \{1, 2, 3\}$

$P(x) = \{ \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

Base: $\{\} \in P(x)$

Recursive Step: $x \in X$

$X = \{1,2\} \cup \{3\}$ $X = \{1\} \cup \{2\}$

$X = \{\} \cup \{1\}$

$P(x) = \{ \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

Closure: $n \in P(x)$ only if it can be obtained from the base using a finite number of the recursive step.

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2. Prove by induction on n that $\sum_{i=0}^{n} n = \frac{n(n+1)}{2}$

Basis: $0 = 0$ $\frac{0(0+1)}{2} = \frac{0}{2} = 0$

Induction hypothesis: $\frac{K(K+1)}{2}$

Inductive Step:

1. $\sum = \frac{(K+1)((K+1)+1)}{2}$

2. $0 + 1 + 2 + \ldots + K + K+1 = \frac{(K+1)(K+2)}{2}$

3. $\frac{K(K+1)}{2}$ + K + 1

(3) $\frac{K(K+1)}{2} + K + 1$

(4) $\frac{K(K+1)}{2} + \frac{2K}{2} + \frac{2}{2}$

(5) $\frac{K(K+1) + 2K + 2}{2}$

(6) $\frac{K(K+1) + 2(K+1)}{2}$

(7) $\frac{(K+1)(K+2)}{2}$

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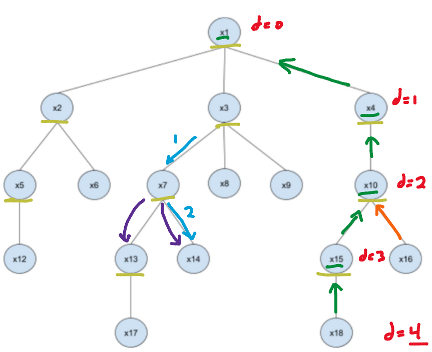
3. Using the tree below, give the values of each of the items:
   a. the depth of the tree
   b. the ancestors of x_{18}
   c. the internal nodes of the tree
   d. the length of the path from x_3 to x_{14}
   e. the vertex that is the parent of x_{16}
   f. the vertices children of x_7



a. Depth of the tree = 4

b. Ancestors of X_{18} = X_{15}, X_{10}, X_4, X_1

c. Internal nodes = $X_1, X_2, X_3, X_4, X_5, X_7, X_{10}, X_{13}, X_{15}$

d. Length from X_3 to X_{14} = 2 arcs in the Path

e. Parent of X_{16} = X_{10}

f. Children of X_7 = X_{13}, X_{14}