Technical University of Denmark

46750 - Optimization in Modern Power Systems

Assignment 1: Demand-Side Flexibility in Active Distribution Grids

Deadline: October 11, 2025 at 00:01am

Instructions and deliverables:

This assignment should be carried out in **groups of two to three students**. Each group should provide a single submission, including the following:

- **Concise report** using the recommended Latex Template, submitted in DTU Learn. The report should include:
  - Concise answers to each question, including: rigorous and compact mathematical formulations, with definition of all *new* notations and description of all *new* equations, motivation and description of relevant modeling steps, relevant numerical analysis and insights, adequate visual aids and tables.
  - Individual participation table, summarizing the contribution (in %) of each student, including coding, model formulation, writing, result analysis, data visualization, review and formatting, etc.
- **Signed group contract**, submitted in DTU Learn.
- **Working and well-documented code**, in the programming language of your choice, submitted in Github classroom.

Grading:

Each group will receive a grade between **0-100 points**, as follows:

This grade will be adjusted individually based on each student's overall participation, as reported in the participation table, and the adjustment rules illustrated in Figure 1. This individual grade will count towards **40%** of the final grade.

Learning objectives:

This assignment aims at evaluating the understanding fundamental optimization concepts, and their application to simple decision-making problems in power systems (Modeling steps 1-5). Figure 2 summarizes the learning objectives (LOs) assessed.

		Question 1	Question 2
(a)	i	6points	7points
	ii	5points	8points
	iii	6points	-
	iv	3points	-
	v	6points	-
(b)	i	4points	7points
	ii	4points	7points
	iii	4points	3points
	iv	1points	8points
	v	4points	-
(c)	i	4points	-
	ii	4points	-
	iii	4points	-
	iv	1points	-
	v	4points	-
Total		**60 points**	**40 points**

Table 1: Points repartition

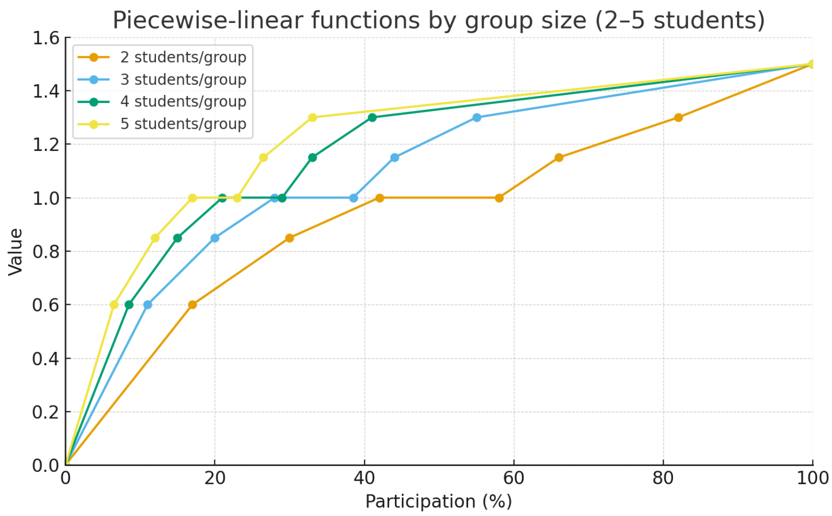

Figure 1: Individual grade adjustment ratios, depending on % of total participation and number of students/group

Data provided:

All relevant data is provided in a single file in Github classroom, including the following individual and system-level techno-economic parameters:

- Techno-economic characteristics of flexible appliances and loads: maximum and minimum hourly load (in kWh/h), hourly PV production profile (in kWh/h), marginal PV production cost (in DKK/kWh) energy storage capacity (in KWh), maximum charging/discharging power (in kWh/h) and charging/discharging efficiencies (-) of battery, and capital investment cost (in DKK/kWh of energy storage capacity).

- Flexibility preferences (multiple scenarios): minimum daily required energy consumption (in kWh), reference hourly profile of preferred energy consumptions (in kWh/h).

Learning Objective (LO):	Bloom's Taxonomy	How it is assessed:
LO1: Describe the fundamental principles and properties of convex optimization, complementarity, and optimization under uncertainty.	Understand	Demonstrate understanding of these concepts by identifying the key components (input data, decision variables, objective(s), constraints) of the optimization problems considered and their fundamental properties (e.g. feasibility, restriction, relaxation, approximation, convexity, optimality, etc.) in this context, clearly indicating where each property appears in your models (e.g., DC power flow → approximation of feasibility set) and why that matters for solvability and interpretation.
LO3: Analyze the structure of optimization models by examining how assumptions, input data and modelling choices influence robustness, feasibility, accuracy, optimality, and computational complexity.	Analyze	Demonstrate model-based reasoning by qualitatively predicting (before solving) how the structure of the consumers' optimization problems (e.g. constraints, objective, etc.) and input data (e.g. grid tariff, comfort preferences) will push the solutions, clearly identifying causal links from inputs to outputs (e.g., stronger comfort preferences → flatter load near baseline).
LO4: Formulate the dual problem and optimality conditions of linear and convex optimization problems, and explain their mathematical properties and practical implications.	Apply	Demonstrate that you can apply known techniques to form the dual formulation and derive the KKT conditions of the optimization problems considered, and state the mathematical properties (e.g., necessary and sufficient conditions for strong duality) of these models, based on the structure of the primal optimization problem. Briefly explain what these properties (e.g. complementary slackness and stationarity conditions) reveal about the solutions of the optimization problems (e.g. binding constraints, feasibility, and optimality) in this context.
LO5: Interpret the meaning, from a geometric and techno-economic perspective, of the optimality conditions, dual formulation and dual variables of power system optimization problems, by linking them to marginal costs, resource valuations, and operational constraints.	Understand, Analyze	Interpret the optimality conditions and dual formulations of the optimization problems considered (geometric and techno-economic meaning), by analyzing its structure and mapping each key dual variable to a techno-economic quantity in power systems (congestion, willingness to pay). Use these to explain (in your own words) intuitions about the characteristics of solutions of the primal/dual optimization problems (e.g. when/why marginal prices spike).
LO7: Interpret and decompose real-life power system decision-making problems (described in natural language) into the structured elements (objectives, constraints, decision variables, input data).	Understand, Analyze	Interpret domain-specific language in both power systems and mathematical optimization fields, by translating the narrative in the assignment's problem statement, into structured elements, including sets, variables (with units/domains), input parameters, objective(s) components, and constraint families (device physics, flexibility settings, network limits, etc.), and noting key interactions between these components (e.g., how flexibility constraints and objectives interact with load scheduling variables).
LO8: Design tailored optimization models to address specific decision-making problems in power systems, by formulating mathematically the constraints, objective and decision variables in precise and efficient way.	Apply	Complete the next step in mathematical modelling by translating the identified components into formal mathematical structures, writing a complete, consistent optimization model, defining all notations, boundary conditions and units, individually explaining each objective(s) and constraints, and stating the model class (e.g. LP, NLP, MILP, etc.).
LO9: Collaboratively develop and implement scientific code to efficiently solve real-life decision-making problems in power systems using suitable solution algorithms, effectively integrating contributions and documenting workflows.	Apply, Create	Implement modular, runnable code (separate data loading, model build, solve, plotting) with a single entry command and configuration files. Clearly document your code, and provide all files required, and run minimum checks to ensure reproducibility (e.g. fixed seeds where relevant, environment/versions noted).
LO11: Conduct numerical experiments on optimization models in a systematic and reproducible manner.	Apply	Carry out planned simulations by design a small case-study scenario matrix (e.g., varied DSO incentives, comfort penalties, device availability) and executing it systematically. Save and clearly report configurations and outputs so results can be exactly reproduced and verified.
LO12: Analyze numerical results to derive practical insights for operational or planning decisions.	Analyze	Analyze relevant KPIs (e.g. baseline vs. optimized loads, line loading, bid/offer curves) and explain observed patterns, trends, and causal links (e.g., higher DSO tariffs reduce congestion in certain lines) via model mechanics (e.g., alignment of congestion hours with input data or dual variables). Discuss trade-offs (e.g., comfort vs. cost, peak reduction vs. welfare) using marginal values where relevant to extract meaning and valuable insights for DSOs, aggregators, or flexible consumers.
LO14: Effectively communicate the solutions of complex decision-making problems in power systems through clear and compelling narratives and visualization aids.	Create	Deliver a concise report, including detailed answers to each question, with compact, rigorous and well-described mathematical formulations, a structured narrative and clear progression of ideas, adequate visual aids and tables (clear units, captions, readable axes), and a short executive summary clearly stating assumptions and limitations and presenting conclusions tied to the assignment's objectives.

Figure 2: List of LOs assessed in Assignment 1 and instructions to demonstrate them.

- System-level cost structures (multiple scenarios): hourly profiles of *non-negative* distribution grid tariffs and electricity prices (in DKK/kWh)

Problem Statement

The objective of this assignment is to model and analyze an active distribution network with flexible residential consumers. You will build consumer-level flexibility models, analyze the structure of these optimization problems and derive meaningful techno-economic insights for flexible consumers.

1. **Consumer-level flexibility models**

   We consider the individual load scheduling problem of a single flexible consumer, and analyze how different types of flexible assets, flexibility preferences, and cost structures may impact its flexibility and profits.

   (a) We first consider that this consumer has a single (fully) flexible load and a rooftop PV panel. They want to utilize their flexible load and rooftop PV production to minimize their daily energy procurement cost. Their maximum hourly PV production is perfectly known, but can be fully curtailed. Their load is fully flexible in each hour, however, the consumer wants to satisfy a minimum (total) energy consumption at the end of the day. They can sell/buy electricity to/from the distribution grid at an hourly electricity price, but must pay an hourly grid tariff fee for all net imports and exports. We analyze how these cost structures can impact their flexibility and profits.

   i. (6 points) Formulate the optimization problem of a consumer, describing its main components (input data, decision variables, objective function, constraints) and properties.

   ii. (5 points) Formulate the dual problem, Lagrangian, and KKT conditions of this optimization problem. Describe their properties.

   iii. (6 points) Without numerically solving this optimization problem, analyze its structure and discuss qualitatively the characteristics of its optimal solutions. Which cost components and constraints have the most impact on the solutions? Which constraint are binding at optimality? How does the input data impact the optimal solutions? In particular, compare the cases in which: the electricity prices and grid tariffs are constant through the day or vary hourly. For which values of the cost components, can you conclude that this optimization problem is equivalent to a simpler optimization one? *Tip: you can use the dual formulation and KKT conditions to motivate your analysis*

   iv. (3 points) Implement this optimization problem in a programming language of your choice. Provide a well-documented working code.

   v. (6 points) Construct relevant scenarios of system-level cost structures based on the data provided, solve this optimization problem over these scenarios, analyze its optimal solutions (primal and dual) and answer the following question: How do cost structures impact the consumers flexibility and profits? Present your insights in a compact manner using adequate visual aids. Do these solutions align with your analysis from Question 1.(a).iii.?

(b) We now consider that the consumer described in Question 1.(a) does not have a strict daily energy consumption requirement. Instead, it is interested, in minimizing the discomfort uncured from shifting its flexible load at each hour compared to a reference hourly load profile, in addition to minimizing its energy procurement cost through the day. And we analyze the impact on its flexibility and profits.

i. (4 points) Adapt the optimization problem formulated in Question 1.(a) to reflect these new flexibility preferences. Introduce (if necessary) new constraints objective(s), or input parameters. Define all new notations, and motivate these modeling choices.

ii. (4 points) Formulate the dual problem, Lagrangian, and KKT conditions of this updated optimization problem. Describe their properties.

iii. (4 points) Without numerically solving this optimization problem, discuss qualitatively how these changes impact the structure of this optimization problem, and the characteristics of its optimal solutions. Do the findings in Question 1.(a).iii still hold? How do different flexibility preferences and cost structures impact the optimal solutions? What can you infer about which constraints are binding at optimality?

iv. (1 point) Implement this optimization problem in a programming language of your choice. Provide a well-documented working code.

v. (4 points) Construct relevant scenarios of consumer flexibility preferences based on the data provided[1], solve this optimization problem over these scenarios, analyze its optimal solutions (primal and dual) and answer the following question: How do the flexibility preferences of consumers impact their flexibility and profits? Present your insights in a compact manner using adequate visual aids. Do these solutions align with your analysis from Question 1.(b).iii.?

(c) We now consider that this consumer has installed a battery, and analyze the impact on its flexibility and profits.

i. (4 points) Adapt the optimization problem formulated in Question 1.(b) to include this new flexible asset. Describe how to handle boundary conditions on the state of charge of the battery, and avoid that it is fully depleted at the end of the day. Motive these modeling choices.

ii. (4 points) Formulate the dual problem, Lagrangian, and KKT conditions of this updated optimization problem. Describe their properties.

iii. (4 points) Discuss qualitatively what economic benefits the consumers can derive from this battery and how it can utilize it. Discuss in which cases these benefits

[1] If you introduced new input parameters, specify the value(s) chosen

are expected to be more or less significant, e.g., depending on the hourly profiles of reference load, PV production, grid tariffs and electricity prices, or different relative cost or capacity values.

iv. (1 point) Implement this optimization problem in a programming language of your choice. Provide a well-documented working code.

v. (4 points) Solve this updated optimization problem over relevant scenarios of consumer flexibility preferences and cost structures (consistent with Questions 1.(a) and 1.(b)), analyze its optimal solutions (primal and dual) and answer this question: How does this additional flexibility impact the consumer's load scheduling and profits? Present your insights in a compact manner using adequate visual aids. Do these solutions align with your analysis from Question 1.(c).iii.?

2. **Value of demand-side flexibility**

Building on the individual models derived in Question 1, we further analyze the value of demand-side flexibility for various types of consumers, and derive actionable operational and investment insights.

(a) We consider the flexible consumer described in Question 1.(b), and conduct a techno-economic analysis of the value of flexibility, through duality.

i. (7 points) Describe in your own words the techno-economic meaning of the dual formulation of the cost-minimization optimization problem of this consumer, derived in Question 1.(b), including the dual variables associated with key constraints, and the Lagrange function. What economic signal does the value at optimality of these dual variables provide?

ii. (8 points) Considering that this consumer wants to participate in the day-ahead electricity market, derive their hourly demand (or supply) curve, representing their marginal willingness to pay (or marginal opportunity cost) for purchasing (selling) 1 additional MWh of electricity in the market. *Tip: you can use the KKT conditions derived in Question 1.(b) to find conditions linking the optimal quantity purchased/sold on the market for varying electricity prices.*

(b) This flexible consumer is now considering investing in a battery, and wants to evaluate which investments are more profitable. We consider the following assumptions: i) the consumer can only invest in a battery which technical characteristics (charging/discharging power and energy storage capacity) scale linearly and uniformly with the ones of the battery considered in Question 1.(b); ii) charging/discharging efficiencies remain the same regardless of the battery size; iii) the capital cost of purchasing and installing a battery scales linearly with its energy storage capacity; iv) the battery has a life of 10 years, and has no degradation over this period; iii) all input parameters (e.g. reference load profiles, hourly electricity prices, PV production, etc.) remain the same, and no discount factors are applied to future costs/profits over this time horizon.

i. (7 points) Based on the stated assumptions, formulate this consumer's investment decision-making problem as an optimization problem.

ii. (7 points) Discuss qualitatively how these assumptions impact the structure of the optimization problem and its optimal solutions. How could you lift these assumptions (no mathematical formulation required)? How do you expect the optimal solutions to change if these assumptions are lifted? What computational challenges may lifting these assumptions raise?

iii. (3 points) Implement this optimization problem in a programming language of your choice. Provide a well-documented working code.

iv. (8 points) Design and conduct a numerical experiment to quantify the profitability of installing a battery, over relevant scenarios of consumer preferences and cost structures. *Tip: you can leverage the analysis in Questions 1.(a) - 1.(c).* Describe and motivate the scenarios considered and the metrics to be evaluated in order to derive actionable insights for various types of flexible consumers. Present these insights in a concise manner, using adequate visual aids to support them. Discuss the potential limitations of this analysis.