Poisson distribution

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In probability theory and statistics, the **Poisson distribution** (French pronunciation [pwasɔ̃]; in English usually /ˈpwɑːsɒn/), named after French mathematician Siméon Denis Poisson, is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event.[1] The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

For instance, an individual keeping track of the amount of mail they receive each day may notice that they receive an average number of 4 letters per day. If receiving any particular piece of mail doesn't affect the arrival times of future pieces of mail, i.e., if pieces of mail from a wide range of sources arrive independently of one another, then a reasonable assumption is that the number of pieces of mail received per day obeys a Poisson distribution.[2] Other examples that may follow a Poisson: the number of phone calls received by a call center per hour or the number of decay events per second from a radioactive source.

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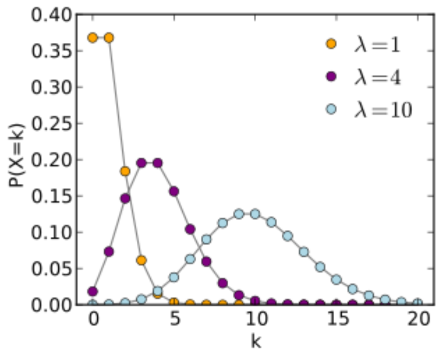
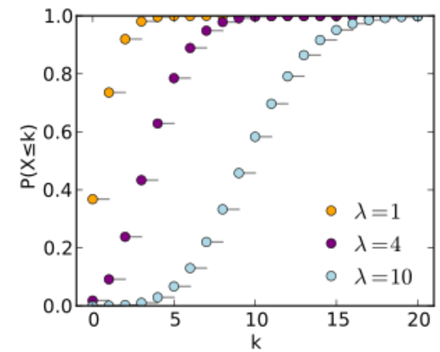


The horizontal axis is the index k, the number of occurrences. λ is the expected number of occurrences. The vertical axis is the probability of k occurrences given λ. The function is defined only at integer values of k. The connecting lines are only guides for the eye.

Cumulative distribution function



The horizontal axis is the index k, the number of occurrences. The CDF is discontinuous at the integers of k and flat everywhere else because a variable that is Poisson distributed takes on only integer values.

Parameters	$\lambda > 0$ (real)

Support	$k \in \mathbb{N} \cup 0$;
pmf	$\frac{\lambda^k e^{-\lambda}}{k!}$
CDF	$\frac{\Gamma(\lfloor k+1 \rfloor, \lambda)}{\lfloor k \rfloor!}$, or $e^{-\lambda} \sum_{i=0}^{\lfloor k \rfloor} \frac{\lambda^i}{i!}$, or $Q(\lfloor k+1 \rfloor, \lambda)$ (for $k \geq 0$, where $\Gamma(x, y)$ is the incomplete gamma function, $\lfloor k \rfloor$ is the floor function, and Q is the regularized gamma function)
Mean	λ
Median	$\approx \lfloor \lambda + 1/3 - 0.02/\lambda \rfloor$
Mode	$\lceil \lambda \rceil - 1, \lfloor \lambda \rfloor$
Variance	λ
Skewness	$\lambda^{-1/2}$
Ex. kurtosis	λ^{-1}
Entropy	$\lambda[1 - \log(\lambda)] + e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k \log(k!)}{k!}$ (for large λ) $\frac{1}{2}\log(2\pi e\lambda) - \frac{1}{12\lambda} - \frac{1}{24\lambda^2} - \frac{19}{360\lambda^3} + O\left(\frac{1}{\lambda^4}\right)$
MGF	$\exp(\lambda(e^t - 1))$
CF	$\exp(\lambda(e^{it} - 1))$
PGF	$\exp(\lambda(z - 1))$
Fisher information	$\frac{1}{\lambda}$

Introduction to the Poisson distribution

The Poisson distribution is popular for modelling the number of times an event occurs in an interval of time or space.

Examples

The Poisson distribution may be useful to model events such as

- The number of meteors greater than 1 meter diameter that strike earth in a year
- The number of patients arriving in an emergency room between 10 and 11 pm