Exponential distribution

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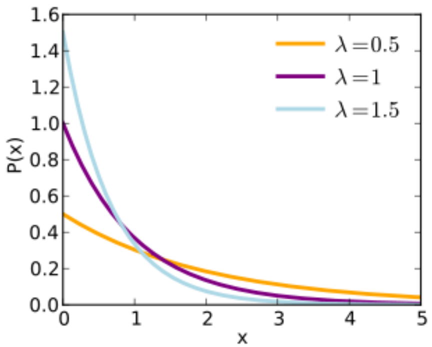
In probability theory and statistics, the **exponential distribution** (a.k.a. **negative exponential distribution**) is the probability distribution that describes the time between events in a Poisson process, i.e. a process in which events occur continuously and independently at a constant average rate. It is a particular case of the gamma distribution. It is the continuous analogue of the geometric distribution, and it has the key property of being memoryless. In addition to being used for the analysis of Poisson processes, it is found in various other contexts.

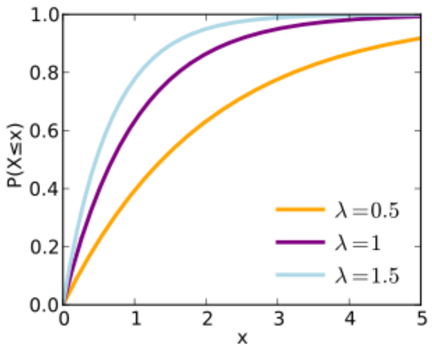
The exponential distribution is not the same as the class of exponential families of distributions, which is a large class of probability distributions that includes the exponential distribution as one of its members, but also includes the normal distribution, binomial distribution, gamma distribution, Poisson, and many others.

Exponential

Probability density function



Cumulative distribution function



Parameters	$\lambda > 0$ rate, or inverse scale
Support	$x \in [0, \infty)$
PDF	$\lambda e^{-\lambda x}$
CDF	$1 - e^{-\lambda x}$
Quantile	$-\ln(1 - F) / \lambda$
Mean	$\lambda^{-1}\ (= \beta)$
Median	$\lambda^{-1} \ln(2)$
Mode	0
Variance	$\lambda^{-2}\ (= \beta^2)$
Skewness	2
Ex. kurtosis	6
Entropy	$1 - \ln(\lambda)$
MGF	$\frac{\lambda}{\lambda - t}$, for $t < \lambda$
CF	$\frac{\lambda}{\lambda - it}$
Fisher information	λ^{-2}

Contents

Characterization

Probability density function

The probability density function (pdf) of an exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

Alternatively, this can be defined using the right-continuous Heaviside step function, $H(x)$ where $H(0)=1$:

$$f(x;\lambda) = \lambda e^{-\lambda x} H(x)$$

Here $\lambda > 0$ is the parameter of the distribution, often called the *rate parameter*. The distribution is supported on the interval $[0, \infty)$. If a random variable X has this distribution, we write $X \sim \mathrm{Exp}(\lambda)$.

The exponential distribution exhibits infinite divisibility.

Cumulative distribution function

The cumulative distribution function is given by

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

Alternatively, this can be defined using the Heaviside step function, $H(x)$.

$$F(x;\lambda) = (1 - e^{-\lambda x}) H(x)$$

Alternative parameterization

A commonly used alternative parametrization is to define the probability density function (pdf) of an exponential distribution as

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

where $\beta > 0$ is mean, standard deviation, and scale parameter of the distribution, the reciprocal of the *rate parameter*, λ, defined above. In this specification, β is a *survival parameter* in the sense that if a random variable X is the duration of time that a given biological or mechanical system manages to survive and $X \sim$