732A90 - Exam - Task 1

Assignment 1

Inverse CDF

1. Calculate the normalization constant:

$$P(x) = \int_0^1 a^x dx \tag{1}$$
$$= \int_0^1 e^{xlog(a)} dx \tag{2}$$
$$= \left[\frac{1}{log(a)} e^{xlog(a)}\right]_0^1 \tag{3}$$
$$= \frac{a-1}{log(a)} \tag{4}$$
$$=> c = \frac{log(a)}{a-1} \tag{5}$$

2. Calculate the CDF

$$P(y) = \int_0^y ca^x dx \tag{6}$$
$$= c\int_0^y e^{xlog(a)} dx \tag{7}$$
$$= c\left[\frac{1}{log(a)} e^{xlog(a)}\right]_0^y \tag{8}$$
$$= \frac{c}{log(a)}(e^{ylog(a)} - 1) \tag{9}$$

3. Inverse it

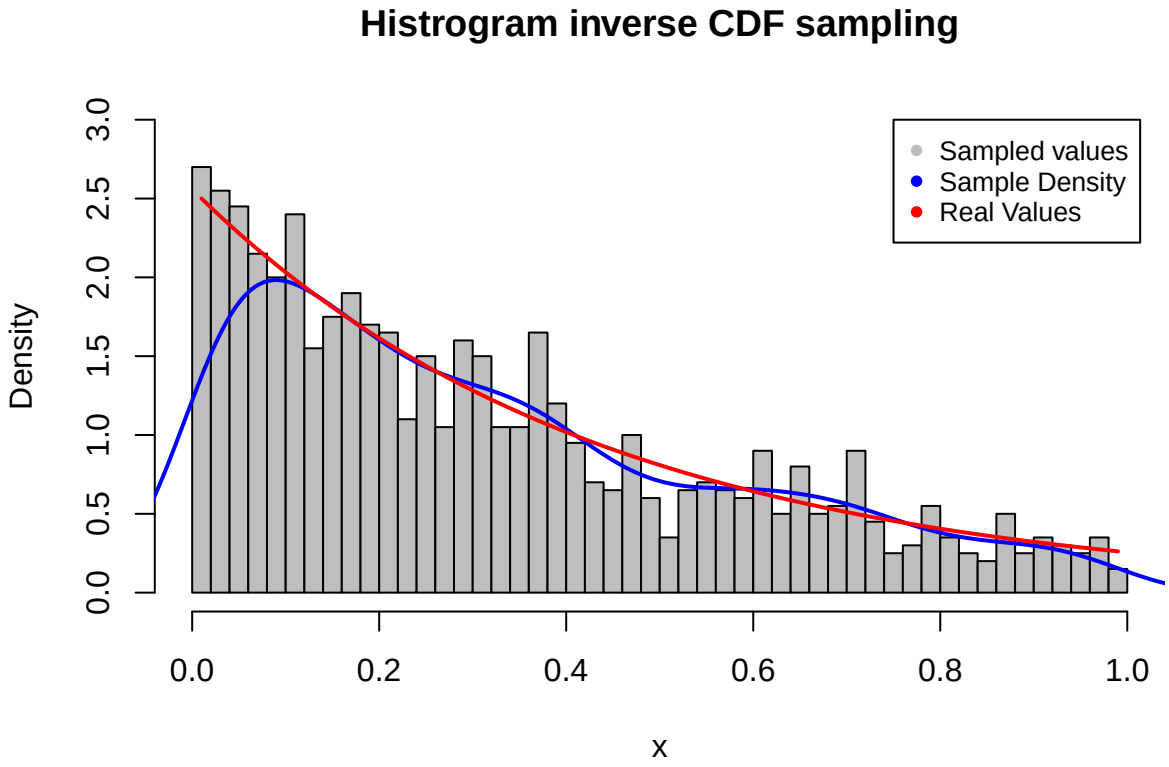
$$y = \frac{1}{a-1}(e^{xlog(a)} - 1) \tag{10}$$
$$x = \frac{1}{a-1}(e^{ylog(a)} - 1) \tag{11}$$
$$\frac{log(x(a-1)+1)}{log(a)} = y \tag{12}$$

```
inverse_cdf <- function(x, a) {
  log(x * (a - 1) + 1) / log(a)
}

u <- runif(1000)
res_11 <- sapply(u, inverse_cdf, a = 0.1)
```

```r
a <- 0.1
c <- log(a) / (a-1)
x <- seq(0.01, 0.99, 0.01)
y <- a^x * c

hist(res_11, freq = FALSE, breaks = 40,
     xlim = c(0,1 ), ylim = c(0, 3),
     xlab = "x", ylab = "Density",
     main = "Histrogram inverse CDF sampling",
     col = "grey")
lines(density(res_11), col = "blue", lwd = 2)
lines(x, y, col = "red", lwd = 2)
legend(x = 0.75, y = 3,
       legend = c("Sampled values", "Sample Density", "Real Values"),
       col = c("grey", "blue", "red"),
       pch = c(16, 16, 16),
       cex = 0.8)
```



Sampled values look like the values from $p(y)$. The blue line will converge towards the red line if the sample size increases.