Log-normal distribution

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In probability theory, a **log-normal (or lognormal) distribution** is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable X is log-normally distributed, then $Y = \ln(X)$ has a normal distribution. Likewise, if Y has a normal distribution, then $X = \exp(Y)$ has a log-normal distribution. A random variable which is log-normally distributed takes only positive real values. The distribution is occasionally referred to as the **Galton distribution** or **Galton's distribution**, after Francis Galton.[1] The log-normal distribution also has been associated with other names, such as McAlister, Gibrat and Cobb–Douglas.[1]

A log-normal process is the statistical realization of the multiplicative product of many independent random variables, each of which is positive. This is justified by considering the central limit theorem in the log domain. The log-normal distribution is the maximum entropy probability distribution for a random variate X for which the mean and variance of $\ln(X)$ are specified.[2]

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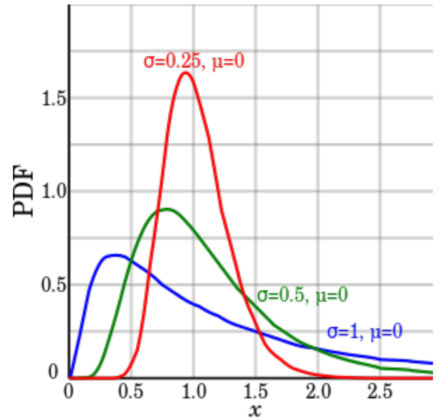
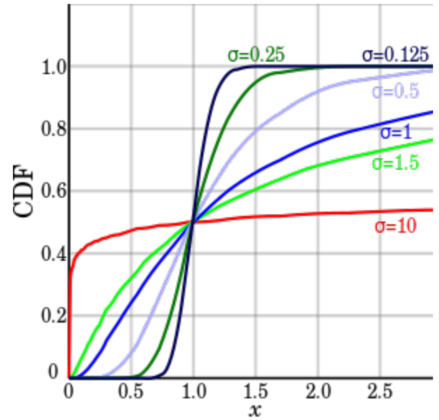


Some log-normal density functions with identical location parameter μ but differing scale parameters σ

Cumulative distribution function



Cumulative distribution function of the log-normal distribution (with $\mu = 0$)

Notation	$\ln\mathcal{N}(\mu,\,\sigma^2)$
Parameters	$\mu \in \mathbb{R}$ — location, $\sigma > 0$ — scale of associated normal
Support	$x \in (0, +\infty)$
PDF	$\frac{1}{x\sigma\sqrt{2\pi}}\ e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$
CDF	$\frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\Big[\frac{\ln x-\mu}{\sqrt{2}\sigma}\Big]$
Mean	$e^{\mu+\sigma^2/2}$
Median	e^{μ}
Mode	$e^{\mu-\sigma^2}$
Variance	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Skewness	$(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$
Ex. kurtosis	$e^{4\sigma^2}+2e^{3\sigma^2}+3e^{2\sigma^2}-6$
Entropy	$\log(\sigma e^{\mu+\frac{1}{2}}\sqrt{2\pi})$
MGF	defined only on the negative half-axis, see text
CF	representation $\sum_{n=0}^{\infty}\frac{(it)^n}{n!}e^{n\mu+n^2\sigma^2/2}$ is asymptotically divergent but sufficient for numerical purposes
Fisher information	$\begin{pmatrix}1/\sigma^2 & 0\\ 0 & 2/\sigma^2\end{pmatrix}$

Notation

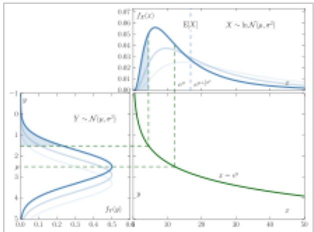
Given a log-normally distributed random variable X and two parameters μ and σ that are, respectively, the mean and standard deviation of the variable's natural logarithm, then the logarithm of X is normally distributed, and we can write X as

$$X = e^{\mu+\sigma Z}$$

with Z a standard normal variable.

This relationship is true regardless of the base of the logarithmic or exponential function. If $\log_a(Y)$ is normally distributed, then so is $\log_b(Y)$, for any two positive numbers $a, b \neq 1$. Likewise, if e^X is log-normally distributed, then so is a^X, where a is a positive number $\neq 1$.

On a logarithmic scale, μ and σ can be called the *location parameter* and the *scale parameter*, respectively.



Relation between normal and lognormal distribution. If $Y = \mu + \sigma Z$ is normally distributed, then $X \sim e^Y$ is lognormally distributed.

In contrast, the mean, standard deviation, and variance of the non-logarithmized sample values are respectively denoted m, *s.d.*, and v in this article. The two sets of parameters can be related as (see also Arithmetic moments below)[3]

$$\mu = \ln\left(\frac{m}{\sqrt{1+\frac{v}{m^2}}}\right), \qquad \sigma = \sqrt{\ln\left(1+\frac{v}{m^2}\right)}.$$

Characterization

Probability density function