732A90 - Exam - March 2016

Assignment 1

1.1 Inverse CDF

We can use the inverse CDF method to sample from the distribution. Assuming we have a PDF of the target distribution we can follow the following schema:

1. If the target distribution is a proper distribution (area under curve = 1) go to step 3, otherwise to step 2.
2. Integrate the function and calculate the area under the curve, lets call this value β; The normalization factor is then given by $\frac{1}{\beta}$; Multiply the original distribution with $\frac{1}{\beta}$ to get a proper density function.
3. Integrate the density function to get its primitive function (cumulative distribution function (CDF)).
4. Calculate the inverse of the CDF.
5. Sample from the inverse with $u \sim Unif(0, 1)$

$$F(x) = \int_{-\infty}^{x} 1.5\sqrt{s}\ ds = \int_{0}^{x} 1.5\sqrt{s}\ ds = \left[s^{\frac{3}{2}}\right]_0^x = x^{\frac{3}{2}}$$

Now the inverse can be calculated:

$$F(x)^{-1} : x = y^{\frac{3}{2}} y = x^{\frac{2}{3}}$$

We can now sample from this distribution.

```r
# Define the function of the inverse cdf
inverse_cdf <- function(x) {
  return(x^(2/3))
}

# Generate 10000 random values uniform(0, 1) and
# sample from the inverse cdf
u <- runif(10000)
res_11 <- inverse_cdf(u)

# Generate the values of the real distribution
x <- seq(0.001, 0.999, 0.001)
y <- 1.5 * sqrt(x)

# Plot the distributions in a histogram
hist(res_11, freq = FALSE,
     breaks = 30,
     xlab = "x", ylab = "Density",
     main = "Distribution of f(x)",
     col = "grey")

# Plot the density estimate of the sample (blue) and the true values
# of the function (red)
lines(density(res_11), col = "blue", lwd = 2)
lines(x, y, col = "red", lwd = 2)
```

```
# Add a legend
legend(x = 0, y = 1.5,
       legend = c("sample", "sample density", "true density"),
       col = c("grey", "blue", "red"),
       pch = c(16, 16, 16),
       cex = 0.8)
```