732A96 Lab 2

Emil K Svensson

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Question 1

```
states <- paste("z",1:10,sep = "")
symbols <- paste("s",1:10,sep = "")
start <- rep(0.1,10)

#Just to see the structure
#initHMM(states, symbols)

# The transition probabilities, since we only can move
trans <- matrix(0,ncol = 10, nrow = 10)
diag(trans) <- 0.5
diag(trans[,-1]) <- 0.5
trans[10,1] <- 0.5
#trans[1,10] <- 0.5

# Making sure the probabilities sum to 1 in each row
#apply(trans, MARGIN = 1, FUN = sum)

emission <- matrix(0,ncol = 10, nrow = 10)
diag(emission) <- 0.5

# The Emission probabilities are our uncertaintiec in the position of the robot.

# Brute force set up, Nick, Rasmus and Sascha probably has a nicer ways of solving this
diag(emission[,-1]) <- 0.2
diag(emission[-1,]) <- 0.2
diag(emission[c(-1:-2),]) <- 0.2
diag(emission[,c(-1:-2)]) <- 0.2
emission[10,2] <- 0.2
emission[2,10] <- 0.2
emission[2,10] <- 0.2
emission[1,10] <- 0.2
emission[9:10,1] <- 0.2
emission[1,9] <- 0.2
```

```
# The different places the robot can be in
symbols
```

```
##  [1] "s1"  "s2"  "s3"  "s4"  "s5"  "s6"  "s7"  "s8"  "s9"  "s10"
```

```
# The Hidden States
states
```

```
##  [1] "z1"  "z2"  "z3"  "z4"  "z5"  "z6"  "z7"  "z8"  "z9"  "z10"
```

```
# Where they start
start
```

```
##  [1] 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1
```

```
# Transition matrix is the probabilities the robot will move
trans
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]  0.5  0.5  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0
##  [2,]  0.0  0.5  0.5  0.0  0.0  0.0  0.0  0.0  0.0   0.0
##  [3,]  0.0  0.0  0.5  0.5  0.0  0.0  0.0  0.0  0.0   0.0
##  [4,]  0.0  0.0  0.0  0.5  0.5  0.0  0.0  0.0  0.0   0.0
##  [5,]  0.0  0.0  0.0  0.0  0.5  0.5  0.0  0.0  0.0   0.0
##  [6,]  0.0  0.0  0.0  0.0  0.0  0.5  0.5  0.0  0.0   0.0
##  [7,]  0.0  0.0  0.0  0.0  0.0  0.0  0.5  0.5  0.0   0.0
##  [8,]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.5  0.5   0.0
##  [9,]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.5   0.5
## [10,]  0.5  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.5
```

```
# Emission matrix states the uncertainties we have about the robots position
emission
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]  0.5  0.2  0.2  0.0  0.0  0.0  0.0  0.0  0.2   0.2
##  [2,]  0.2  0.5  0.2  0.2  0.0  0.0  0.0  0.0  0.0   0.2
##  [3,]  0.2  0.2  0.5  0.2  0.2  0.0  0.0  0.0  0.0   0.0
##  [4,]  0.0  0.2  0.2  0.5  0.2  0.2  0.0  0.0  0.0   0.0
##  [5,]  0.0  0.0  0.2  0.2  0.5  0.2  0.2  0.0  0.0   0.0
##  [6,]  0.0  0.0  0.0  0.2  0.2  0.5  0.2  0.2  0.0   0.0
##  [7,]  0.0  0.0  0.0  0.0  0.2  0.2  0.5  0.2  0.2   0.0
##  [8,]  0.0  0.0  0.0  0.0  0.0  0.2  0.2  0.5  0.2   0.2
##  [9,]  0.2  0.0  0.0  0.0  0.0  0.0  0.2  0.2  0.5   0.2
## [10,]  0.2  0.2  0.0  0.0  0.0  0.0  0.0  0.2  0.2   0.5
```

With everything defined we can initialize the HMM-robot.

```
robot <- initHMM(states,symbols,start,trans,emission)
```

Question 2

The function below is built for answering Questions 2 to 7

```
mrRobot <- function(hmm,simobs = 100, what = "acc"){

  # Simulation the robot simobs times
  sim_rob <- simHMM(hmm,length = simobs)
```

```
  #extract the observations (x_t)
  observations <- sim_rob$observation

  #extract the hidden states z_t
  state_place <- sim_rob$states

### Filter
    # Forward function computes the filtering with log
    log_filter <- forward(hmm = hmm, observation = observations)

    # Remove the log-transformation
    filter <- exp(log_filter)

    # Normalizing
    norm_filter <-prop.table(filter, margin = 2)

    # Checking which probability in each column is the highest
    most_prob_filter <- apply(norm_filter, MARGIN = 2, FUN = which.max)

    # Accuracy for the filter
    accuracy_filter <-  sum(paste("z",most_prob_filter, sep = "")
                         == state_place) / length(state_place)



### Smoothed (in this package called the posterior)
    smoothed <- posterior(hmm=hmm,observations)

    # Normalizing
    norm_smoothed <-prop.table(smoothed, margin = 2)

    most_prob_smoothed <- apply(norm_smoothed, MARGIN = 2, FUN = which.max)

    # Accuracy for the smoothed
    accuracy_smoothed <-  sum(paste("z",most_prob_smoothed, sep = "")
                              ==  state_place) / length(state_place)

  #cat("The accuracy of the smoothed is",accuracy_smoothed)


### Most probable path (Viterbi)
    mpp <- viterbi(hmm,observations)

    accuracy_mpp <-  sum(mpp == state_place) / length(state_place)
    #cat("The accuracy of the Viterbi is",accuracy_mpp)




  #Just logical statements on what to return.
  if(what == "acc"){
  return( c(accuracy_filter = accuracy_filter,
            accuracy_smoothed = accuracy_smoothed,
```

```
            accuracy_mpp = accuracy_mpp))
  }

  if(what == "filter"){
    return(norm_filter)
  }

  if(what == "smooth"){
    return(norm_smoothed)
  }

  if(what == "mpp"){
    return(mpp)
  }

}
```

Filter

```
mrRobot(hmm = robot,simobs = 100, what = "filter")
```

To much to print out, so run the code if you want to see the distribution

Smoothed

```
mrRobot(hmm = robot,simobs = 100, what = "smooth")
```

To much to print out, so run the code if you want to see the distribution

Most probable path

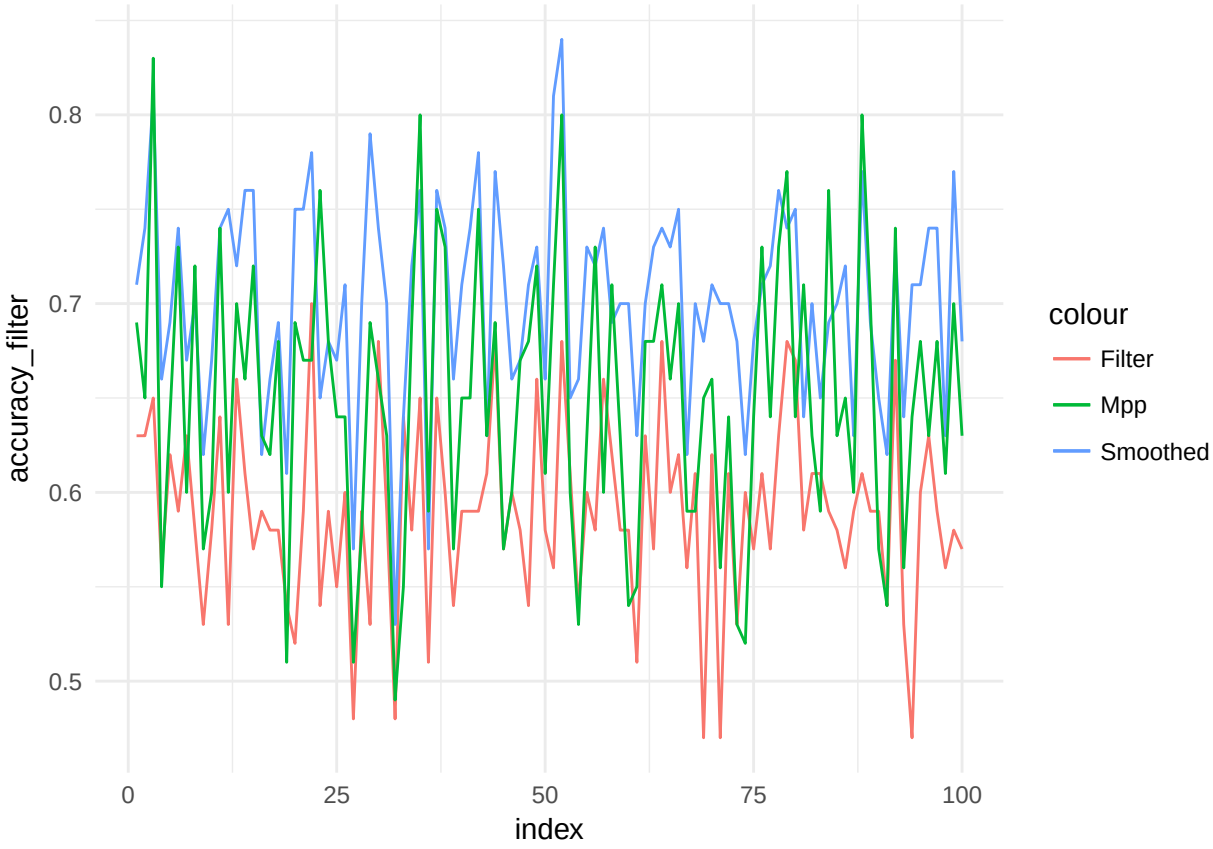
```
mrRobot(hmm = robot,simobs = 100, what = "mpp")
```

```
##   [1] "z1"  "z1"  "z2"  "z3"  "z4"  "z4"  "z4"  "z5"  "z6"  "z7"  "z8"
##  [12] "z8"  "z9"  "z9"  "z10" "z1"  "z1"  "z2"  "z2"  "z2"  "z2"  "z3"
##  [23] "z4"  "z4"  "z4"  "z4"  "z4"  "z4"  "z4"  "z5"  "z6"  "z7"  "z8"
##  [34] "z9"  "z10" "z1"  "z1"  "z2"  "z3"  "z4"  "z5"  "z5"  "z5"  "z5"
##  [45] "z6"  "z7"  "z7"  "z8"  "z9"  "z9"  "z10" "z1"  "z1"  "z1"  "z2"
##  [56] "z3"  "z3"  "z3"  "z4"  "z4"  "z4"  "z4"  "z4"  "z5"  "z6"  "z6"
##  [67] "z7"  "z8"  "z8"  "z8"  "z9"  "z9"  "z10" "z1"  "z2"  "z3"  "z3"
##  [78] "z3"  "z4"  "z5"  "z5"  "z5"  "z6"  "z7"  "z7"  "z8"  "z9"  "z9"
##  [89] "z10" "z1"  "z1"  "z2"  "z3"  "z3"  "z3"  "z3"  "z3"  "z3"  "z4"
## [100] "z4"
```

```
total_acc <- sapply(1:100,FUN = function(x){mrRobot(robot,100, what = "acc")} )

total_acc <- as.data.frame(t(total_acc))
total_acc$index <- 1:100


ggplot(data = total_acc) + geom_line(aes(x=index,y=accuracy_filter , col = "Filter")) +  geom_line(aes(
```




```
colMeans(total_acc[,-4])
```

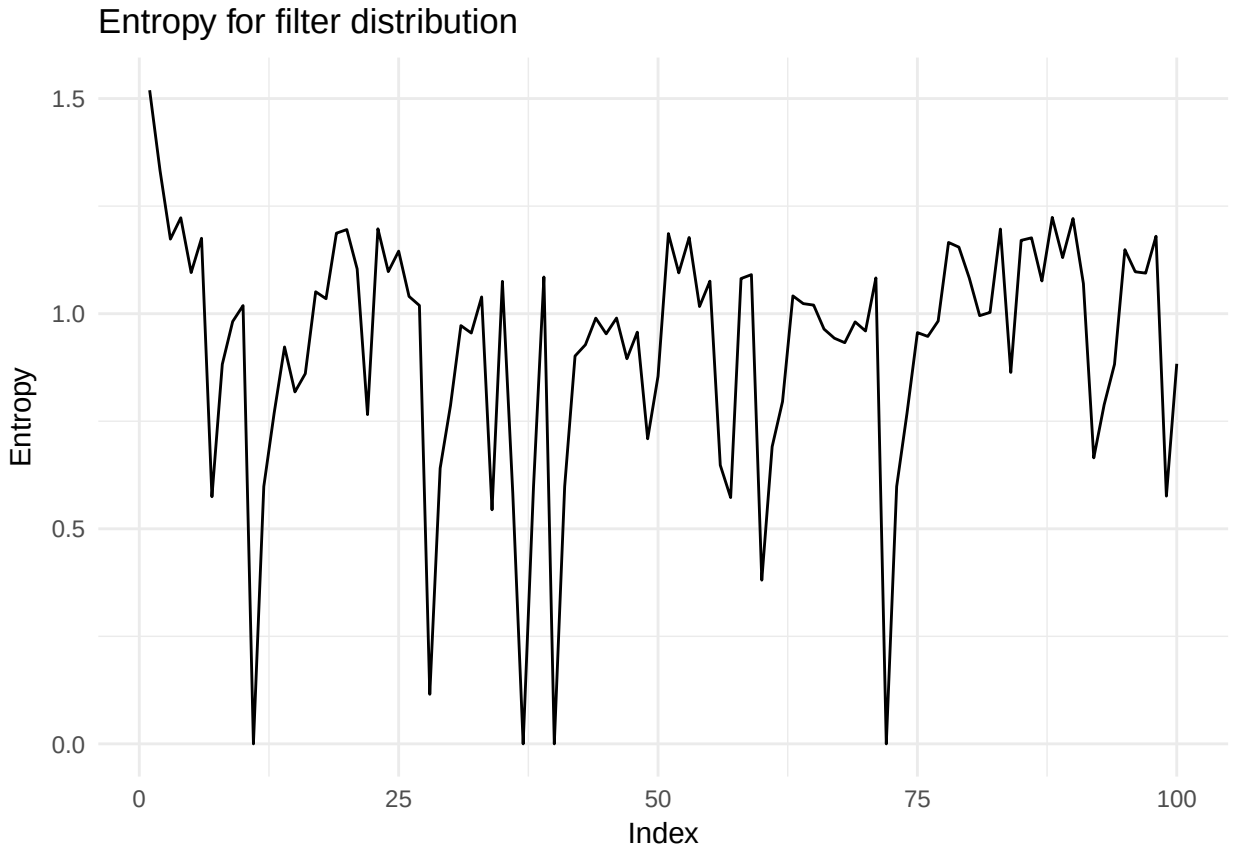
```
##   accuracy_filter accuracy_smoothed      accuracy_mpp
##            0.5897            0.7007            0.6505
```

Question 6

```
# filter_data <- sapply(1,FUN = function(x){
#   mrRobot(robot,100, what = "filter")
#   })

filter_data <- mrRobot(robot,100, what = "filter")
filter_entropy <- data.frame(Index = 1:100 ,
                  Entropy =
                  apply(filter_data, MARGIN = 2, FUN =entropy.empirical))

ggplot(data = filter_entropy, aes(x = Index, y = Entropy)) + geom_line() + ggtitle("Entropy for filter
```



No, the entropy remains random even while increasing the number of observations added to the hmm. This is because it is markovian and only depends on the previous observation.

Question 7

```
prior <- filter_data[,100] # The last information of the robot aka the prior
transition <- robot$transProbs
transition %*% prior
```

```
##
## from        [,1]
##   z1  0.00000000
##   z2  0.00000000
##   z3  0.04230482
##   z4  0.20329260
##   z5  0.45769518
##   z6  0.29670740
##   z7  0.00000000
##   z8  0.00000000
##   z9  0.00000000
##   z10 0.00000000
```

Since we have a markovian assumption that the only relevant state is the one we are in now and since all previous states feeds forward through z100 and provides it with information about previout observations and states we can just multiply z100 probabilities for each state with the transition probabilities to ge at prediction of how z101 and s101 will be like. In this case we were asked to get the hidden states probabilites

but if you want the observation you can just do a argmax over these probabilities to get were the next state should be.