732A96 Lab 3

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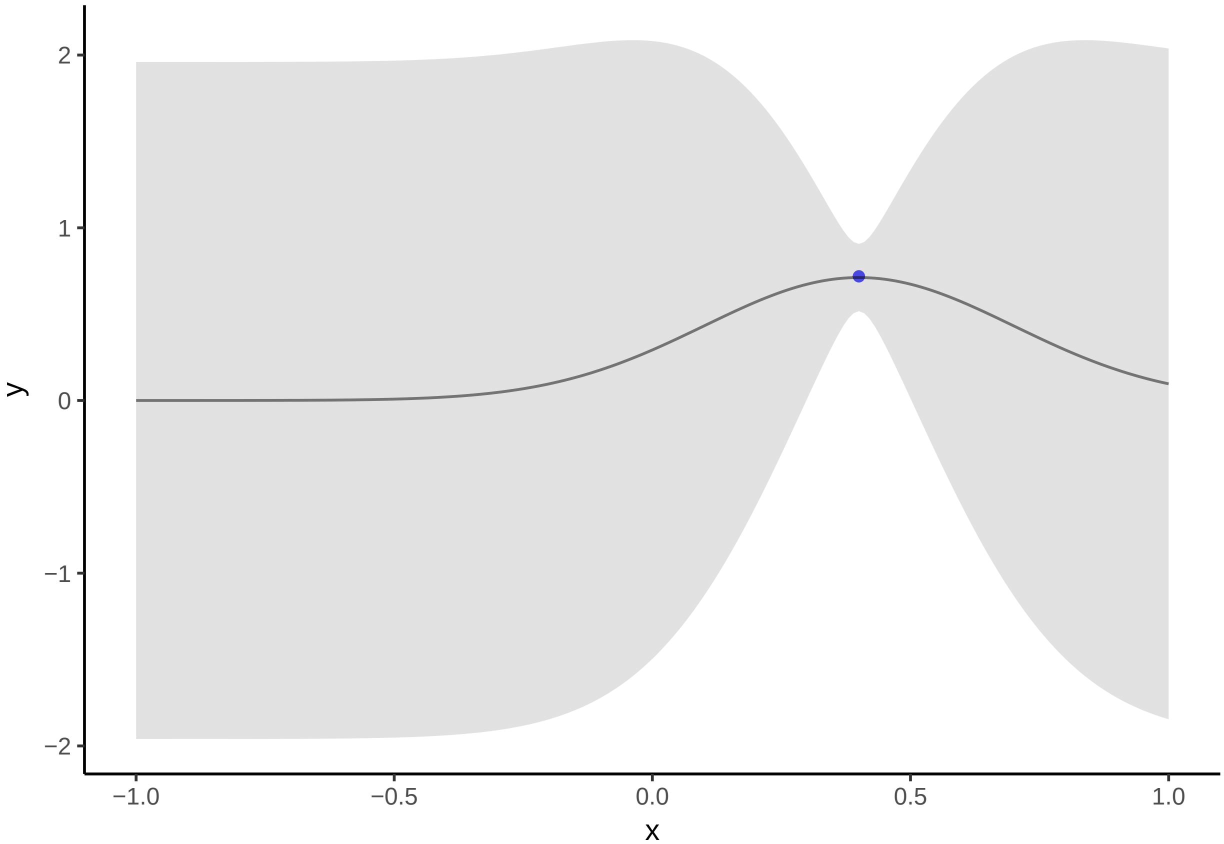
```r
# The kernel function

exp_kern <- function(x,xi,l, sigmaf ){

  return((sigmaf^2)*exp(-0.5*( (x - xi) / l )^2))

}


linear_gp <- function(x,y,xStar,hyperParam,sigmaNoise,kernel){

n <- length(x)
kernel_f <- kernel
# K = Covariance matrix calculation
  K <- function(X, XI,...){

    kov <- matrix(0,nrow = length(X), ncol = length (XI))

    for(i in 1:length(XI)){

      kov[,i]<- kernel_f(X,XI[i],...)

    }
    return(kov)
  }

l <-hyperParam[1]
sigmaf <- hyperParam[2]

#K(X,X)
K_xx   <- K(x,x, l = l, sigmaf = sigmaf) #, kernel = exp_kern

#K(X*,X*)
K_xsxs <- K(xStar,xStar, l = l, sigmaf = sigmaf) # kernel = exp_kern,

#K(X,X*)
K_xxs  <- K(x,xStar, l = l, sigmaf = sigmaf)  #kernel = exp_kern,

# Algorithm in page 19 of the Rasmus/Williams book


sI <- sigmaNoise^2 * diag(dim(as.matrix(K_xx))[1])
# L is transposed according to a definition in the R & W book
L_transposed <- chol(K_xx + sI)
L <- t(L_transposed)

alpha <- solve(t(L), solve(L,y))
```

```r
f_bar_star <- t(K_xxs) %*% alpha

v <- solve(L,K_xxs)

V_fs <- K_xsxs - t(v) %*% v

log_mlike <- -0.5 %*% t(y) %*% alpha - sum( diag(L) - n/2 * log(2*pi) )

return(list(fbar = f_bar_star, vf = V_fs, log_post= log_mlike))

}
```

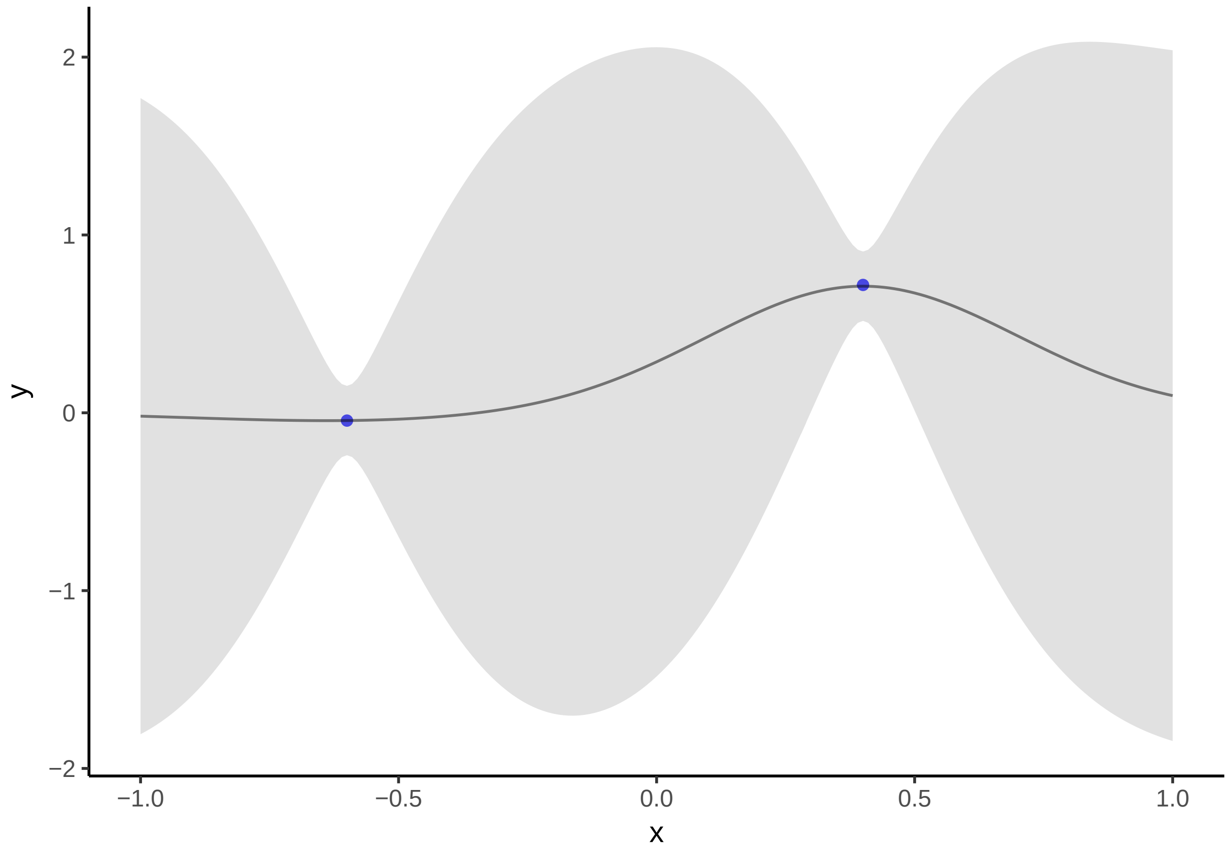
```r
plot_gp<- function(plot_it,band_it){

ggplot() +

  geom_point(
    aes(x = x, y = y),
    data = plot_it,
    col = "blue",
    alpha = 0.7) +

  geom_line(
    aes(x = xs, y = fbar),
    data = band_it,
    alpha = 0.50) +

  geom_ribbon(
    aes(ymin = low, ymax = upp, xs),
    data = band_it,
    alpha = 0.15) +

  theme_classic()

# plot(x,y,ylim = c(-2,2), xlim = c(-1,1))
# lines(xs,res$fbar)
# lines(xs, upp, lty = 3)
# lines(xs, low, lty = 3)
}
```

```r
x <- c(-1.0, -0.6, -0.2, 0.4, 0.8)
y <- c(0.768, -0.044, -0.940, 0.719 ,-0.664)
sn <- 0.1
xs <- seq(-1,1,0.01)
hyperParam <- c(0.3, 1)

repeter <- function(x,y,xs,sn,hyperParam,kernel){

res <- linear_gp(x,y,xs,hyperParam,sn,kernel)

upp <- res$fbar + 1.96*sqrt(diag(res$vf))
low <- res$fbar - 1.96*sqrt(diag(res$vf))

plot_it <- data.frame(x = x, y = y)
```

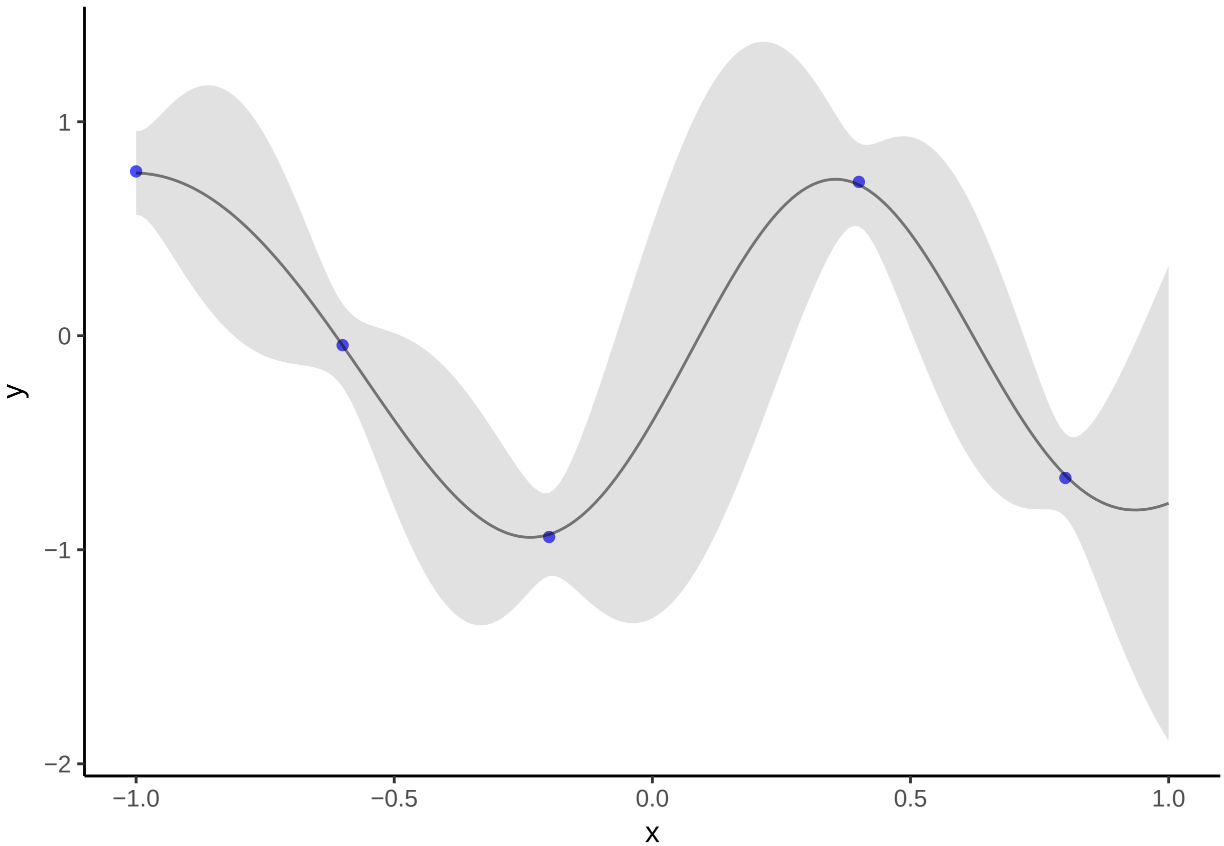
```r
band_it <- data.frame(xGrid = xs, fbar = res$fbar,  upp = upp, low = low)

plot_gp(plot_it,band_it)
}

repeter(x = x[4], y = y[4],xs,sn,hyperParam, kernel = exp_kern)
```




```r
repeter(x = x[c(2,4)], y = y[c(2,4)],xs,sn,hyperParam, kernel = exp_kern)
```




```
repeter(x = x, y = y,xs,sn,hyperParam, kernel = exp_kern)
```



Assignment 2

Data preparations

```
tullinge$time <- 1:nrow(tullinge)
tullinge$day <- rep(1:365,6)

time_sub <- tullinge$time %in% seq(1,2190,5)
tullinge <- tullinge[time_sub,]
```

```
kern_maker <- function(l,sigmaf){

  exp_k <- function(x,y = NULL){

   return((sigmaf^2)*exp(-0.5*( (x - y) / l )^2))
  }

  class(exp_k) <- "kernel"
  return(exp_k)
}
```

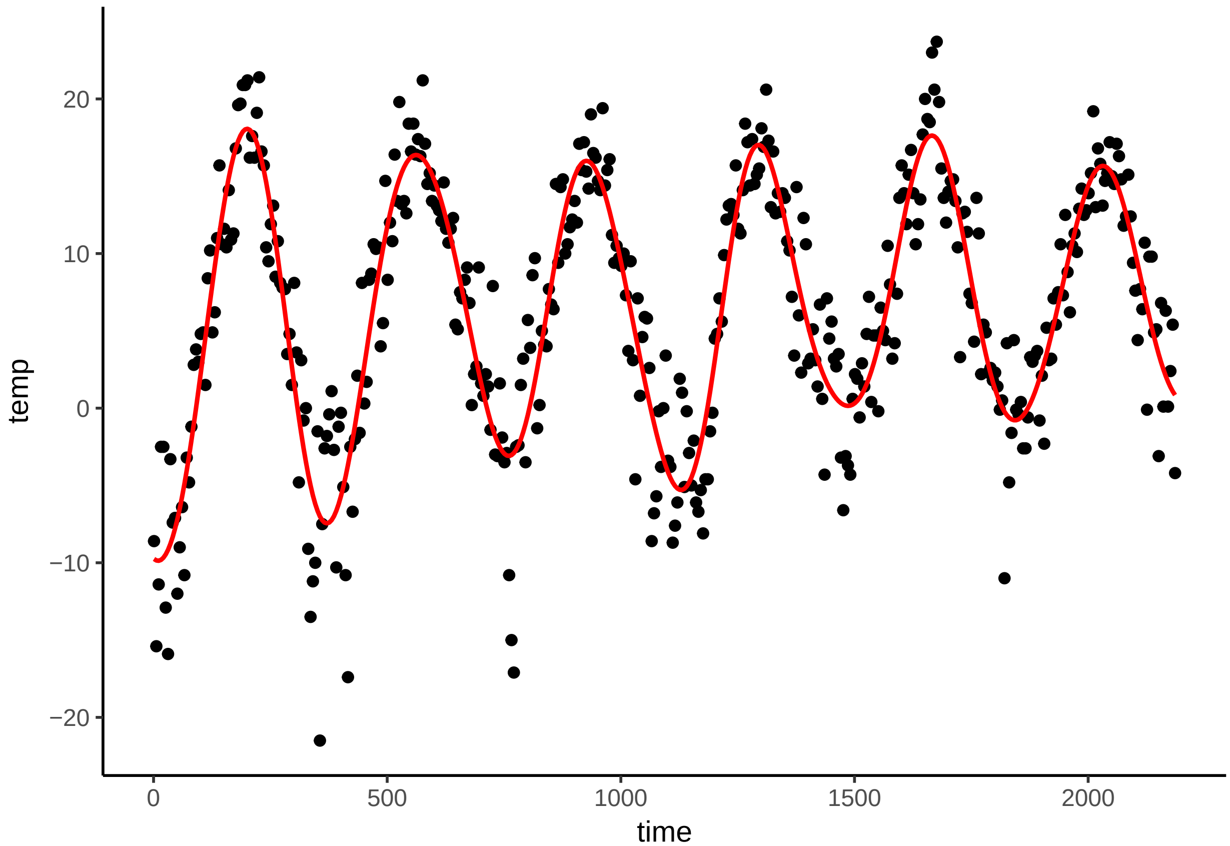
a)

```r
# gausspr()
# kernelMatrix()


ell <- 1
# SEkernel <- rbfdot(sigma = 1/(2*ell^2)) # Note how I reparametrize the rbfdo (which is the SE kernel)
# SEkernel(1,2)

my_exp <- kern_maker(l = 10, sigmaf =20)

x <- c(1,3,4)
x_star <- c(2,3,4)
#my_exp(x,x_star)
kernelMatrix(my_exp,x,x_star)
```

```
## An object of class "kernelMatrix"
##          [,1]     [,2]    [,3]
## [1,] 398.0050 392.0795 382.399
## [2,] 398.0050 400.0000 398.005
## [3,] 392.0795 398.0050 400.000
```

b)

```r
lm_tull <- lm(temp ~ time + I(time^2), data = tullinge)

sigma_2n <- var(resid(lm_tull))


a2b_kern <- kern_maker(l = 0.2, sigmaf = 20 )
gp_tullinge <- gausspr(x = tullinge$time, y = tullinge$temp, kernel = a2b_kern)


ggplot(data = tullinge, aes(x= time, y = temp)) +
  geom_point() +
  geom_line(aes(y = predict(gp_tullinge)), col = "red", size = 0.8) +
  theme_classic()
```



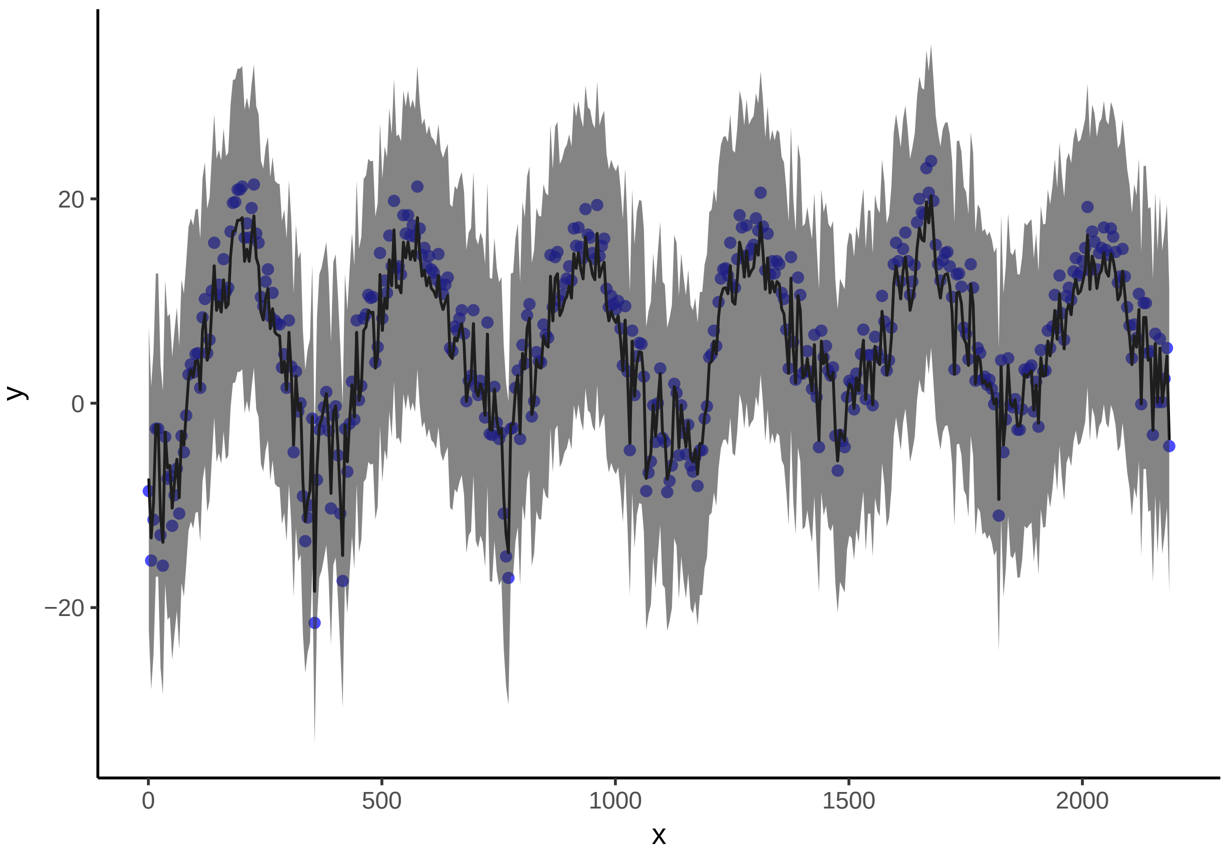

```
# plot(y = tullinge$temp, x = tullinge$time)
# #lines(x = tullinge$time, y = fitted(lm_tull), col = "red")
# lines(x = tullinge$time, y = predict(gp_tullinge), col = "red" , lwd = 1)
```

c)

```
sn_2c <- sqrt(sigma_2n)
xs_2c <- tullinge$time
hyperParam_2c <- c(0.2, 20)


res_2c<- linear_gp(x = tullinge$time,
        y = tullinge$temp,
        xStar = xs_2c,
        sigmaNoise  = sn_2c,
        hyperParam = hyperParam_2c,
        kernel = exp_kern)

upp2c <- res_2c$fbar + 1.96*sqrt(diag(res_2c$vf))
low2c <- res_2c$fbar - 1.96*sqrt(diag(res_2c$vf))

plot_it1 <- data.frame(x = tullinge$time, y = tullinge$temp)
band_it1 <- data.frame(xGrid = xs_2c, fbar = res_2c$fbar,  upp = upp2c, low = low2c)
```

```
ggplot() +
```

```
  geom_point(
    aes(x = x, y = y),
    data = plot_it1,
    col = "blue",
    alpha = 0.7) +

  geom_line(
    aes(x = xGrid, y = fbar),
    data = band_it1,
    alpha = 1) +

  geom_ribbon(
    aes(ymin = low2c, ymax = upp2c, x = xGrid),
    data = band_it1,
    alpha = 0.6) +

  theme_classic()
```

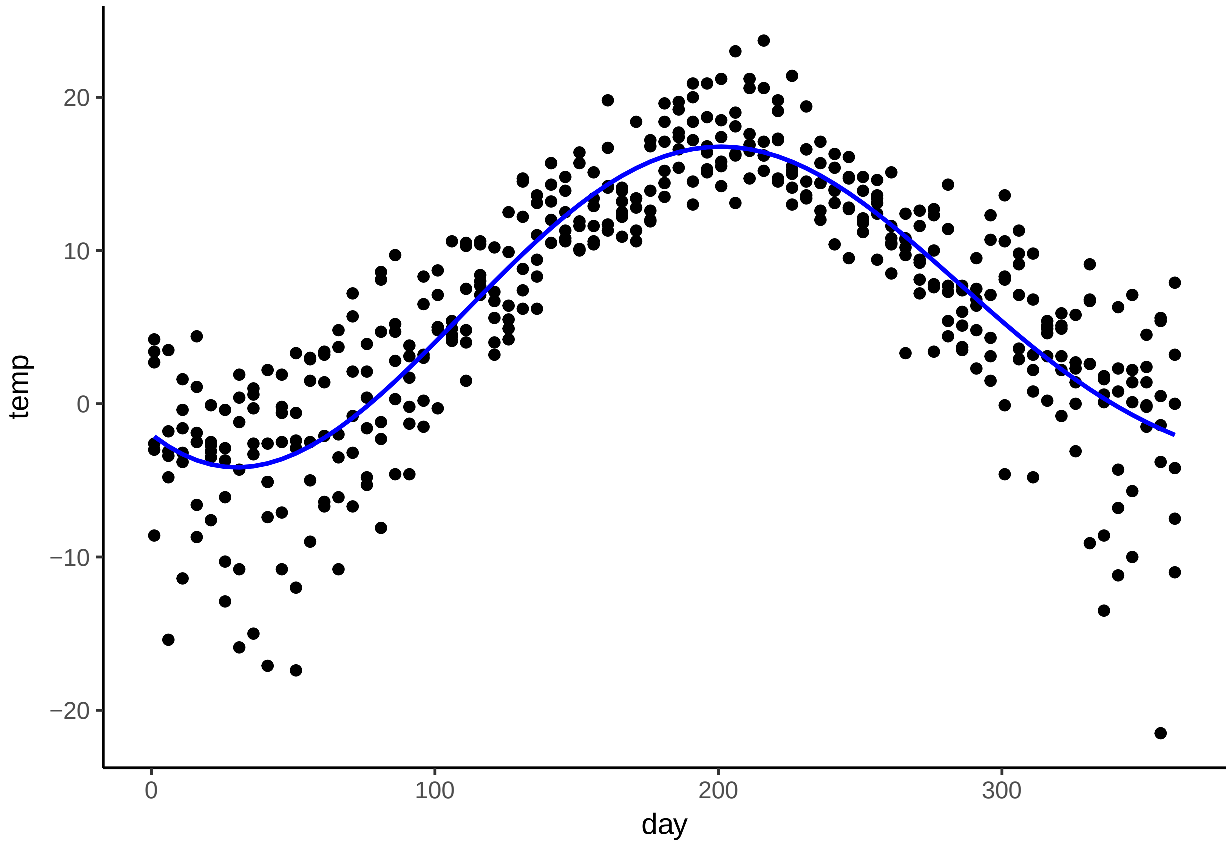



```
#plot(x=band_it1$xGrid, y=band_it1$fbar, type = "l")
```

d)

```
a2d_kern <- kern_maker(l = 1.2, sigmaf = 20 )
gp_tullinge_d <- gausspr(x = tullinge$day, y = tullinge$temp, kernel = a2d_kern)
```

```r
ggplot(data = tullinge, aes(x= day, y = temp)) +
  geom_point() +
  geom_line(aes(y = predict(gp_tullinge_d)), col = "blue", size = 0.8) +
  theme_classic()
```




```r
# plot(y = tullinge$temp, x = tullinge$day)
# #lines(x = tullinge$time, y = fitted(lm_tull), col = "red")
# lines(x = tullinge$day, y = predict(gp_tullinge_d), col = "red" , lwd = 1)
```

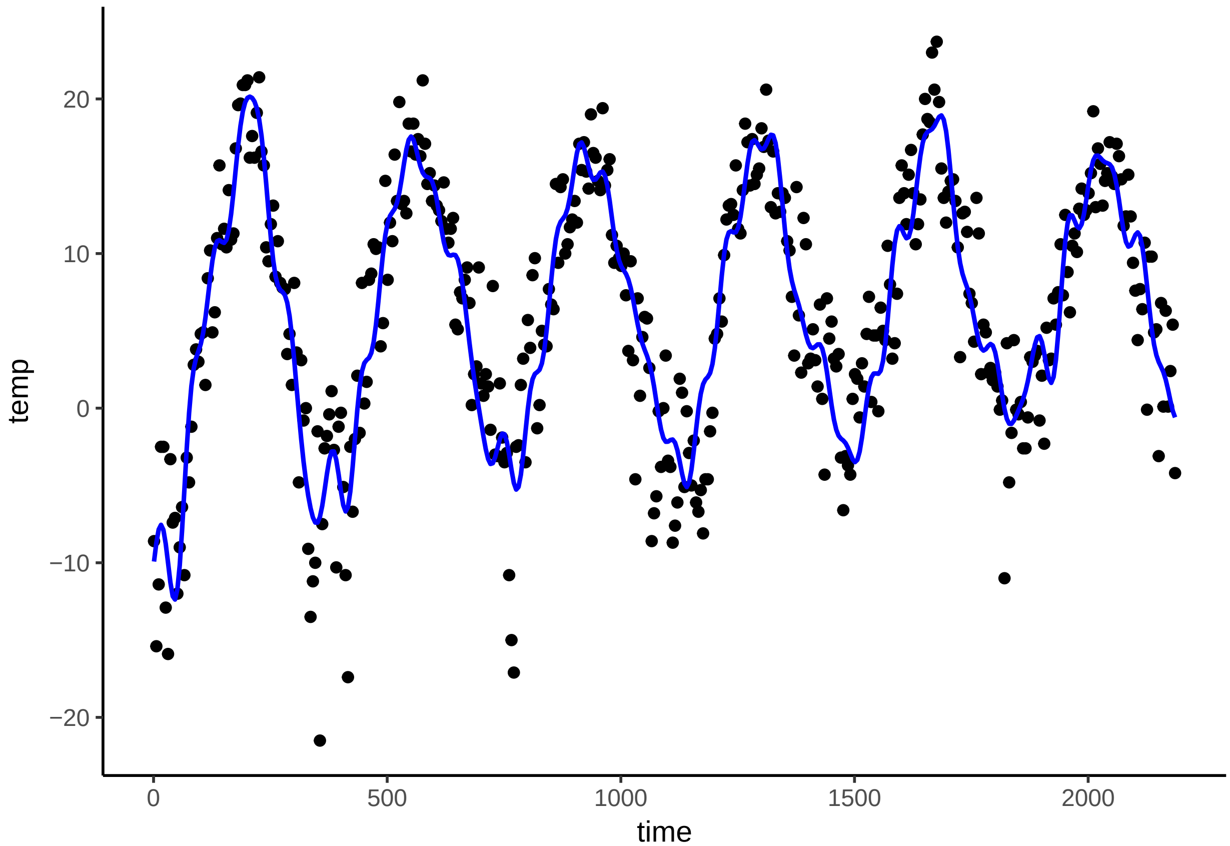
e)

```r
# periodic_kernel <- function(x,xi,sigmaf,d, l_1, l_2){
#
#   part1 <- exp(2 * sin(pi * abs(x - xi) / d)^2 / l_1^2 )
#   part2 <- exp(-0.5 * abs(x - xi)^2 / l_2)
#
#   sigmaf^2 * part1 * part2
#
# }



kern_maker2 <- function(sigmaf,d, l_1, l_2){

 periodic_kernel <- function(x,y = NULL){
```

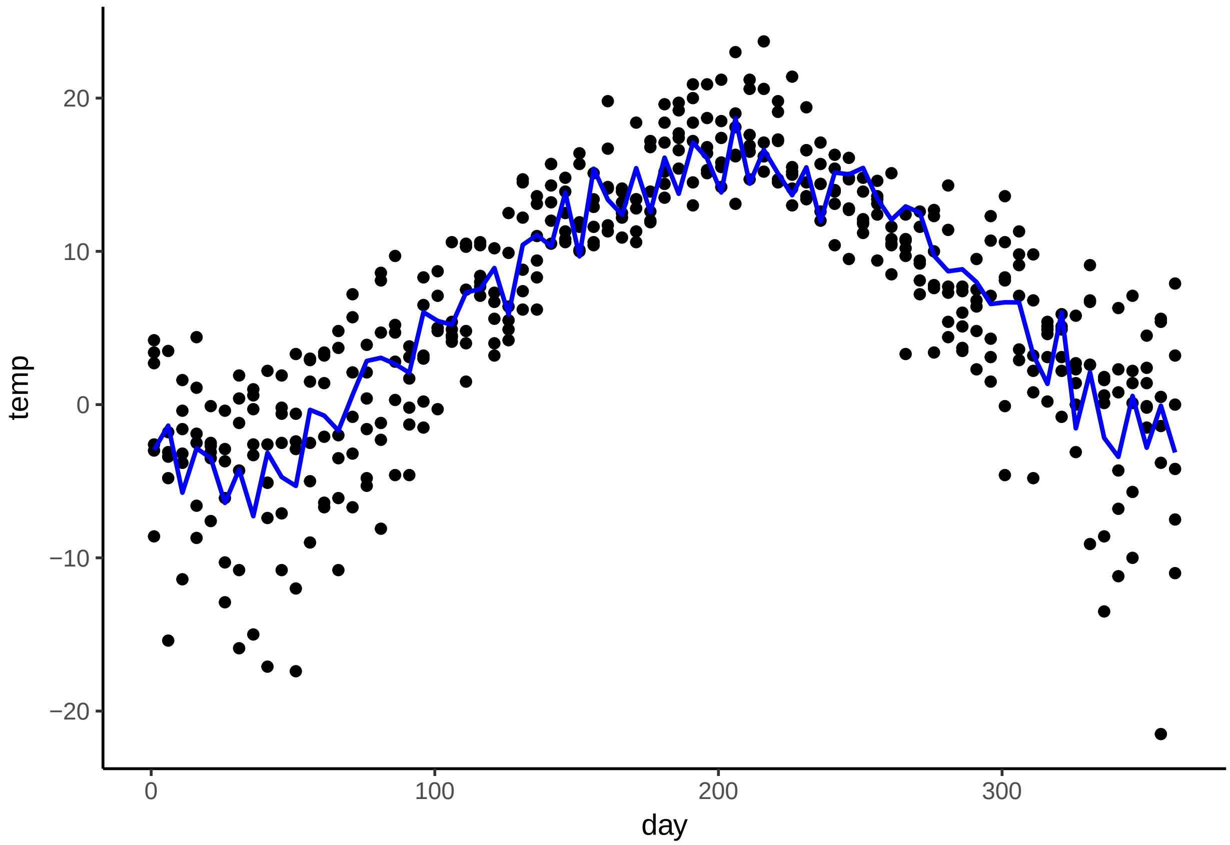
```
  part1 <- exp(2 * sin(pi * abs(x - y) / d)^2 / l_1^2 )
  part2 <- exp(-0.5 * abs(x - y)^2 / l_2)

  sigmaf^2 * part1 * part2

}


  class(periodic_kernel) <- "kernel"
  return(periodic_kernel)
}
```

```
sigmaff <- 20
l1 <- 1
l2 <- 10
d_est <- 365 / sd(tullinge$time)

periodic_kernel <- kern_maker2(sigmaf = sigmaff,
                               d = d_est ,
                               l_1 = l1,
                               l_2 = l2)

gp_tullinge_et <- gausspr(x = tullinge$time,
                      y = tullinge$temp,
                      kernel = periodic_kernel)


gp_tullinge_ed <- gausspr(x = tullinge$day,
                      y = tullinge$temp,
                      kernel = periodic_kernel)
```

```
ggplot(data = tullinge, aes(x= time, y = temp)) +
  geom_point() +
  geom_line(aes(y = predict(gp_tullinge_et)), col = "blue", size = 0.8) +
  theme_classic()
```




```r
ggplot(data = tullinge, aes(x= day, y = temp)) +
  geom_point() +
  geom_line(aes(y = predict(gp_tullinge_ed)), col = "blue", size = 0.8) +
  theme_classic()
```



Question 3

```r
names(data) <- c("varWave","skewWave","kurtWave","entropyWave","fraud")
data[,5] <- as.factor(data[,5])
set.seed(111)
SelectTraining <- sample(1:dim(data)[1], size = 1000, replace = FALSE)
train <- data[SelectTraining,]
test <- data[-SelectTraining,]
```

a)

```r
colnames(data)
```

```
## [1] "varWave"     "skewWave"    "kurtWave"    "entropyWave" "fraud"
```

```r
GPfitFraud <- gausspr(fraud ~  varWave + skewWave, data = train)
```

```
## Using automatic sigma estimation (sigest) for RBF or laplace kernel
```

```r
GPfitFraud
```

```
## Gaussian Processes object of class "gausspr"
## Problem type: classification
##
## Gaussian Radial Basis kernel function.
```

```
##  Hyperparameter : sigma =  1.2043047635594
##
## Number of training instances learned : 1000
## Train error : 0.068
```

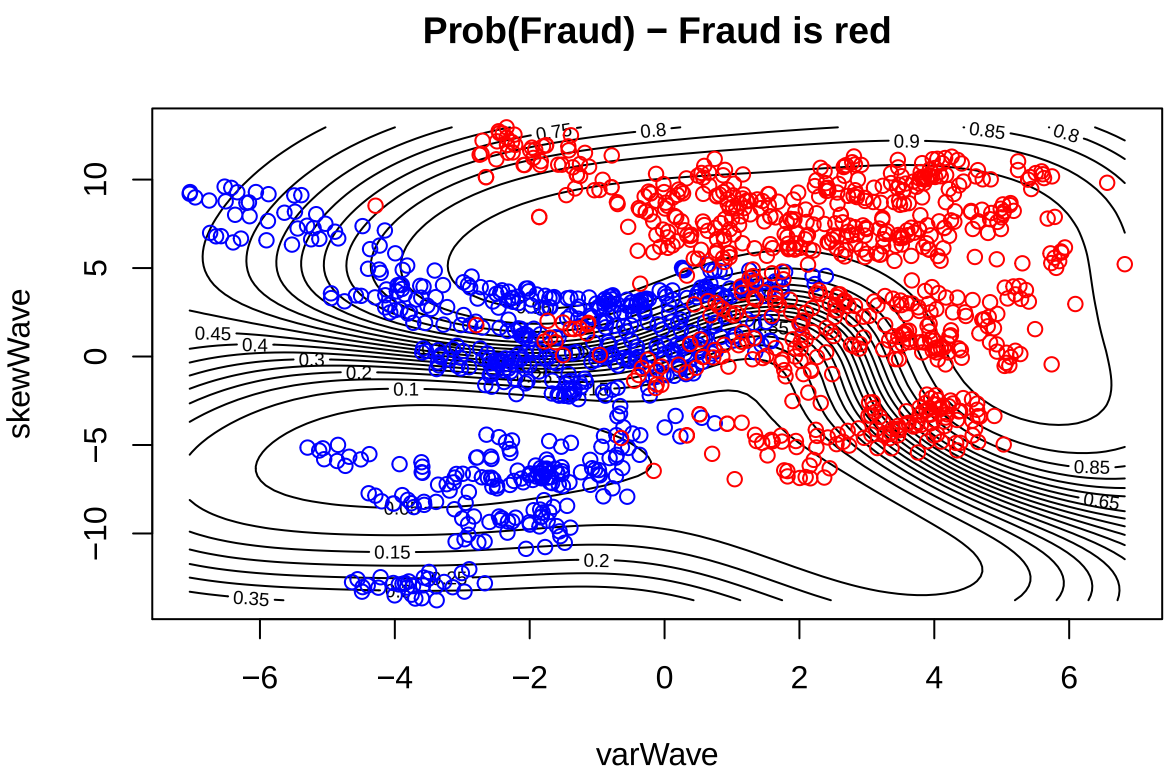
```
# predict on the test set
fit_train<- predict(GPfitFraud,train[,c("varWave","skewWave")])
table(fit_train, train$fraud) # confusion matrix
```

```
##
## fit_train   0   1
##         0 512  24
##         1  44 420
```

```
mean(fit_train == train$fraud)
```

```
## [1] 0.932
```

```
# probPreds <- predict(GPfitIris, iris[,3:4], type="probabilities")
x1 <- seq(min(data[,"varWave"]),max(data[,"varWave"]),length=100)
x2 <- seq(min(data[,"skewWave"]),max(data[,"skewWave"]),length=100)
gridPoints <- meshgrid(x1, x2)
gridPoints <- cbind(c(gridPoints$x), c(gridPoints$y))

gridPoints <- data.frame(gridPoints)
names(gridPoints) <- c("varWave","skewWave")
probPreds <- predict(GPfitFraud, gridPoints, type="probabilities")

contour(x1,x2,matrix(probPreds[,1],100), 20,
        xlab = "varWave", ylab = "skewWave",
        main = 'Prob(Fraud) - Fraud is red')

points(data[data[,5]== 1,"varWave"],data[data[,5]== 1,"skewWave"],col="blue")
points(data[data[,5]== 0,"varWave"],data[data[,5]== 0,"skewWave"],col="red")
```



b)

```
# predict on the test set
fit_test<- predict(GPfitFraud,test[,c("varWave","skewWave")])
table(fit_test, test$fraud) # confusion matrix
```

```
##
## fit_test   0   1
##        0 191   9
##        1  15 157
```

```
mean(fit_test == test$fraud)
```

```
## [1] 0.9354839
```

c)

```
GPfitFraudFull <- gausspr(fraud ~ ., data = train)
```

```
## Using automatic sigma estimation (sigest) for RBF or laplace kernel
```

```
GPfitFraudFull
```

```
## Gaussian Processes object of class "gausspr"
## Problem type: classification
##
## Gaussian Radial Basis kernel function.
##  Hyperparameter : sigma =  0.399933221120042
##
```

```
## Number of training instances learned : 1000
## Train error : 0.004
```

```
# predict on the test set
fit_Full<- predict(GPfitFraudFull,test[,-ncol(test)])
table(fit_Full, test$fraud) # confusion matrix
```

```
##
## fit_Full   0   1
##        0 205   0
##        1   1 166
```

```
mean(fit_Full == test$fraud)
```

```
## [1] 0.9973118
```