Oblig 10

Oppgave 10.2.1 c

Matrisen $C = \begin{bmatrix} -2 & 1 & -6 & 9 \\ 0 & -8 & -7 & -3 \end{bmatrix}$ er på formen 2×4

$a_{12} = 1$
$a_{22} = -8$
$a_{31} =$ Eksisterer ikke
$a_{24} = -3$

Oppgave 10.2.2

a) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ Er kvadratisk ettersom den er på formen 2×2.
Er triangulær ettersom alle elementene under diagonalen er 0

b) $\begin{bmatrix} -2 & 3 & 0 \\ 0 & -1 & 4 \end{bmatrix}$ Er ingen av de nevnte

c) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Er både kvadratisk og diagonal ettersom matrisen er 3×3 og alle elementene over og under diagonalen er 0.

d) $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \\ 3 & 0 & 5 \end{bmatrix}$ Er kvadratisk og triangulær

Oppgave 10.2.3 c

$A = \begin{bmatrix} 4 & 2 & 0 \\ -2 & 5 & -1 \\ -1 & 0 & 2 \end{bmatrix}$ $A^T = \begin{bmatrix} 4 & -2 & -1 \\ 2 & 5 & 0 \\ 0 & -1 & 2 \end{bmatrix}$

Matrisen er ikke symmetrisk

Oppgave 10.2.5 b

$2B - 3A^T$ $2B = \begin{bmatrix} 4 & 2 \\ -6 & 4 \\ 2 & 8 \end{bmatrix}$ $3A^T = \begin{bmatrix} 3 & 6 \\ 0 & 9 \\ 12 & -6 \end{bmatrix}$ $2B - 3A^T = \begin{bmatrix} 4-3 & 2-6 \\ -6 & 4-9 \\ 2-12 & 8+6 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ -6 & -5 \\ -10 & 14 \end{bmatrix}$

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Oppgave 10.2.6 a)

$$AB = \quad A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \cdot B = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 0\cdot 3 & 2\cdot(-2) + 0\cdot(-1) \\ 1\cdot 1 + 3\cdot 3 & 1\cdot(-2) + 3\cdot(-1) \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 10 & -5 \end{bmatrix}$$

Oppgave 10.2.7

a) AB, BA

$$\underset{3\times 3 \;\; 3\times 2}{AB} \quad 3\times 2$$

$$AB = A \begin{bmatrix} 3 & 1 & -1 \\ -1 & 2 & 0 \\ -3 & 0 & -2 \end{bmatrix} \cdot B \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3\cdot 2 + 1\cdot 1 - 1\cdot 3 & 3\cdot(-1) + 1\cdot 0 + (-1)\cdot(-2) \\ -1\cdot 2 + 2\cdot 1 + 0\cdot 3 & -1\cdot(-1) + 2\cdot 0 + 0\cdot(-2) \\ -3\cdot 2 + 0\cdot 1 + (-2)\cdot 3 & -3\cdot(-1) + 0\cdot 0 - 2\cdot(-2) \end{bmatrix} = \begin{bmatrix} 6+1-3 & -3+0+2 \\ 0 & 1 \\ -6-6 & 3+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -1 \\ 0 & 1 \\ -12 & 7 \end{bmatrix}$$

$$\underset{3\times 2 \;\; 3\times 3}{BA} = \text{Ikke løslig}$$

d) $\underset{3\times 2 \;\; 1\times 3}{BC^T} = \text{Ikke løslig}$

$$\underset{1\times 3 \;\; 3\times 2}{C^T B} \quad 1\times 2$$

$$C^T B = C^T \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \cdot B \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 2\cdot 2 - 1\cdot 1 + 3\cdot 3, & 2\cdot(-1) - 1\cdot 0 + 3\cdot(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 - 1 + 9, & -2 - 6 \end{bmatrix}$$

$$C^T B = \begin{bmatrix} 12 & -8 \end{bmatrix}$$

Oppgave 10.2.8a

$$A \cdot B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 6-5 & -15+15 \\ 2-2 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6-5 & 10-10 \\ -3+3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Ved å bestemme begge matrisene ser vi at AB og BA = I

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Oppgave 10.2.12a

$A = \begin{bmatrix} 2 & -3 \\ 4 & -1 \end{bmatrix}$ $\quad b = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ $\quad A^{-1} = \frac{1}{2\cdot(-1)+3\cdot 4}\begin{bmatrix} -1 & 3 \\ -4 & 2 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -1 & 3 \\ -4 & 2 \end{bmatrix}$

$A^{-1}\cdot b = \frac{1}{10}\begin{bmatrix} -1 & 3 \\ -4 & 2 \end{bmatrix}\cdot\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+12 \\ 4+8 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 13 \\ 12 \end{bmatrix}$

$Ax = b \iff \underline{\frac{1}{10}\begin{bmatrix} 13 \\ 12 \end{bmatrix}}$

Oppgave 10.3.1 f

$A = \begin{bmatrix} 4 & 1 & 4 \\ 2 & -2 & 1 \\ -2 & 1 & -3 \end{bmatrix}$ $\quad \det A = \left[\begin{array}{ccc|cc} 4 & 1 & 4 & 4 & 1 \\ 2 & -2 & 1 & 2 & -2 \\ -2 & 1 & -3 & -2 & 1 \end{array}\right]$

$4\cdot(-2)\cdot(-3) + 1\cdot 1\cdot(-2) + 4\cdot 2\cdot 1 - 1\cdot 2\cdot(-3) - 4\cdot 1\cdot 1 - 4\cdot(-2)\cdot(-2)$

$= 24 - 2 + 8 + 6 - 4 - 16$

$\underline{\det A = 16}$

Oppgave 10.3.9

b) $B = \begin{bmatrix} 2 & 4 & -3 \\ 1 & 3 & -1 \\ -3 & 0 & 2 \end{bmatrix}$ $\quad \det B = \left[\begin{array}{ccc|cc} 2 & 4 & -3 & 2 & 4 \\ 1 & 3 & -1 & 1 & 3 \\ -3 & 0 & 2 & -3 & 0 \end{array}\right]$

$2\cdot 3\cdot 2 + 4\cdot(-1)\cdot(-3) - 3\cdot 1\cdot 0$
$- 4\cdot 1\cdot 2 - 2\cdot(-1)\cdot 0 + 3\cdot 3\cdot(-3)$

$= 12 + 12 + 0$
$- 8 - 0 - 27$

$\underline{\det B = -11}$ $\quad$ Ikke singulær ettersom $\det B \neq 0$

c) $C = \begin{bmatrix} 3 & 1 & -2 \\ -2 & 3 & 7 \\ 4 & 5 & 3 \end{bmatrix}$ $\quad \det C = \left[\begin{array}{ccc|cc} 3 & 1 & -2 & 3 & 1 \\ -2 & 3 & 7 & -2 & 3 \\ 4 & 5 & 3 & 4 & 5 \end{array}\right]$

$3\cdot 3\cdot 3 + 1\cdot 7\cdot 4 - 2\cdot(-2)\cdot 5$
$- 1\cdot(-2)\cdot 3 - 3\cdot 7\cdot 5 + 2\cdot 3\cdot 4$

$= 27 + 28 + 20$
$+ 6 - 105 + 24$

$\det C = 0$ $\quad$ Er singulær ettersom $\det C = 0$

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Oppgave 10.3.10 a

$A: \begin{bmatrix} 4-t & -3 \\ 1 & -t \end{bmatrix}$ $\quad \det A = (4-t)(-t)+3 = t^2-4t+3 \Rightarrow t^2-4t+3=0$

$$\frac{4 \pm \sqrt{16-4\cdot 1\cdot 3}}{2} = \frac{4 \pm \sqrt{16-12}}{2}$$

$$\frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} \qquad t_1 = \frac{6}{2} = 3$$

$$t_2 = \frac{2}{2} = 1$$

Svar: $t = 1, 3$

Oppgave 1:

a)

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ -2x_1 + 0 + x_3 &= -7 \\ x_1 - x_2 + 0 &= 5 \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ -2 & 0 & 1 & -7 \\ 1 & -1 & 0 & 5 \end{array}\right] \qquad A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -7 \\ 5 \end{bmatrix}$$

b)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & 1 & 0 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right] \begin{aligned} R_2' &= R_2 + 2R_1 \\ R_3' &= R_3 - R_1 \end{aligned}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{2}{4} & \frac{1}{4} & 0 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right] R_2' = \frac{1}{4}R_2 \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{2}{4} & \frac{1}{4} & 0 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right] R_1' = R_1 - 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{2}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & 1 \end{array}\right] R_3' = R_3 + 3R_2 \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{2}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right] R_3' = 4 \cdot R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & -\frac{1}{4} & \frac{2}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right] R_1' = R_1 + \frac{1}{2}R_3 \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right] R_2' = R_2 + \frac{1}{4}R_3$$

Følgelig er $A^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

c) $A^{-1} \cdot b$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -7 \\ 5 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 1\cdot(-7) + 2\cdot(5) \\ 1\cdot 1 + 1\cdot(-7) + 1\cdot 5 \\ 2\cdot 1 + 3(-7) + 4\cdot 5 \end{bmatrix} = \begin{bmatrix} 1-7+10 \\ 1-7+5 \\ 2-21+20 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$