Oblig 7

Oppgave 3.6.1

c) $1+5+9+\dots+(4n+1)=(n+1)(2n+1)$, for $n\in\mathbb{N}$

1 $n=0$

$\hookrightarrow 4\cdot 0+1=1$

$(0+1)(2\cdot 0+1)=1\cdot 1=1$ } VS = HS ok!

2 $k+1$

$\hookrightarrow (k+1)(2k+1)+4k+5 = ((k+1)+1)(2(k+1)+1$

$2k^2+k+2k+1+4k+5 = (k+2)(2k+3)$

$2k^2+7k+6 = 2k^2+7k+6$

VS = HS

d) $1+3^1+3^2+\dots+3^n=\frac{3^{n+1}-1}{2}$, for $n\in\mathbb{N}$

1 $n=0$

$\hookrightarrow 1^0=1$

$\frac{3^{0+1}-1}{2}=\frac{2}{2}=1$ } VS = HS ok!

2 $k+1$

$\hookrightarrow 3^{k+1}+\frac{3^{k+1}-1}{2}=\frac{3^{k+2}-1}{2}$

$\frac{2\cdot 3^{k+1}}{2}+\frac{3^{k+1}-1}{2}=\frac{3^{k+2}-1}{2}$

$\frac{3\cdot 3^{k+1}-1}{2}=\frac{3^{k+2}-1}{2}$

$\frac{3^{k+2}-1}{2}=\frac{3^{k+2}-1}{2}$

VS = HS

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$\boxed{\text{oppgave } 3.6.4}$

c) $11^n - 4^n$, delbar med 7, for $n \in \mathbb{Z}^+$

1 $n=1$

$\rightarrow 11 - 4 = 7 \rightarrow 7 \mid 7$

2 $k+1$

$\rightarrow 11^k - 4^k = 7m$

$11^{k+1} - 4^{k+1} = 11 \cdot 11^k - 4 \cdot 4^k$

$11 \cdot 11^k - 4 \cdot 4^k = 11(7m + 4^k) - 4 \cdot 4^k = 77m + 11 \cdot 4^k - 4 \cdot 4^k$

$= 77m + 7 \cdot 4^k = \underline{\underline{7(11m + 4^k)}}$

$\boxed{\text{oppgave } 3.7.1}$

b) $y_n = 4y_{n-1} - 1$

$y_2 = 4 \cdot 1 - 1 = 3$
$y_3 = 4 \cdot 3 - 1 = 11$
$y_4 = 4 \cdot 11 - 1 = 43$
$y_5 = 4 \cdot 43 - 1 = 171$

c) $y_n = 2^{y_{n-1}}$

$y_2 = 2^1 = 2$
$y_3 = 2^2 = 4$
$y_4 = 2^4 = 16$
$y_5 = 2^{16} = 65536$

$\boxed{\text{oppgave } 3.7.2}$

b) $y_{n+1} = -3y_n + 2$

$y_1 = -3 \cdot 3 + 2 = -7$
$y_2 = -3 \cdot (-7) + 2 = 23$
$y_3 = -3 \cdot 23 + 2 = -67$
$y_4 = -3 \cdot (-67) + 2 = 203$

c) $y_{n+1} = -y_n^2 + 2y_n + 3$

$y_1 = -3^2 + 2 \cdot 3 + 3 = -9 + 9 = 0$
$y_2 = 0^2 + 2 \cdot 0 + 3 = 3$
$y_3 = -3^2 + 2 \cdot 3 + 3 = -9 + 9 = 0$
$y_4 = 0 + 0 + 3 = 3$

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Oppgave 3.7.8

a) $a_n = 3n$, for $n \in \mathbb{Z}^+$

3, 6, 9, 12, 15 . . . (øker med 3)

$y_{n-1} = 3(n-1) = 3n-3$

$y_n - y_{n-1} = 3n - (3n-3) = \underline{3}$

$\underline{\underline{y_n = y_{n-1} + 3}}$

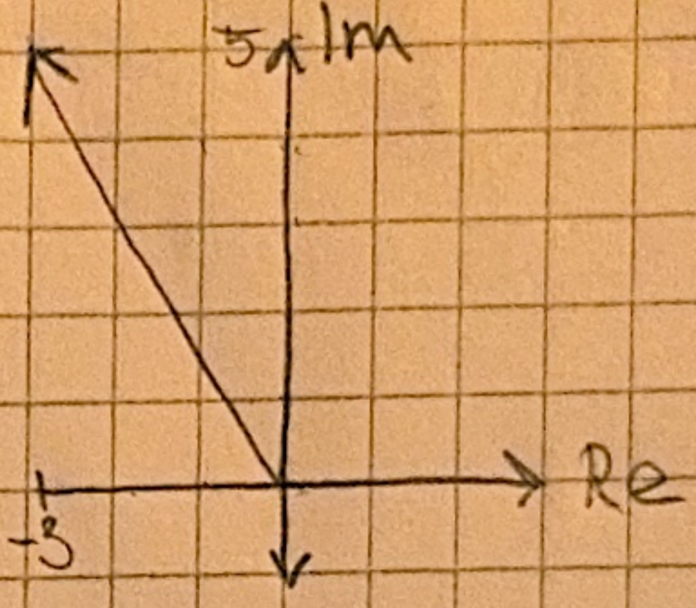
b) $a_n = 2n-1$, for $n \in \mathbb{Z}^+$

1, 3, 5, 7 . . . (øker med 2)

$y_{n-1} = 2(n-1) - 1 = 2n-2-1 = 2n-3$

$y_n - y_{n-1} = (2n-1) - (2n-3) = \underline{2}$

$\underline{\underline{y_n = y_{n-1} + 2}}$



Oppgave 4.4.1 b

b) $-3 + \sqrt{-25} = -3 + \sqrt{25} \cdot \sqrt{-1} = \underline{\underline{-3+5i}}$

Oppgave 4.4.2

b) $x^2 + 7 = 0 \to x^2 = -7 \to x = \pm\sqrt{-7} \to x = \pm\sqrt{7} \cdot \sqrt{-1} \to \underline{\underline{x = \pm\sqrt{7}\, i}}$

c) $x^2 - 2x + 2 = 0 \to$ abc formel $\to x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$

$x = \frac{2 \pm \sqrt{4-8}}{2}$ $\quad x = \frac{2 \pm \sqrt{-4}}{2}$

$x = \frac{2 \pm \sqrt{4} \cdot \sqrt{-1}}{2}$ $\quad x = \frac{2 \pm 2i}{2}$ $\quad \underline{\underline{x = 1 \pm i}}$

d) $3x^2 + 2x + 1 \to$ abc formel

$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 3}}{2 \cdot 3} \to x = \frac{-2 \pm \sqrt{4-12}}{6} \to x = \frac{-2 \pm \sqrt{8} \cdot \sqrt{-1}}{6}$

$x = \frac{2 \pm \sqrt{4 \cdot 2} \cdot \sqrt{-1}}{6}$ $\quad x = \frac{2 \pm 2i\sqrt{2}}{6}$ $\quad x = \frac{2}{6} \pm \frac{2i\sqrt{2}}{6}$ $\quad \underline{\underline{x = \frac{1}{3} \pm \frac{i\sqrt{2}}{3}}}$

Oblig 7

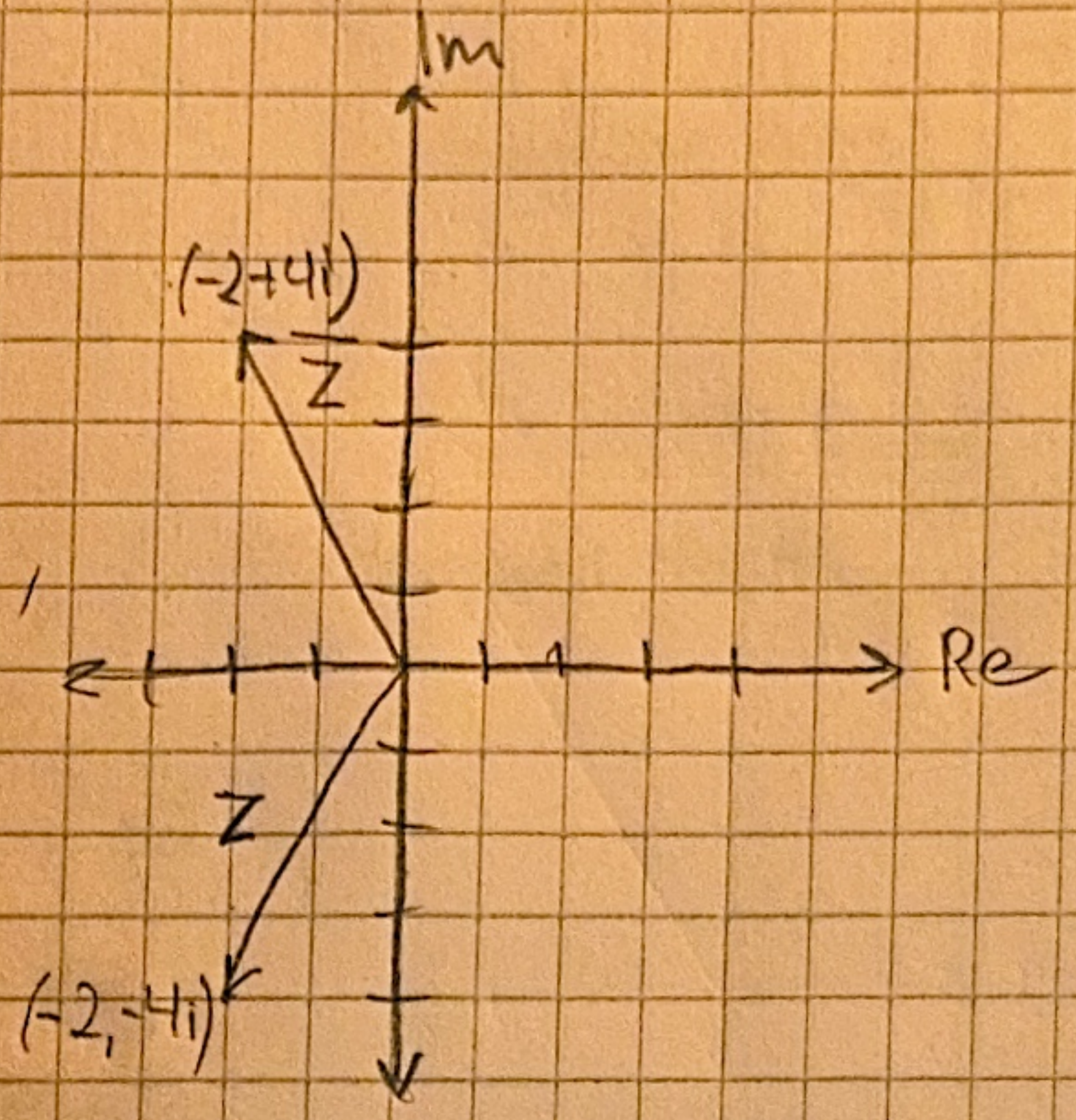
oppgave 4.4.4

a) $3+\sqrt{-16}-(2+5i) = 3+\sqrt{16}\cdot\sqrt{-1}-(2+5i) = 3+4i-2-5i = \underline{\underline{1-i}}$

c) $(3+2i)\cdot(-i) = -3i-2i^2 = -3i-2\cdot(-1) = \underline{-3i+2} \iff \underline{2-3i}$

oppgave 4.4.5 d

d) $z = -2-4i$, $\underline{\bar{z} = -2+4i}$



oppgave 4.4.6

a) $\frac{2}{i} = \frac{2}{i}\cdot\frac{(-i)}{(-i)} = \underline{\underline{-2i}}$

c) $\frac{i}{3-i} = \frac{i}{3-i}\cdot\frac{3+i}{3+i} = \frac{i(3+i)}{(3-i)(3+i)} = \frac{3i+i^2}{9+\cancel{3i}-\cancel{3i}-i^2} = \frac{3i-1}{9+1} = \frac{3i-1}{10} = \underline{\underline{\frac{3i}{10}-\frac{1}{10}}}$

oppgave 4.4.8

b) $-\sqrt{3}+i \rightarrow r = \sqrt{(-\sqrt{3})^2+1^2} = \sqrt{3+1} = \sqrt{4} = \underline{2}$

$\cos\theta = \frac{a}{r} = \frac{-\sqrt{3}}{2} = \frac{5\pi}{6}$

polarform: $z = r(\cos\theta + i\sin\theta) \iff \underline{\underline{z = 2\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)}}$

Oblig 7

Oppgave 4.4.10

a) $e^{i\left(\frac{\pi}{3}\right)} \rightarrow \left.\begin{array}{l} a = 1 \cdot \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \\ b = 1 \cdot \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \end{array}\right\}$ Rektangulær form: $\underline{\underline{\frac{1}{2} + \frac{\sqrt{3}}{2} i}}$

b) $e^{i\left(\frac{5\pi}{6}\right)} \rightarrow \left.\begin{array}{l} a = 1 \cdot \cos\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2} \\ b = 1 \cdot \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2} \end{array}\right\}$ Rektangulær form: $\underline{\underline{\frac{-\sqrt{3}}{2} + \frac{1}{2} i}}$

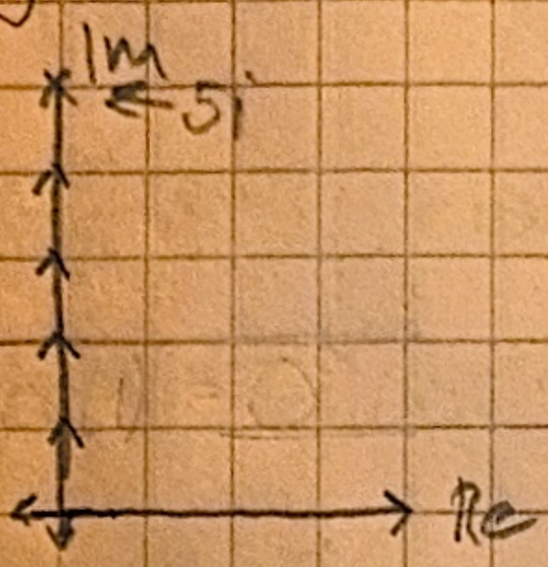
c) $e^{i\left(-\frac{\pi}{6}\right)} \rightarrow \left.\begin{array}{l} a = 1 \cdot \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \\ b = 1 \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2} \end{array}\right\}$ Rektangulær form: $\underline{\underline{\frac{\sqrt{3}}{2} - \frac{1}{2} i}}$

Oppgave 4.4.11

a) $5i \rightarrow r = \sqrt{5^2} = 5$

$\sin\theta = \frac{b}{r} = \frac{5}{5} = 1$

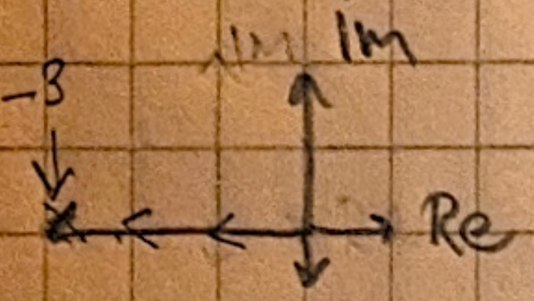
$\sin^{-1}(1) = 90°$



$\theta = 90° = \frac{\pi}{2}$ Eksponentiell form: $\underline{\underline{z = 5e^{i\frac{\pi}{2}}}}$ ← kvadrant 1

b) $-3 \rightarrow r = \sqrt{(-3)^2} = 3$

$\cos\theta = \frac{a}{r} = \frac{-3}{3} = -1$



$\theta = 180° = \pi$ Eksponentiell form: $\underline{\underline{z = 3e^{i(\pi)}}}$ ← kvadrant 2

c) $1+\sqrt{3}i \rightarrow r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$

$\cos\theta = \frac{a}{r} = \frac{1}{2} = \frac{\pi}{3}$

$\sin\theta = \frac{b}{r} = \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ Eksponentiell form: $\underline{\underline{z = 2e^{i\left(\frac{\pi}{3}\right)}}}$ ← kvadrant 1

d) $-\sqrt{3} + 3i \rightarrow r = \sqrt{(-\sqrt{3})^2 + 3^2} = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$

$\cos\theta = \frac{a}{r} = \frac{-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2}$ $= \frac{2\pi}{3}$

$\sin\theta = \frac{b}{r} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$ ← kvadrant 2

Eksponentiell form $= \underline{\underline{2\sqrt{3}e^{i\frac{2\pi}{3}}}}$

Oblig 7

$\boxed{\text{oppgave 1}}$

Fjerde kvadrant

a) $z \cdot w =$ Andre kvadrant

$r_z = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$r_w = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\cos\theta_z = \frac{3}{r_z} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2} \Leftrightarrow \frac{\pi}{6}$

$\sin\theta_z = \frac{-\sqrt{3}}{r_z} = \frac{-\sqrt{3}}{2\sqrt{3}} \Leftrightarrow -\frac{1}{2}$

$\cos\theta_w = \frac{-2}{4} = -\frac{1}{2} \Leftrightarrow \frac{2\pi}{3}$

$\sin\theta_w = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Polarform $z \cdot w = 2\sqrt{3} \cdot 4 \left(\cos\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right) + i\sin\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right)\right)$

$= 8\sqrt{3}\left(\cos\left(\frac{3\pi}{6}\right) + i\sin\left(\frac{3\pi}{6}\right)\right) = 8\sqrt{3}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$\cos\left(\frac{\pi}{2}\right) = 0$ og $\sin\left(\frac{\pi}{2}\right) = 1$

$\underline{\underline{= 8\sqrt{3}\, i}}$

b) $z : w$

$r_z = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$

$r_w = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$

$\cos\theta_z = \frac{3}{2\sqrt{3}}$ -ll- $= \frac{\pi}{6}$

$\sin\theta_z =$ -ll- $= -\frac{1}{2}$

$\cos\theta_w = \frac{2\pi}{3} = \frac{4\pi}{6}$

$\sin\theta_w = \frac{\sqrt{3}}{2}$

Polarform $\frac{z}{w} = \frac{2\sqrt{3}}{4}\left(\cos\left(\frac{\pi}{6} - \frac{4\pi}{6}\right) + i\sin\left(\frac{\pi}{6} - \frac{4\pi}{6}\right)\right)$

$\underline{\underline{= \frac{\sqrt{3}}{2} e^{-i\frac{5\pi}{6}}}}$

Oblig 7

Oppgave 2

a) $Z \cdot W = 9e^{\frac{5\pi}{6}i} \cdot 3e^{\frac{\pi}{6}i}$

$\hookrightarrow 9 \cdot 3 \cdot e^{i\left(\frac{5\pi}{6} + \frac{\pi}{6}\right)} = 27e^{i\left(\frac{6\pi}{6}\right)} = \underline{\underline{27e^{i\pi}}}$

b) $\frac{Z}{W} = \frac{9e^{\frac{5\pi}{6}i}}{3e^{\frac{\pi}{6}i}}$

$\hookrightarrow \frac{9}{3}e^{i\left(\frac{5\pi}{6} - \frac{\pi}{6}\right)} = 3e^{i\left(\frac{4\pi}{6}\right)} = \underline{\underline{3e^{i\left(\frac{2\pi}{3}\right)}}}$

c) $Z = r(\cos\theta + i\sin\theta)$

$Z = 9\left(\cos\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)$

$Z = 9\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$Z = \underbrace{-\frac{9\sqrt{3}}{2}}_{Re} + \underbrace{\frac{9}{2}i}_{Im}$

Realdelen til $Z = \underline{\underline{\frac{9\sqrt{3}}{2}}}$

Imaginærdelen til $Z = \underline{\underline{\frac{9}{2}i}}$