Oblig 8

$\boxed{\text{oppgave } 6.2.6}$

a) y_n med n bokstaver som har jevnt antall b-er.

$\hookrightarrow y_n = \underbrace{2y_{n-1}}_{\text{partall}} + \underbrace{y_{n-1}}_{\text{oddetall}} \rightarrow y_n + y_n = 3^n$

$\underline{\underline{y_n - y_{n-1} = 3^{n-1}}}$

b) Startbetingelse: $n=1$

$\hookrightarrow \underline{\underline{y_1 = 2}}$, fordi det kun er a og c som gir partall antall b-er.

$\boxed{\text{oppgave } 6.2.8}$

a) Et kodeord er gyldig dersom det inneholder et like antall 4-ere.

$\hookrightarrow$ 1. Siste siffer ikke 4

$\hookrightarrow y_n = 7 \cdot y_{n-1}$

$\hookrightarrow$ 2. Siste siffer er 4

$\hookrightarrow y_n = 8^{n-1} - y_{n-1}$

$y_n = 6y_{n-1} + 8^{n-1} \quad | -6y_{n-1}$

$\underline{\underline{y_n - 6y_{n-1} = 8^{n-1}}}$

b) Startbetingelse:

$\underline{\underline{y_0 = 1}}$, $y_1 = 7$

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Oppgave 6.3.2

b) $y_n + 2y_{n-1} = 0$

KL: $\lambda + 2 = 0$

$\lambda = -2$

Generell løsning: $\underline{\underline{y_n = A \cdot (-2)^n}}$

d) $y_n - 5y_{n-1} + 6y_{n-2} = 0$

KL: $\lambda^2 - 5\lambda + 6 = 0$

ABC $\dfrac{-(-5) \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2} = \dfrac{5 \pm \sqrt{25-24}}{2} = \dfrac{5 \pm \sqrt{1}}{2} = \dfrac{5 \pm 1}{2}$

$x_1 = \dfrac{6}{2} = \underline{3}$ $x_2 = \dfrac{4}{2} = \underline{2}$

Generell løsning: $\underline{\underline{y_n = A \cdot 3^n + B \cdot 2^n}}$

f) $4y_n + 8y_{n-1} + 3y_{n-2} = 0$

KL: $4\lambda^2 + 8\lambda + 3 = 0$

ABC $\dfrac{-8 \pm \sqrt{64 - 4 \cdot 4 \cdot 3}}{2 \cdot 4} = \dfrac{-8 \pm \sqrt{64-48}}{8} = \dfrac{-8 \pm \sqrt{16}}{8}$

$= \dfrac{-8 \pm 4}{8}$

$x_1 = \dfrac{-12}{8} = \dfrac{-6}{4} = \dfrac{-3}{2}$

$x_2 = \dfrac{-4}{8} = \dfrac{-2}{4} = \dfrac{-1}{2}$

Generell løsning:

$\underline{\underline{y_n = A \cdot \left(-\frac{3}{2}\right)^n + B \cdot \left(-\frac{1}{2}\right)^n}}$

Oblig 8

$\boxed{\text{Oppgave 6.3.3}}$

b) $y_n - 2y_{n-1} + y_{n-2} = 0$

KL: $\lambda^2 - 2\lambda + 1 = 0$

ABC $\frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2} = \frac{2 \pm \sqrt{4-4}}{2} = \frac{2 \pm 0}{2} = \frac{2}{2} = 1$ (én reel rot)

Generell løsning: $\underline{\underline{y_n = A + Bn}}$

d) $y_n + 4y_{n-2} = 0$

KL: $\lambda^2 + 0 + 4 = 0$

ABC $\frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot 4}}{2} = \frac{0 \pm \sqrt{-16}}{2} = \frac{0 \pm 4 \cdot (-1)}{2} = \frac{0 \pm 4i}{2}$

$x_1 = 2i$, $x_2 = -2i$

(To komplekse røtter)

Generell løsning:

$\underline{\underline{y_n = 2^n \left(A \cos\left(\frac{n\pi}{2}\right) + B \sin\left(\frac{n\pi}{2}\right)\right)}}$

f) $y_n - y_{n-1} - 4y_{n-2} + 4y_{n-3} = 0$

KL: $\lambda^3 - \lambda^2 - 4\lambda + 4 = 0$

1) Prøver $\lambda = 1$

↳ $1 - 1 - 4 + 4 = 0$ OK

2) Polynomdivisjon:

↳ $(\lambda^3 - \lambda^2 - 4\lambda + 4) : (\lambda - 1) = \underline{\lambda^2 - 4}$

$$\begin{array}{l} -(\lambda^3 - \lambda^2) \\ \quad 0 \quad\quad 0 - 4\lambda + 4 \\ \quad\quad\quad -(-4\lambda + 4) \\ \quad\quad\quad\quad\quad\quad 0 \end{array}$$

3) ABC-formel

↳ $\frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot 4}}{2} = \frac{0 \pm 4}{2}$

$x_1 = 2$
$x_2 = -2$
$x_3 = 1$

4) Generell løsning:

$\underline{\underline{y_n = A \cdot 2^n + B \cdot (-2)^n + C^n}}$

Oblig 8

$\boxed{\text{Oppgave } 6.3.4}$

a) $y_n - 6y_{n-1} = 0$, $n \in \mathbb{Z}^+$, $y_0 = 3$

KL: $\lambda - 6 = 0$
KR: $\lambda = 6$

Generell løsning: $y_n = A \cdot 6^n$

$y_0 = A \cdot 6^0 = 3 \iff A \cdot 1 = 3 \iff \underline{A = 3}$

$$\underline{\underline{y_n = 3 \cdot 6^n}}$$

c) $y_n - 2y_{n-1} - 3y_{n-2} = 0$, $n \geq 2$, $y_0 = 3$, $y_1 = 5$

KL: $\lambda^2 - 2\lambda - 3 = 0$

ABC: $\frac{-(-2) \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2} = \frac{-(-2) \pm \sqrt{4 + 12}}{2} = \frac{2 \pm 4}{2}$ $\quad X_1 = 3$, $X_2 = -1$

$\boxed{\text{Generell løsning: } y_n = A \cdot 3^n + B \cdot (-1)^n}$

$y_0 = A\, 3^0 + B \cdot (-1)^0 = 3 \iff A + B = 3 \quad \boxed{A = 3 - B}$

$y_1 = A \cdot 3^1 + B \cdot (-1)^1 = 5 \iff 3A - B = 5$

$3(3 - B) - B = 5$

$9 - 4B = 5 \quad | : (-9)$

$-4B = -4 \quad | : (-4)$

$\underline{B = 1}$

$A = 3 - B$
$A = 3 - 1$
$\underline{A = 2}$

$y_n = \overset{A}{2} \cdot 3^n + \overset{B}{1} \cdot (-1)^n$

$$\underline{\underline{y_n = 2 \cdot 3^n + (-1)^n}}$$

c) $y_n + 4y_{n-1} + 4y_{n-2} = 0$, $n \geq 2$, $y_0 = 1$, $y_1 = 2$

KL: $\lambda^2 + 4\lambda + 4 = 0$

ABC $\rightarrow \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4 \pm 0}{2}$ $x_1 = -2$

Generell løsning: $y_n = (A + Bn)(-2)^n$

$y_0 = (A + B \cdot 0)(-2)^0 = 1 = A = 1$

$y_1 = (A + B \cdot 1)(-2)^1 = 2 = (1+B)(-2) = 2 \rightarrow -2(1+B) = -2$

Generell løsning: $\underline{\underline{y_n = (1+2n)(-2)^n}}$

d) $y_n - 2y_{n-1} + 2y_{n-2} = 0$, $n \geq 2$, $y_0 = 1$, $y_1 = 1$

KL: $\lambda^2 - 2\lambda + 2 = 0$

ABC $\longrightarrow \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = \underline{1 \pm i}$ $r_1 = 1+i$, $r_2 = 1-i$

Generell løsning: $A(1+i)^n + B(1+i)^n$

$y_n = 2^n (A\cos(n\theta) + B\sin(n\theta)) \rightarrow \theta = \frac{\pi}{4}$

$y_n = 2^{\frac{n}{2}} (A\cos(\frac{n\pi}{4}) + B\sin(\frac{n\pi}{4}))$

$y_0 = 2^{\frac{0}{2}} (A\cos(\frac{0\pi}{4}) + B\sin(\frac{0\pi}{4})) = A \cdot 1 + B \cdot 0 = \underline{A}$ $A = 1$

$y_1 = 2^{\frac{1}{2}} (A\cos(\frac{1\pi}{4}) + B\sin(\frac{1\pi}{4})) \rightarrow 1 = \sqrt{2}(1 \cdot \cos(\frac{\pi}{4}) + B\sin(\frac{\pi}{4})) = \cancel{\sqrt{2}}(\frac{1}{\cancel{\sqrt{2}}} + B \cdot \frac{1}{\cancel{\sqrt{2}}})$

$1 = (1 + B) \rightarrow B = 0$

$\underline{\underline{y_n = 2^{\frac{n}{2}} \cos(\frac{n\pi}{4})}}$

Oblig 8

$\boxed{\text{oppgave } 6.3.6\ a}$

$y_n + 2y_{n-1} = 3n^2 - n \; ; \; n \in \mathbb{Z}^+ \; , y_0 = 1$ $\qquad$ $\underline{y_n^h = A \cdot (-2)}$

Homogen løsning: $y_n + 2y_{n-1} = 0$

$KL: \lambda + 2 = 0$

$\lambda = -2$

Partikulær løsning: $y_n^p = A \cdot n^2 + Bn + C$

$(An^2 + Bn + C) + 2(A(n-1)^2 + B(n-1) + C) = 3n^2 - n$

$An^2 + Bn + C + 2An^2 - 4An + 2A + 2Bn - 2B + 2C = 3n^2 - n$

$(A + 2A)n^2 + (B - 4A + 2B) + (C + 2A - 2B + 2C) = 3n^2 - n$

1) $A + 2A = 3 \rightarrow 3A = 3 \rightarrow A = 1$

2) $B - 4A + 2B = -1 \rightarrow 3B - 4 = -1 \rightarrow B = 1$

3) $C + 2A - 2B + 2C = 0 \rightarrow 3C + 2 - 2 = 0 \rightarrow 3C = 0 \rightarrow C = 0$

Partikulær løsning: $y_n^p = n^2 + n$

$$\underline{\underline{y_n = (-2)^n + n^2 + n}}$$

Oblig 8

$\boxed{\text{oppgave } 6.3.12}$

a) $\underbrace{y_n = 3 \cdot y_{n-1}}$, $y_0 = 120$

$\underline{\underline{y_n - 3y_{n-1} = 0}}$

b) $y_n = y_0 \cdot 3^n \rightarrow \underline{\underline{y_n = 120 \cdot 3^n}}$

$\boxed{\text{Oppgave } 6.3.13a}$

a) Startbetingelse: $y_0 = 1$
$y_1 = 2$ (1,0)
$y_2 = 4$ (00,11,01,10)

$y_n = y_{n-1} + y_{n-2} + y_{n-3}$

$\underline{\underline{y_n - y_{n-1} - y_{n-2} - y_{n-3} = 0}}$