

# Oblig 9

## Oppgave 6.3.8 A)

$$y_n - 5y_{n-1} + 6y_{n-2} = 5 \cdot 4^n$$

Homogen løsning:

$$y_n - 5y_{n-1} + 6y_{n-2} = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2} \rightarrow \frac{5 \pm \sqrt{25 - 24}}{2} \rightarrow \frac{5 \pm 1}{2}$$

$$\lambda_1 = 3$$

$$\lambda_2 = 2$$

$$y_n^h = A \cdot 3^n + B \cdot 2^n$$

$$y_n^p - 5y_{n-1}^p + 6y_{n-2}^p = 5 \cdot 4^n$$

$$\text{Prøv med } y_n^p = C \cdot 4^n$$

$$\rightarrow C \cdot 4^n - 5C \cdot 4^{n-1} + 6C \cdot 4^{n-2} = 5 \cdot 4^n \quad | : 4^n$$

$$C - \frac{5C}{4} + \frac{6C}{16} = 5$$

$$C - \frac{5C}{4} + \frac{3C}{8} = 5$$

$$C - \frac{10C}{8} + \frac{3C}{8} = 5 \quad | \cdot 8$$

$$8C - 10C + 3C = 40$$

$$C = 40$$

$$y_n^p = 40 \cdot 4^n$$

$$y_n = y_n^h + y_n^p = A \cdot 3^n + B \cdot 2^n + 40 \cdot 4^n$$

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## Oppgave 6.3.8 D)

$$y_n - 2y_{n-1} + y_{n-2} = 6n$$

$$THL: y_n - 2y_{n-1} + y_{n-2} = 0$$

$$KL \text{ og } KE: \lambda^2 - 2\lambda + 1 \rightarrow \frac{2 \pm \sqrt{4-4}}{2} \rightarrow \frac{2 \pm 0}{2} \lambda_1 = 1$$

$$\underline{y_n} = A + Bn$$

$$(k_3 n^3 + k_2 n^2 + k_1 n + k_0) - 2(k_3(n-1)^2 + k_2(n-1)^2 + k_1(n-1) + k_0 + (k_3(n-2)^3 + k_2(n-2)^2 + k_1(n-2) + k_0)) = 6n$$

$$k_3 n^3 + k_2 n^2 + k_1 n + k_0 - 2(k_3(n^3 - 3n^2 + 3n - 1) + k_2(n^2 - 2n + 1) + k_1(n-1) + k_0 \\ + k_3(n^3 - 6n^2 + 12n - 8) + k_2(n^2 - 4n + 4) + k_1(n-2) + k_0)$$

$$k_1: 0$$

$$y_n = y_n^h + y_n^p = A + Bn + n^3 + 3n^2$$

$$k_2: 3$$

$$k_3: 1$$

$$k_0: 0$$

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## Oppgave 1

$$\begin{array}{l} -3x_1 - 2x_2 = -5 \\ 4x_1 + x_2 = 10 \end{array} \quad P = \left[ \begin{array}{cc|c} 3 & -2 & -5 \\ 4 & 1 & 10 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -1 & 5 \\ 4 & 1 & 10 \end{array} \right] R'_1 = R_1 + R_2 \rightarrow \left[ \begin{array}{cc|c} 1 & -1 & 5 \\ 4 & 1 & 10 \end{array} \right] \begin{array}{l} +3 \\ \hline 1 \end{array} \begin{array}{l} -2 \\ \hline 1 \end{array} \begin{array}{l} -5 \\ \hline 10 \end{array}$$

$$\left[ \begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 5 & 10 \end{array} \right] R'_2 = R_2 - 4R_1 \rightarrow \left[ \begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 5 & 10 \end{array} \right] \begin{array}{l} -4(1) \\ \hline 0 \end{array} \begin{array}{l} +1 \\ \hline 5 \end{array} \begin{array}{l} 10 \\ \hline -4(5) \end{array}$$

$$\left[ \begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & 2 \end{array} \right] \rightarrow 5x_2 = -10 \mid :5$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array} \right] R'_3 = R_1 + R_2 \rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array} \right] \begin{array}{l} +0 \\ \hline 1 \end{array} \begin{array}{l} -1 \\ \hline 1 \end{array} \begin{array}{l} 3 \\ \hline 2 \end{array}$$

Løsning:  $x_1 = 3, x_2 = -2$

## Oppgave 2

$$\begin{array}{l} x_1 + x_2 + 5x_3 = 6 \\ 2x_1 - 3x_2 + 4x_3 = 16 \\ 4x_1 + 2x_2 - 5x_3 = 3 \end{array} \quad \left| \begin{array}{ccccc} 1 & 1 & 5 & 6 \\ 2 & -3 & 4 & 16 \\ 4 & 2 & -5 & 3 \end{array} \right|$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 5 & 6 \\ 0 & -5 & -6 & 4 \\ 0 & 2 & -5 & 3 \end{array} \right| R'_2 = R_2 - 2R_1 \rightarrow \left| \begin{array}{ccc|c} 1 & 1 & 5 & 6 \\ 0 & 0 & -16 & -12 \\ 0 & 2 & -5 & 3 \end{array} \right|$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 5 & 6 \\ 0 & 0 & -16 & -12 \\ 0 & -2 & -25 & -21 \end{array} \right| R'_3 = R_3 - 4R_1 \rightarrow \left| \begin{array}{ccc|c} 1 & 1 & 5 & 6 \\ 0 & 0 & -16 & -12 \\ 0 & 0 & -29 & -21 \end{array} \right|$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 5 & 6 \\ 0 & 1 & \frac{5}{8} & -\frac{6}{5} \\ 0 & 0 & -25 & -21 \end{array} \right| R'_2 = -\frac{1}{5}R_2$$

$$\left| \begin{array}{ccc|c} 1 & 0 & \frac{19}{5} & \frac{34}{5} \\ 0 & 1 & \frac{5}{8} & -\frac{6}{5} \\ 0 & 0 & -25 & -21 \end{array} \right| R'_1 = R_1 - R_2$$

$$\left| \begin{array}{ccc|c} 1 & 0 & \frac{19}{5} & \frac{34}{5} \\ 0 & 1 & \frac{5}{8} & -\frac{6}{5} \\ 0 & 0 & -\frac{113}{5} & -\frac{113}{5} \end{array} \right| R'_3 = R_3 + 2R_2$$

$$\left| \begin{array}{ccc|c} 1 & 0 & \frac{19}{5} & \frac{34}{5} \\ 0 & 1 & \frac{5}{8} & -\frac{6}{5} \\ 0 & 0 & 1 & 1 \end{array} \right| R_3 = R_3 \cdot \left( -\frac{5}{113} \right)$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right| R'_1 = R_1 - \frac{19}{5} \cdot R_3 \\ R'_2 = R_2 - \frac{5}{8} \cdot R_3$$

Resultat:  $x_1 = 3, x_2 = -2, x_3 = 1$

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## Oppgave 3)

$$\begin{array}{l} x_1 + x_2 + 2x_3 = 7 \\ 2x_1 + 4x_2 - 3x_3 = -9 \\ 3x_1 + 6x_2 - 5x_3 = -15 \end{array} \quad \left| \begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 2 & 4 & -3 & -9 \\ 3 & 6 & -5 & -15 \end{array} \right.$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & 2 & -7 & -23 \\ 3 & 6 & -5 & -15 \end{array} \right. \quad R_2' = R_2 - 2R_1$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & 2 & -7 & -23 \\ 0 & 3 & -11 & -36 \end{array} \right. \quad R_3' = R_3 - 3R_1$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & 1 & -\frac{7}{2} & -\frac{23}{2} \\ 0 & 3 & -11 & -36 \end{array} \right. \quad R_2 = R_2 : 2$$

$$\left| \begin{array}{ccc|c} 1 & 0 & -\frac{7}{2} & -\frac{23}{2} \\ 0 & 1 & -\frac{7}{2} & -\frac{23}{2} \\ 0 & 3 & -11 & -36 \end{array} \right. \quad R_1 = R_1 - R_2 \quad \rightarrow \quad \left| \begin{array}{ccc|c} 1 & 0 & 0 & \frac{11}{2} \\ 0 & 1 & -\frac{7}{2} & -\frac{23}{2} \\ 0 & 3 & -11 & -36 \end{array} \right. \quad \begin{aligned} -\frac{1}{2} + \frac{7}{2} &= \frac{11}{2} \\ + \frac{7}{2} &= \frac{37}{2} \end{aligned}$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & \frac{11}{2} \\ 0 & 1 & -\frac{7}{2} & -\frac{23}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array} \right. \quad R_3' = R_3 - 3R_2$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & \frac{11}{2} \\ 0 & 1 & 0 & -\frac{23}{2} \\ 0 & 0 & 1 & 3 \end{array} \right. \quad R_3' = R_3 \cdot (-2)$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & \frac{11}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right. \quad R_2' = R_2 + \frac{1}{2}R_3$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & \frac{11}{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right. \quad R_1 = R_1 - \frac{11}{2} \cdot R_3$$

Løsning:  $x_1 = 2, x_2 = -1, x_3 = 3$

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## [Oppgave 4]

$$\begin{array}{l} -3x_1 + x_2 - x_3 = 12 \\ 2x_1 + x_3 = -9 \\ 2x_1 - 3x_2 + x_3 = -13 \end{array}$$

$$\left[ \begin{array}{ccc|c} -3 & 1 & -1 & 12 \\ 2 & 0 & 3 & -9 \\ 2 & -3 & 1 & -13 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{3} & -4 \\ 0 & 0 & 3 & -9 \\ 2 & -3 & 1 & -13 \end{array} \right] R_1' = -\frac{1}{3}R_1$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{3} & -4 \\ 0 & \frac{2}{3} & \frac{1}{3} & -1 \\ 2 & -3 & 1 & -13 \end{array} \right] R_2' = R_2 - 2R_1 \quad + \frac{0}{\frac{2}{3}} - \frac{3}{\frac{2}{3}} = \frac{9}{3} - \frac{2}{3} = -9 + 8$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{3} & -4 \\ 0 & \frac{2}{3} & \frac{1}{3} & -1 \\ 0 & -\frac{2}{3} & \frac{1}{3} & -5 \end{array} \right] R_3' = R_3 - 2R_1 \quad + \frac{-\frac{2}{3}}{\frac{2}{3}} = \frac{-9}{3} + \frac{2}{3} = -\frac{7}{3} = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3} + \frac{1}{8} = -\frac{5}{8}$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{3} & -4 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & -\frac{2}{3} & \frac{1}{3} & -5 \end{array} \right] R_2' = R_2 \cdot \frac{3}{2} R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{3} & -4 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{7}{2} \end{array} \right] R_3' = R_3 + \frac{7}{3} \cdot R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{3} & -4 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{array} \right] R_3' = \frac{2}{17} R_3$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & 0 & -\frac{11}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{3}{2} \\ 0 & 0 & 1 & -1 \end{array} \right] R_1' = R_1 + \frac{1}{3} R_3$$

$$\left[ \begin{array}{ccc|c} 1 & -\frac{1}{3} & 0 & -\frac{11}{3} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right] R_2' = R_2 - \frac{7}{2} R_3$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right] R_1' = R_1 + \frac{1}{3} R_2$$

$$\begin{array}{l} x_1 = -3 \\ x_2 = 2 \\ \underline{x_3 = -1} \end{array}$$

$$R_1: -3 + 2 + (-1) = 0 + 1 = 1 = 9 + 1 = 12$$

# Oblig a

[Oppgave 10.1.6 a og d]

a)  $\begin{aligned} 2x_1 + 5x_2 &= 0 \\ 3x_1 - 2x_2 &= 0 \end{aligned}$

$$\left[ \begin{array}{cc|c} 2 & 5 & 0 \\ 3 & -2 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & \frac{5}{2} & 0 \\ 3 & -2 & 0 \end{array} \right] R_1' = \frac{1}{2}R_1$$

$$\left[ \begin{array}{cc|c} 1 & \frac{5}{2} & 0 \\ 0 & -\frac{19}{2} & 0 \end{array} \right] R_2' = R_2 - 3R_1$$

$$-\frac{2}{3} \cdot \frac{5}{2} = -\frac{10}{6} = -\frac{5}{3} = -\frac{19}{2} = \frac{19}{2}$$

$$\left[ \begin{array}{cc|c} 1 & \frac{5}{2} & 0 \\ 0 & 1 & 0 \end{array} \right] R_2' = -\frac{2}{9}R_2$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right] R_1 = R_1 - \frac{5}{2}R_2$$

$$\underline{\underline{x_1 = 0, x_2 = 0}}$$

d)  $\begin{aligned} 3x_1 + 2x_2 - 5x_3 &= 0 \\ 2x_1 - x_3 &= 0 \\ 5x_1 - 2x_2 + x_3 &= 0 \end{aligned}$

$$\left[ \begin{array}{ccc|c} 3 & 2 & -5 & 0 \\ 2 & 0 & -1 & 0 \\ 5 & -2 & 1 & 0 \end{array} \right]$$

$$\left( \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 2 & 0 & -1 & 0 \\ 5 & -2 & 1 & 0 \end{array} \right) R_1' = R_1 - R_2$$

$$\left( \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -4 & 7 & 0 \\ 5 & -2 & 1 & 0 \end{array} \right) R_2' = R_2 - 2R_1$$

$$\left( \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -4 & 7 & 0 \\ 0 & -12 & 21 & 0 \end{array} \right) R_3' = R_3 - 5R_1$$

$$\left( \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -\frac{7}{4} & 0 \\ 0 & -12 & 21 & 0 \end{array} \right) R_2' = R_2 - 4R_1$$

$$\left( \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -\frac{7}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) R_3' = R_3 + 12R_2 \rightarrow R_3 = -12$$

$$\left( \begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{7}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) R_1' = R_1 - 2R_2$$

Resultat:  $x_1 = -\frac{1}{2}t$   
 $x_2 = -\frac{7}{4}t$   
 $x_3 = -\frac{1}{2}t$

$$\underline{\underline{x_1, x_2, x_3 = 0}}$$