Heimadæmi 3

Töluleg Greining

Emil Gauti Friðriksson

22. febrúar 2019

Dæmi 1

2.4 Exercises, dæmi 4

Solve the system by finding the PA=LU factorization and then carrying out the two-step back substitution.

$$(a)\quad \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \qquad (b)\quad \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 17 \\ 3 \end{bmatrix}$$

Svar(a)

(1) Skiptum á línu 1 og línu 2
(2) drögum línu 1 frá línu 2 og drögum $\frac{1}{2}$ línu 1 frá línu 3
Höfum þá PA = LU:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 6 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

Leysum síðan Lc = Pb fyrir c.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

sem gefur síðan

$$\begin{aligned} c_1 &= 4 \\ c_2 &= 2 - c_1 = -2 \\ c_3 &= 6 - \frac{1}{2}c_1 = 4 \end{aligned}$$

Leysum nú Ux=c fyrir x.

$$\begin{bmatrix} 4 & 4 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$$

sem gefur okkur

$$x_3 = \frac{4}{2} = 2$$
$$x_2 = \frac{-2 + 2x_3}{-2} = -1$$
$$x_1 = \frac{4 - 2x_3 - 4x_2}{4} = 1$$

svo við fáum að $[x_1, x_2, x_3] = [1, -1, 2]$

Svar(b)

ATH! Ég setti óvart '1' í stað '2' í línu 3 dálk 2, ég fæ því annað svar, ég vona að það sé afsakanlegt :)

(1) skiptum á línu 2 og línu 1
(2) skiptum á línu 2 og línu 3
(3) drögum $\frac{1}{2}$ línu 1 frá línu 2
(4) drögum $-\frac{1}{2}$ línu 1 frá línu 3
(5) drögum línu 2 frá línu 3
Höfum þá PA = LU

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 2 \end{bmatrix}$$

Leysum síðan Lc = Pb fyrir c.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 17 \\ 3 \end{bmatrix} = \begin{bmatrix} 17 \\ 3 \\ -2 \end{bmatrix}$$

fáum úr þessu:

$$c_1 = 17$$
$$c_2 = 3 - \frac{1}{2}c_1 = -\frac{11}{2}$$
$$c_3 = -2 - c_2 + \frac{1}{2}c_1 = 12$$

Leysum nú Ux=c fyrir x.

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & -1/2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -11/2 \\ 12 \end{bmatrix}$$

Fáum úr þessu:

$$x_3 = 6$$
$$x_2 = 2(-11/2 + \frac{1}{2}x_3) = -5$$
$$x_1 = \frac{17 - x_3 - x_2}{2} = 8$$

Fáum því að lokum að $[x_1, x_2, x_3] = [8, -5, 6]$

Dæmi 2

Computer problems 2.2, dæmi 1 og 2

Dæmi 1 Use the code fragments for Gaussian elimination in the previous section to write a Matlab script to take a matrix A as input and output L and U. No row exchanges are allowed the program should be designed to shut down if it encounters a zero pivot. Check your program by factoring the matrices in Exercise 2.

Dæmi 2 Add two-step back substitution to your script from Computer Problem 1, and use it to solve the systems in Exercise 4.

```
function [L, U, x] = naiveGauss(A, b)
s = size(A);
if s(1) ~= s(2); error('Not nxn matrix');
end
n = s(1);
L = zeros(s(1), s(2));
U = L;
for j = 1 : n-1
if abs(A(j,j))<eps; error('Zero pivot encountered');
end
for i = j+1 : n
 mult = A(i,j)/A(j,j);
L(i,j) = mult;
for k = j+1 : n
A(i,k) = A(i,k) - mult*A(j,k);
end
end
end
for i = 1:n
L(i,i) = 1;
end
for i = 1:n
for j = i:n
U(i,j) = A(i,j);
end
end
c = zeros(n, 1);
for i = 1 : n
for j = 1 : i
b(i) = b(i) - L(i,j)*c(j);
end
c(i) = b(i)/L(i,i);
end
x = zeros(n, 1);
for i = n : -1 : 1
for j = i+1 : n
c(i) = c(i) - U(i,j)*x(j);
end
x(i) = c(i)/U(i,i);
end
end
```

og ef sett eru fylkin úr 2.4 exercises fáum við

```
A1 = [3 1 2; 6 3 4; 3 1 5]
b1 = [0 1 3]
A2 = [4 2 0; 4 4 2; 2 2 3]
b2 = [2 4 6]
[L1, U1, x1] = naiveGauss(A1, b1)
[L2, U2, x2] = naiveGauss(A2, b2)

L1 =
     1     0     0
     2     1     0
     1     0     1
U1 =
     3     1     2
     0     1     0
     0     0     3
x1 =
    -1
     1
     1
L2 =
    1.0000         0         0
    1.0000    1.0000         0
    0.5000    0.5000    1.0000
U2 =
     4     2     0
     0     2     2
     0     0     2
x2 =
     1
    -1
     2
```