

Homework 3, P8130

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```
library(tidyverse)

## -- Attaching packages -----
## v ggplot2 3.3.2      v purrr  0.3.4
## v tibble  3.0.3      v dplyr  1.0.2
## v tidyr   1.1.2      v stringr 1.4.0
## v readr   1.3.1      v forcats 0.5.0

## -- Conflicts -----
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

library("animation")

theme_set(theme_minimal() + theme(legend.position = "bottom"))
options(
  ggplot2.continuous.colour = "viridis",
  ggplot2.continuous.fill = "viridis"
)

scale_colour_discrete = scale_colour_viridis_d
scale_fill_discrete = scale_fill_viridis_d

exercise_df =
  read_csv(
    "./data/exercise.csv")

## Parsed with column specification:
## cols(
##   Group = col_double(),
##   Age = col_double(),
##   Gender = col_double(),
##   Race = col_double(),
##   HTN = col_double(),
##   T2DM = col_double(),
##   Depression = col_double(),
##   Smokes = col_double(),
##   Systolic_PRE = col_double(),
##   Systolic_POST = col_double()
## )
```

Problem 1

For each question, make sure to state the formulae for hypotheses, test-statistics, decision rules/p-values, and provide interpretations in the context of the problem. Use a type I error of 0.05 for all tests.

Problem 1.a

Perform appropriate tests to assess if the Systolic BP at 6 months is significantly different from the baseline values for the intervention group.

*****Check for normality, consider a bonferroni adjustment, state that it's a a paired two-tailed test using an estimate. Then provide the formulas for the test statistic needed, critical value needed, the value, and then get these. and then interpret in context.*****

H_0 , H_A , μ_1 , μ_2 , test statistic formula, critical value needed, interpret

Problem 1.a.i In order to determine an appropriate test, we check for normality using visual inspection of the plot of systolic blood pressure in the Intervention group at both baseline and endline, using raw data. This is explored in Problem 1.c.i and utilizes mean μ_{pre} and μ_{post} respectively.

We first consider the changes in the intervention group between Baseline and Endline first. We determine, given that these are the same patients with data collected at two different timepoints and we do not have reason to test a specific directionality, to use a two-sided Paired t-test.

The H_0 is that $\mu_{pre} - \mu_{post} = 0$ or $\Delta = 0$. The H_A is $\mu_{pre} - \mu_{post} \neq 0$ or $\Delta \neq 0$.

The test statistic is $t = \frac{\bar{d}-0}{s_d/\sqrt{n}}$ where \bar{d} is the point estimate of the mean difference, s_d/\sqrt{n} is the estimated standard error of the differences, and we use the critical value of $t_{n-1, 1-\alpha/2}$. We could use a Bonferroni correction or Tukey's, considering that we will be implementing multiple significance tests, but we say it is not necessary for the case of this homework problem. Additionally, the standard deviation for the difference between Baseline and Endline is given and thus the variance is known.

Using $t = \frac{\bar{d}-0}{s_d/\sqrt{n}} = t = \frac{-8.58-0}{17.17/\sqrt{36}} = t = \frac{-8.58}{17.17/\sqrt{36}} \approx -2.99825$. The critical value is given the the percentile of the t distribution with (n-1) degrees of freedom, $qt(0.975, 35) \approx 2.03$, such that we find evidence to reject the null hypothesis and conclude that in the intervention group, the mean systolic blood pressure at Endline is significantly different than the mean systolic blood pressure at Baseline.

Problem 1.a.ii Similarly, in order to determine an appropriate test, we check for normality using visual inspection of the plot of systolic blood pressure in the Control group at both Baseline and Endline, using raw data. This is explored in Problem 1.c.i and utilizes mean μ_{pre} and μ_{post} respectively.

We determine, given that these are the same patients with data collected at two different timepoints and we do not have reason to test a specific directionality, to use a two-sided Paired t-test.

The H_0 is that $\mu_{pre} - \mu_{post} = 0$ or $\Delta = 0$. The H_A is $\mu_{pre} - \mu_{post} \neq 0$ or $\Delta \neq 0$.

The test statistic is $t = \frac{\bar{d}-0}{s_d/\sqrt{n}}$ where \bar{d} is the point estimate of the mean difference, s_d/\sqrt{n} is the estimated standard error of the differences, and we use the critical value of $t_{n-1, 1-\alpha/2}$. We could use a Bonferroni correction, Tukey's or others, considering that we will be implementing multiple significance tests, but we say it is not necessary for the case of this homework problem. Additionally, the standard deviation for the difference between Baseline and Endline is given and thus the variance is known.

Using $t = \frac{\bar{d}-0}{s_d/\sqrt{n}} = t = \frac{-3.33-0}{14.81/\sqrt{36}} \approx -1.3491$. The critical value is given the the percentile of the t distribution with (n-1) degrees of freedom, $qt(0.975, 35) \approx 2.03$, such that we fail to reject the null hypothesis and conclude that in the Control group, the mean systolic blood pressure at Endline is not significantly different than the mean systolic blood pressure at Baseline.

Problem 1.b Now, we assess the systolic blood pressure absolute changes between the two groups using an independent, two-sampled t-test. We assume independence based on the given information on the sampling mechanism. Since we do not know the two population variances, we first check for equality of variances.

The statistic is given by $F = \frac{s_1^2}{s_2^2}$, where the null hypothesis is $\sigma_1^2 = \sigma_2^2$, and the alternate hypothesis is $\sigma_1^2 \neq \sigma_2^2$. In our case, we use $F = \frac{14.81^2}{17.17^2} = 0.74399$ and the critical value is $qf(0.975, 35, 35) = 1.961$, such that we fail to reject the null hypothesis and conclude that the variance of sample 1 is not significantly different from the variance of sample 2.

We use the pooled estimate of the variance (and associated standard deviation) from two independent samples given by: $s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1+n_2-2)} = s^2 = \frac{(35)s_1^2 + (35)s_2^2}{68} = \frac{(35)(14.81^2) + (35)(17.17^2)}{70} = 257.0725$ and $s = \sqrt{257.0725} = 16.0335$

So, we can proceed with the independent two sampled t-test assuming equal variances.

Given the null hypothesis $H_0: \mu_{control} = \mu_{intervention}$ and the alternate, $H_A: \mu_{control} \neq \mu_{intervention}$, we use $t = \frac{\bar{X}_1 - \bar{X}_2}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$ under H_0

Computing this, we have $t = \frac{-3.33 - (-8.58)}{s \sqrt{\frac{1}{36} + \frac{1}{36}}} = \frac{5.25}{s \sqrt{0.055}} = 1.3892$. We can also use `std_pooled <- sqrt(((14.81^2(35)) + (17.17^2(35)))/70)` and `t_stats <- (-3.33 - (-8.58)) / (std_pooled * sqrt((1/36) + (1/36)))`. The critical value is given by $t_{n_1+n_2-2, 0.975} = qt(0.975, 70) = 1.994437$. Therefore, we fail to reject the null hypothesis and conclude that the difference in mean systolic blood pressure change from Baseline to Endline in the Control group is not significantly different from the mean systolic blood pressure change from Baseline to Endline in the Intervention group.

For the confidence interval,

$$\left(\bar{X}_1 - \bar{X}_2 - t_{n_1+n_2-2, 1-\alpha/2} \cdot s \cdot \sqrt{1/n_1 + 1/n_2}, \bar{X}_1 - \bar{X}_2 + t_{n_1+n_2-2, 1-\alpha/2} \cdot s \cdot \sqrt{1/n_1 + 1/n_2} \right) \quad (1)$$

This is to say, the lower limit of the 95% CI is $5.25 - t_{n_1+n_2-2, 1-\alpha} * s \sqrt{\frac{1}{36} + \frac{1}{36}} = 5.25 - 22.27386 = -17.02386$ and the upper limit of the confidence interval is $(-3.33 - (-8.58)) + t_{n_1+n_2-2, 1-\alpha} * s \sqrt{\frac{1}{36} + \frac{1}{36}} = 5.25 + 22.27386 = 27.52386$, thus the 95% CI is $(-17.02386, 27.52386)$.

Problem 1.c

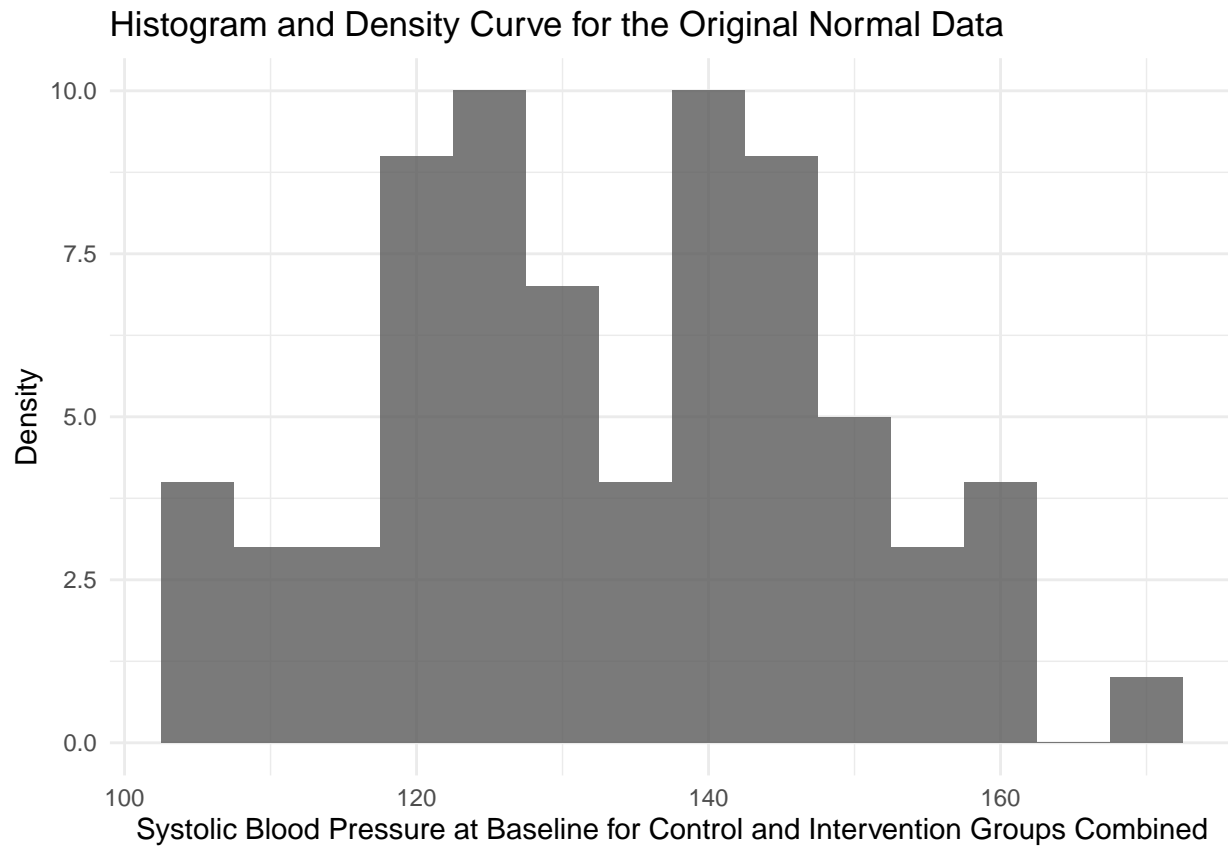
Problem 1.c.i The main assumptions for part a) the two-sided paired t-test, are that we assume the systolic blood pressure measurements are normally distributed with mean $\mu_{baseline}$ $\mu_{endline}$, and that these differences also follow a normal distribution. We therefore also assume that the observed SBP differences constitute a random sample from this normally distributed population of differences. We're assuming the sampled patients have been drawn randomly, and also that each of their sampling is independent from another. We also assume that the variance is known (based on the provided standard deviation; I am also assuming this provided standard deviation was adjusted for the paired t-test (it's already been divided by the square root of (n-1)).

The main assumptions for part b) the two-sided two-sample independent t-test assumes similarly as part a) regarding independence and normality, and then regarding variance instead assume the homogeneity of variance (which we tested with an F-test).

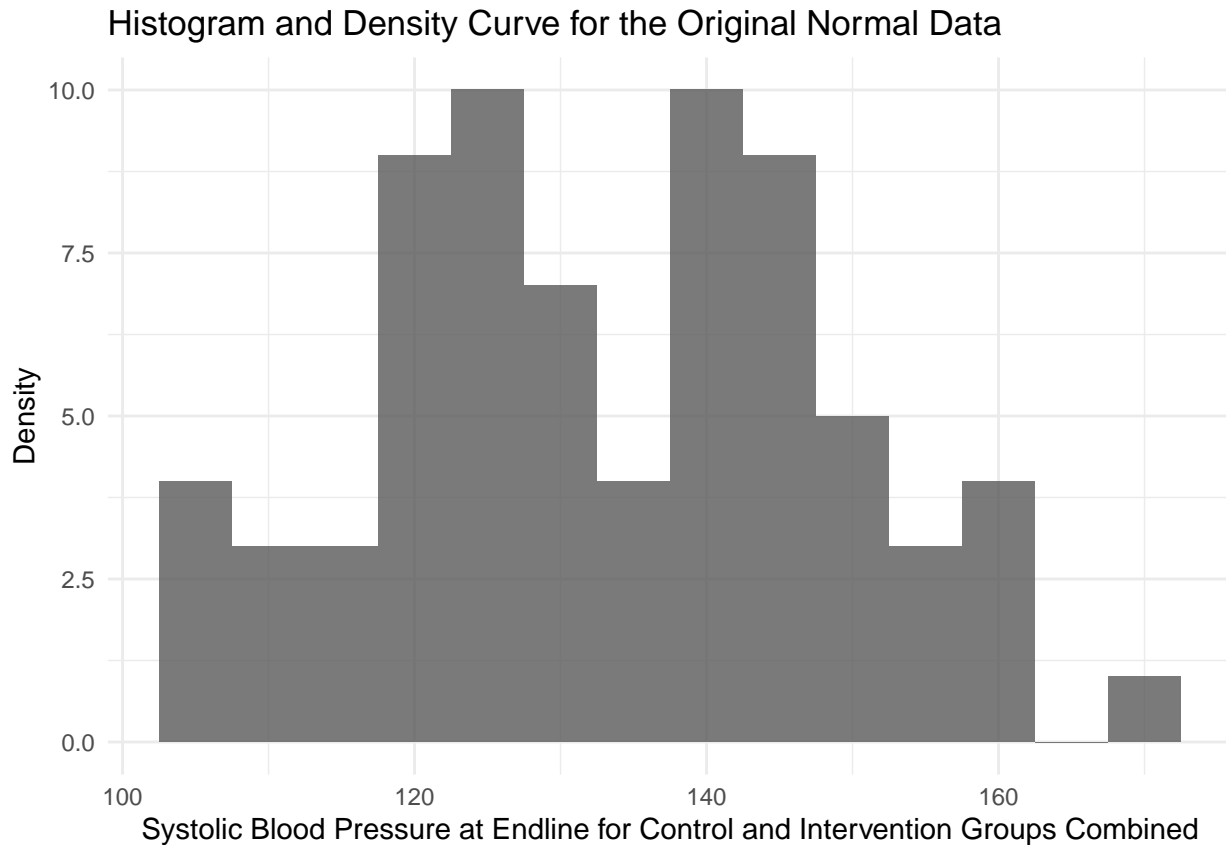
Now, we can examine the normality assumption using graphical displays, for the Intervention group systolic blood pressures at Baseline and at Endline, and similarly for the Control group.

```
#plot of intervention at baseline to examine normality
exercise_df %>%
  ggplot(aes(x = Systolic_PRE)) +
```

```
geom_histogram(binwidth = 5, alpha = 0.8) +
labs(
  x = "Systolic Blood Pressure at Baseline for Control and Intervention Groups Combined",
  y = "Density",
  title = "Histogram and Density Curve for the Original Normal Data") +
scale_fill_viridis_d("") +
theme(legend.position = "none")
```



```
#plot of intervention at endline to examine normality
exercise_df %>%
  ggplot(aes(x = Systolic_PRE)) +
  geom_histogram(binwidth = 5, alpha = 0.8) +
  labs(
    x = "Systolic Blood Pressure at Endline for Control and Intervention Groups Combined",
    y = "Density",
    title = "Histogram and Density Curve for the Original Normal Data") +
  scale_fill_viridis_d("") +
  theme(legend.position = "none")
```



Neither Baseline nor Endline SBP measurements appear to meet the underlying assumption of the normality distribution. While we could use a wider bin width like 10 rather than 5, which makes the data appear more normal (especially near the center of the distribution), but this has the opposite intended effect and hides the problem.

Problem 1.c.ii Unfortunately, this brings the tests' validity into question, as well as brings doubt to the additional assumption that the underlying distribution of the difference between Baseline and Endline scores is normal. This needs to be remedied. One way would be beginning with the Central Limit Theorem and take a larger sample such that the distribution of the sample means will be approximately normally distributed (which holds true even if the population SBP distribution is not normal). Additionally, we could examine if there are measurement errors or outliers in the data that may be uncharacteristically skewing the distribution (though this isn't immediately clear as the case). We could additionally attempt transformation of the data (for example, by taking the logarithm or square root), though this may not be effective. Finally, an alternative is to use an Exact method, to calculate the confidence interval.

Problem 2

In this exercise you learn how to create your own 'true' scenario, simulate corresponding data, and quantify the type I error over many repetitions.

Problem 2.a

Given scenario is that the average IQ score of Ivy League Colleges is 120 and assume this to be the null hypothesis $H_0 : \mu = \mu_0 = 120$. The alternate hypothesis is that the true mean is less than 120, $H_A : \mu < 120$.

The standard deviation and alpha are given as $\sigma = 15$ and $\alpha = 0.05$.

$$X_i \stackrel{i.i.d.}{\sim} N(\mu_1, \sigma^2)$$

Firstly, we generate one random sample of size $n=20$ using `rnorm(20, mean = 120, sd = 15)`.

```
set.seed(4)
rnorm_20 = rnorm(20, mean = 120, sd = 15)
mean_rnorm20 = mean(rnorm_20)
```

We are assuming the normal distribution and compute the test statistic as given by $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$, such that

$$Z = \frac{\bar{X} - 120}{15/\sqrt{20}}$$

= 0.08414241. The critical value is given by $t_{n-1, 1-\alpha} = \text{qnorm}(0.95, \text{lower.tail} = \text{TRUE}) \approx 1.644854$ such that we fail to reject the null hypothesis and conclude that the true mean is not less than 120 units.

Problem 2.b

Using `qnorm(0.05)`, we run a simulation of 100 random samples of size $n=20$ from the underlying null distribution, where for each sample we run a one-sided t-test, and record the result.

```
sampld = rep(NA, 100)
decision = rep(NA, 100)

for (i in 1:100) {
  set.seed(i)
  sampld = rnorm(20, mean = 120, sd = 15)
  t_stats = (mean(sampld) - 120) / (sd(sampld)/sqrt(20))
  decision[i] = ifelse(t_stats > qnorm(0.05), 1, 0)
}
mean(decision)
```

```
## [1] 0.95
```

There is a 95% rate of 1's and 5% rate of 0's produced by this simulation, suggesting that in 5% of the simulations we fail to reject the null hypothesis because the `t_stats` value is greater than the `qnorm(0.05)`, which, since we know the true population is 120 IQ points, this is a Type I error rate.

Problem 2.c

Similarly, using `qnorm(0.05)`, we run a simulation of 1000 random samples of size $n=20$ from the underlying null distribution, where for each sample we run a one-sided t-test, and record the result.

```
sampld = rep(NA, 1000)
decision = rep(NA, 1000)

for (i in 1:1000) {
  set.seed(i)
  sampld = rnorm(20, mean = 120, sd = 15)
  t_stats = (mean(sampld) - 120) / (sd(sampld)/sqrt(20))
  decision[i] = ifelse(t_stats > qnorm(0.05), 1, 0)
}
mean(decision)
```

```
## [1] 0.944
```

There is a 94.4% rate of 1's and 5% rate of 0's produced by this simulation, suggesting that in 6.6% of the simulations we fail to reject the null hypothesis because the `t_stats` value is greater than the `qnorm(0.05)`, which, since we know the true population is 120 IQ points, this is a Type I error rate.

Problem 2.d

The type I error rate in part b) was 5% and in part c) was 6.6%. While we would normally anticipate that the type I error rate should be closer and closer to the initially imposed value of 0.05 as the sampling size increases, this is not the result that we observe.