

INF102 Algorithms and Data Structures

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INF102

- ▶ Lecturer: Marc Bezem, teaching assistants: NN
- ▶ Homepage: [INF102](#) (hyperlinks in red)
- ▶ Textbook: [Algorithms, 4th edition](#), R. Sedgewick and K. Wayne, Pearson, 2011
- ▶ Prerequisites: INF100 + 101 (\approx Ch. 1.1 + 1.2)
- ▶ Syllabus (pensum): Ch. 1.3–1.5, Ch. 2, Ch. 3, Ch. 4
- ▶ Exam: two or three compulsory exercises and a [written exam](#)
- ▶ Old exams: [2004–2013](#), [2014](#)
- ▶ Contents of these slides [here](#)

Didactical stuff

- ▶ Good textbook from USA: many pages, exercises etc.
- ▶ Average speed must be ca 50 pages p/w
- ▶ Lectures focus on the essentials
- ▶ Prepare yourself by reading in advance
- ▶ Workshops about selected exercises
- ▶ Test yourself by trying some exercises in advance
- ▶ If you can do the exercises (incl. compulsory), you are fine

Generic Bags, Queues and Stacks

- ▶ Generic programming in Java, example: **PolyPair.java**
- ▶ Bag, Queue and Stack are generic, iterable collections
- ▶ Queue and Stack: Ch. 9 in textbook INF100/1
- ▶ APIs include: `boolean isEmpty()` and `int size()`
- ▶ All three support adding an element
- ▶ Queue and Stack support removing an element (if any)
- ▶ FIFO Queue, LIFO Stack
- ▶ Dijkstra's Two-Stack Expression Evaluation **Movie**

Implementations

- ▶ `ResizingArray_Stack.java`
- ▶ Resizing takes time and space proportional to size
- ▶ `LinkedList_Stack.java`
- ▶ Pointers take space and dereferencing takes time
- ▶ Programming with pointers: make a picture
- ▶ `LinkedList_Queue.java`

Computation time and memory space

- ▶ Two central questions:
 - ▶ How long will my program take?
 - ▶ Will there be enough memory?
- ▶ Example: **TheeSum.java**
- ▶ Inner loop is important

Methods of Analysis

- ▶ Empirical:
 - ▶ Run program with randomized inputs, measuring time & space
 - ▶ Run program repeatedly, doubling the input size
 - ▶ Measuring time: **StopWatch**
 - ▶ Plot, or log-log plot and **linear regression**
- ▶ Theoretical:
 - ▶ Define a cost model by abstraction (e.g., array accesses, comparisons, operations)
 - ▶ Try to count/estimate/average this cost as function of the input (size)
 - ▶ Use $O(f(n))$ and $f(n) \sim g(n)$

ThreeSum, empirically

- ▶ Input sizes 1K, 2K, 4K, 8K take time 0.1, 0.8, 6.4 ,51.1 sec
- ▶ The log's are 3, 3.3, 3.6, 3.9 and -1, -0.1, 0.8, 1.71
- ▶ Basis of the logarithm should be the same for both
- ▶ Linear regression gives $y \approx 3x - 10$
- ▶ $\lg(f(n)) = 3 \lg(n) - 10$ iff

$$f(n) = 10^{\lg(f(n))} = 10^{3 \lg(n) - 10} = n^3 * 10^{-10}$$

- ▶ Conclusion: cubic in the input size, with constant $\approx 10^{-10}$
- ▶ Strong dependence on input can be a problem
- ▶ Constant 10^{-10} depends on computer, exponent 3 does not

ThreeSum, theoretically

- ▶ Number of different picks of triples: $g(n) = n(n-1)(n-2)/6$
- ▶ Inner loop executed $g(n)$ times
- ▶ $g(n) = n^3/6 - n^2/2 + n/3$
- ▶ Cubic term $n^3/6$ wins for large n
- ▶ Computational model # array accesses: $n^3/2$
- ▶ Cost array access t sec: time $t * n^3/2$ sec
- ▶ Cost models are abstractions! (NB cache)

Big Oh, and \sim

- ▶ Q: 'wins for large n ' uhh???
- ▶ A: Big Oh, and \sim will clear this up
- ▶ Costs are positive quantities, so $f, g, \dots : \mathbb{N} \rightarrow \mathbb{R}^+$
- ▶ MNF130: $f(n)$ is $O(g(n))$ if there exist c, N such that $f(n) \leq cg(n)$ for all $n \geq N$
- ▶ Example: n^2 and even $99n^3$ are $O(n^3)$, but n^3 is not $O(n^{2.9})$
- ▶ INF102: $f(n) \sim g(n)$ if $1 = \lim f(n)/g(n)$
- ▶ If $f(n) \sim g(n)$, then $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$
- ▶ Big Oh and \sim aim to capture 'order of growth'
- ▶ Big Oh abstracts from constant factors, \sim does not
- ▶ Large constant factors are important!

Important orders of growth

- ▶ constant: c ($f(n) = c$ for all n)
- ▶ linear: n (compare all for $n = 20$ sec)
- ▶ linearithmetic: $n \lg n$
- ▶ quadratic: n^2
- ▶ cubic: n^3
- ▶ exponential: 2^n
- ▶ general form: $an^b(\lg n)^c$

Logarithms and Exponents

- ▶ Definition: $\log_x z = y$ iff $x^y = z$ for $x > 0$
- ▶ Inverses: $x^{\log_x y} = y$ and $\log_x x^y = y$
- ▶ Exponent: $x^{(y+z)} = x^y x^z$, $x^{(yz)} = (x^y)^z$
- ▶ Logarithm: $\log_x(yz) = \log_x y + \log_x z$, $\log_x z = \log_x y \log_y z$
- ▶ Double exponent: e.g. $2^{(2^n)}$ (not used in INF102)
- ▶ Double logarithm: $\log(\log n)$ (not used in INF102)

Examples

- ▶ Worst case: guaranteed, independent of input
- ▶ Average case: not guaranteed, dependent of input *distribution*
- ▶ Linked list implementations of Stack, Queue and Bag: all operations take constant time in the worst case
- ▶ Resizing array implementations of Stack, Queue and Bag: adding and deleting take linear time in the worst case (easy)
- ▶ Resizing array implementations of Stack, Queue and Bag: adding and deleting take on average constant time in the worst case (difficult)
- ▶ Special case of resizing array that is only growing:
 $1(2)2(4)3(4)4(8)5(6)6(8)7(16)8(9) \dots 16(32) \dots$, with (n) the new size.
 Resizing to (n) costs $2n$ array accesses, so in total
 $(1+4)+(1+8)+(2+16)+(4+32)+(8+64) \dots$, so 9 per push.

Staying Connected

- ▶ MNF130: relation $R \subseteq V \times V$ is an *equivalence* if
 - ▶ R is *reflexive*: $\forall x \in V. R(x, x)$
 - ▶ R is *symmetric*: $\forall x, y \in V. R(x, y) \rightarrow R(y, x)$
 - ▶ R is *transitive*: $\forall x, y, z \in V. R(x, y) \wedge R(y, z) \rightarrow R(x, z)$
- ▶ We assume connectedness to be an equivalence
- ▶ Dynamic connectivity means that R can grow and shrink
- ▶ Example: if the 'Bergensbanen' is broken, Oslo and Bergen are no longer connected by rail
- ▶ We want efficient algorithms and datastructures for testing whether two objects are connected
- ▶ Clear relationship with paths in graphs, more in Ch. 4
- ▶ Here we take $V = \{0, \dots, N - 1\}$.

Staying Connected

- ▶ MNF130: relation $E \subseteq V \times V$ is an *equivalence* if
 - ▶ E is *reflexive*: $\forall x \in V. E(x, x)$
 - ▶ E is *symmetric*: $\forall x, y \in V. E(x, y) \rightarrow E(y, x)$
 - ▶ E is *transitive*: $\forall x, y, z \in V. E(x, y) \wedge E(y, z) \rightarrow E(x, z)$
- ▶ We assume connectedness to be an equivalence
- ▶ Dynamic connectivity means that R can grow and shrink
- ▶ Example: if the 'Bergensbanen' is broken, Oslo and Bergen are no longer connected by rail
- ▶ We want efficient algorithms and datastructures for testing whether two objects are connected
- ▶ Clear relationship with paths in graphs, (connected) components (MNF130)
- ▶ We take $V = \{0, \dots, N - 1\}$.

Union Find

- ▶ UF, idea: every component has an identifier ('hub'), which has edges ('spokes') to the elements of its component
- ▶ API: **UF**
- ▶ Implementations with `int[] id` containing the identifiers
 - ▶ **SlowUF.java**
 - ▶ **FastUF.java**
 - ▶ **WeightedUF.java**
- ▶ WeightedUF: log depth of tree (Proposition X)

Sorting

- ▶ Sorting: putting objects in a certain order
- ▶ MNF130: relation $R \subseteq V \times V$ is a *total order(ing)* if
 1. R is *reflexive*: $\forall x \in V. R(x, x)$
 2. R is *transitive*: $\forall x, y, z \in V. R(x, y) \wedge R(y, z) \rightarrow R(x, z)$
 3. R is *antisymmetric*: $\forall x, y \in V. R(x, y) \wedge R(y, x) \rightarrow x = y$
 4. R is *total*: $\forall x, y \in V. R(x, y) \vee R(y, x)$
- ▶ Natural orderings:
 - ▶ Numbers of any type: ordinary \leq and \geq
 - ▶ Strings: lexicographic
 - ▶ Objects of a Comparable type: $v.\text{compareTo}(w) < 0$

Sorting (ctnd)

- ▶ Bubble sort: `ExampleSort.java`
- ▶ Certification: `assert isSorted(a)` in `main()`
- ▶ No guarantee against modifying the array (but `exch()` is safe)
- ▶ Costmodel 1: number of `exch()`'s and `less()`'s
- ▶ Costmodel 2: number of array accesses
- ▶ Pitfalls: cache misses, expensive `v.compareTo(w) < 0`
- ▶ Why studying sorting? (`java.util.Arrays.sort()`)
- ▶ Comparing sorting algorithms: `CompareSort.java`

Selection Sort

- ▶ Bubble sort: $\sim n^2/2$ compares, 0 . . $\sim n^2/2$ exchanges
- ▶ Selection sort:
 - ▶ Find index of a minimal value $a[1..n]$, exchange with $a[1]$
 - ▶ Find index of a minimal value $a[2..n]$, exchange with $a[2]$
 - ▶ ... until $n-1$
- ▶ Selection sort: $\sim n^2/2$ compares, $n-1$ exchanges

```
public static void sort(Comparable[] a) {  
    int N = a.length;  
    for (int i=0; i<N-1; i++){  
        int min=i;  
        for (int j=i+1; j<N; j++) if (less(a[j],a[min])) min=j;  
        exch(a,i,min);  
    }  
}
```

Insertion sort

- ▶ Insertion sort:
 - ▶ Insert $a[2]$ on its correct place in (sorted) $a[1..1]$
 - ▶ Insert $a[3]$ on its correct place in (sorted) $a[1..2]$
 - ▶ ... until $a[n]$
- ▶ Very good for partially sorted arrays, costs:
 - ▶ Best case: $n-1$ compares and 0 exchanges
 - ▶ Worst case: $\sim n^2/2$ compares and exchanges
 - ▶ Average case: $\sim n^2/4$ compares and exchanges (distinct keys)

```
public static void sort(Comparable[] a) {  
    int N = a.length;  
    for (int i=1; i<N; i++){  
        for (int j=i; j>0 && less(a[j],a[j-1]); j--)  
            exch(a,j,j-1);  
    }  
}
```

Shell sort

- ▶ Insertion sort:
 - ▶ Very good for partially sorted arrays
 - ▶ Slow in transport: step by step `exch(a,j,j-1)`
- ▶ Idea: h-sort, `a[i], a[i+h], a[i+2h], ...` sorted (any `i`)

```
public static void hsort(int h, Comparable[] a) {  
    int N = a.length;  
    for (int i=h; i<N; i++)  
        for (int j=i; j-h>=0 && less(a[j],a[j-h]); j-=h)  
            exch(a,j,j-h);  
}
```

- ▶ Insertion sort: `hsort(1,a)`
- ▶ Shell sort: e.g., `hsort(10,a); hsort(1,a)`

Shell sort (ctnd)

- ▶ `hsort(10,a); hsort(1,a)` faster than just `hsort(1,a)` !
- ▶ Q: How is this possible?
- ▶ A: `hsort(10,a)` transports items in steps of 10, which would be done by `hsort(1,a)` in 10 steps of 1
- ▶ What about `hsort(100,a); hsort(10,a); hsort(1,a)`?
- ▶ To be expected: depends on the length N of the array
- ▶ Book:

Mergesort

- ▶ Recursive algorithm:
 - ▶ Mergesort left half, mergesort right half
 - ▶ Merge the results
- ▶ Normally with an auxiliary array: `TopDownMergeSort.java`
`Movie`

ToC and topics of general interest

- ▶ Table of Contents on next slide (all items clickable)
- ▶ Practical stuff: slide 2

Introduction

Ch.1.3 Bags, Queues and Stacks

Ch.1.4 Analysis of Algorithms

Ch.1.5 Case Study: Union-Find

Ch.2.1 Elementary Sorts

Ch.2.2 Mergesort

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Ch.2.4 Priority Queues

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Ch.3.3 Balanced Search Trees

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