

INF102

Algorithms, Data Structures and Programming

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INF102, practical stuff

- ▶ Lecturer: Marc Bezem; Team: see homepage
- ▶ Homepage: [INF102](#) (hyperlinks in red)
- ▶ Also: [GitHub](#) (recommended); Dropbox: [slides](#), [schedule](#)
- ▶ Textbook: [Algorithms, 4th edition](#)
- ▶ Prerequisites: INF100 + 101 (\approx Ch. 1.1 + 1.2)
- ▶ Syllabus (pensum): Ch. 1.3 – 1.5, Ch. 2 – 4
- ▶ Exam: three compulsory exercises and a [written exam](#)
- ▶ Old exams: [2004–2013](#), [2014](#)
- ▶ [Table of Contents of these slides](#)

Resources

- ▶ Good textbook, USA-style: many pages, exercises etc.
- ▶ Average speed must be ca 50 pages p/w
- ▶ Lectures (ca 24) focus on the essentials
- ▶ Slides (ca 120, dense!) summarize the lectures
- ▶ Prepare yourself by reading in advance
- ▶ Workshops: selected exercises
- ▶ Test yourself by trying some exercises in advance
- ▶ If you can do the exercises (incl. compulsory), you are fine
- ▶ Review of exercises on Friday morning

Generic Bags, Queues and Stacks

- ▶ Generic programming in Java, example: **PolyPair.java**
- ▶ Bag, Queue and Stack are generic, iterable collections
- ▶ Queue and Stack: Ch. 9 in textbook INF100/1
- ▶ APIs include: `boolean isEmpty()` and `int size()`
- ▶ All three support adding an element
- ▶ Queue and Stack support removing an element (if any)
- ▶ FIFO Queue (en/dequeue), LIFO Stack (push/pop)
- ▶ Dijkstra's Two-Stack Expression Evaluation **Movie**
- ▶ Example: $(1 + ((2 + 3) * (4 * 5)))$

Implementations

- ▶ `ResizingArray_Stack.java`
- ▶ Arrays give direct access, but have fixed size
- ▶ Resizing takes time and space proportional to size
- ▶ `LinkedList_Stack.java`
- ▶ No fixed size, but indirect access
- ▶ Pointers take space and dereferencing takes time
- ▶ Programming with pointers: make a picture
- ▶ `LinkedList_Queue.java`

Computation time and memory space

- ▶ Two central questions:
 - ▶ How long will my program take?
 - ▶ Will there be enough memory?
- ▶ Example: **ThreeSum.java**
- ▶ Inner loop (here $a[i] + a[j] + a[k] == 0$) is important
- ▶ Sorting helps: **ThreeSumOptimized.java**
- ▶ Run some experiments: `1Kints.txt`, `2Kints.txt`, ...

Methods of Analysis

- ▶ Empirical:
 - ▶ Run program with randomized inputs, measuring time & space
 - ▶ Run program repeatedly, doubling the input size
 - ▶ Measuring time: **StopWatch**
 - ▶ Plot, or log-log plot and **linear regression**
- ▶ Theoretical:
 - ▶ Define a cost model by abstraction (e.g., array accesses, comparisons, operations)
 - ▶ Try to count/estimate/average this cost as function of the input (size)
 - ▶ Use $O(f(n))$ and $f(n) \sim g(n)$

ThreeSum, empirically

- ▶ Input sizes 1K, 2K, 4K, 8K take time 0.1, 0.8, 6.4 ,51.1 sec
- ▶ The log's are 3, 3.3, 3.6, 3.9 and -1, -0.1, 0.8, 1.71
- ▶ Basis of the logarithm should be the same for both
- ▶ Linear regression gives $y \approx 3x - 10$
- ▶ $\log(f(n)) = 3 \log(n) - 10$ iff

$$f(n) = 10^{\log(f(n))} = 10^{3 \log(n) - 10} = n^3 * 10^{-10}$$

- ▶ Conclusion: cubic in the input size, with constant $\approx 10^{-10}$
- ▶ Strong dependence on input can be a problem
- ▶ Constant 10^{-10} depends on computer, exponent 3 does not

ThreeSum, theoretically

- ▶ Number of different picks of triples: $g(n) = n(n-1)(n-2)/6$
- ▶ Inner loop $a[i] + a[j] + a[k] == 0$ executed $g(n)$ times
- ▶ $g(n) = n^3/6 - n^2/2 + n/3$
- ▶ Cubic term $n^3/6$ wins for large n
- ▶ Computational model # array accesses: $3 * n^3/6 = n^3/2$
- ▶ Cost array access t sec: time $t * n^3/2$ sec
- ▶ Cost models are abstractions! (NB cache)

Big Oh, and \sim

- ▶ Q: 'wins for large n ' uhh???
- ▶ A: Big Oh, and \sim will clear this up
- ▶ Costs are positive quantities, so $f, g, \dots : \mathbb{N} \rightarrow \mathbb{R}^+$
- ▶ MNF130: $f(n)$ is $O(g(n))$ if there exist $c \in \mathbb{R}^+$, $N \in \mathbb{N}$ such that $f(n) \leq cg(n)$ for all $n \geq N$ (that is,, for n large enough)
- ▶ Example: n^2 and even $99n^3$ are $O(n^3)$, but n^3 is not $O(n^{2.9})$
- ▶ INF102: $f(n) \sim g(n)$ if $1 = \lim f(n)/g(n)$
- ▶ If $f(n) \sim g(n)$, then $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$
- ▶ Big Oh and \sim aim to capture 'order of growth'
- ▶ Big Oh abstracts from constant factors, \sim does not
- ▶ Large constant factors are important!

Important orders of growth

- ▶ constant: c , $f(n) = c$ for all n
- ▶ linear: n (compare all for $n = 20$ sec)
- ▶ linearithmetic: $n \log n$
- ▶ quadratic: n^2
- ▶ cubic: n^3
- ▶ exponential: 2^n
- ▶ general form: $an^b(\log n)^c$

Logarithms and Exponents

- ▶ Definition: $\log_x z = y$ iff $x^y = z$ for $x > 0$
- ▶ Inverses: $x^{\log_x y} = y$ and $\log_x x^y = y$
- ▶ Exponent: $x^{(y+z)} = x^y x^z$, $x^{(yz)} = (x^y)^z$
- ▶ Logarithm: $\log_x(yz) = \log_x y + \log_x z$, $\log_x z = \log_x y \log_y z$
- ▶ Base of logarithm: the x in \log_x
- ▶ Various bases: $\log_2 = \lg$, $\log_e = \ln$, $\log_{10} = \log$
- ▶ Double exponent: e.g. $2^{(2^n)}$ (not used in INF102)
- ▶ Double logarithm: $\log(\log n)$ (not used in INF102)

Worst case, average case, amortized cost

- ▶ Worst case: guaranteed, independent of input; Examples:
 - ▶ Linked list implementations of Stack, Queue and Bag: all operations take constant time in the worst case
 - ▶ Resizing array implementations of Stack, Queue and Bag: adding and deleting take linear time in the worst case (easy)
- ▶ Average case: not guaranteed, dependent of input *distribution*
- ▶ Amortized: worst-case cost *per operation*. E.g., each 10-th operation has cost ≤ 21 , all others cost 1, amortized ≤ 3 p/o.
- ▶ Resizing arrays: adding and deleting take constant time *per operation* in the worst case (proof is difficult)
- ▶ Special case of resizing array that is only growing:
 $1(2)2(4)3(8)4(16)5(32)6(64)7(128)8(256)9(512) \dots 16(32768) \dots$, with (n) the new size.
 Resizing to (n) costs $2n$ array accesses, so in total
 $(1+4)+(1+8)+(2+16)+(4+32)+(8+64) \dots$, so 9 p/push.

Staying Connected

- ▶ We want efficient algorithms and datastructures for testing whether two objects are 'connected'
- ▶ MNF130: relation $E \subseteq V \times V$ is an *equivalence* if
 - ▶ E is *reflexive*: $\forall x \in V. E(x, x)$
 - ▶ E is *symmetric*: $\forall x, y \in V. E(x, y) \rightarrow E(y, x)$
 - ▶ E is *transitive*: $\forall x, y, z \in V. E(x, y) \wedge E(y, z) \rightarrow E(x, z)$
- ▶ We assume connectedness to be an equivalence
- ▶ Dynamic connectivity means (here) that E can grow
- ▶ Clear relationship with paths in graphs, (connected) components (MNF130)
- ▶ Input: N and pairs in $V = \{0, \dots, N-1\}$ defining E
- ▶ Challenge: efficient `boolean connected(int p, int q)`
- ▶ Example: $N = 10$, 4 3, 3 8, ... (`algs4-data/tinyUG.txt`)
- ▶ Picture on blackboard (don't print pairs that are already connected)

Union-Find

- ▶ Find, idea: every component has one element as its identifier, `int find(int n)` computes this identifier
- ▶ Union, idea: for any new pair $n\ m$ that are not already connected, `union(int n, int m)` takes the union of the two components, ensuring `find(n) == find(m)`
- ▶ API: **UF**; Cost model: number of array accesses
- ▶ Implementations:
 - ▶ **SlowUF.java**: `id[p]` identifier of p
`find()` ~ 1 , `union()` \sim between $n+3$ and $2n+1$
 - ▶ **FastUF.java**: `int[] id` pointers, `id[p]==p`: identifier
`find()` $\sim 1+2d$, `union()` $\sim 1 + \text{two find()}'s$
 - ▶ **WeightedUF.java**: `int[] id` pointers, `int[] sz` subtree sizes
`find()` and `union()` both $\sim \lg n$
- ▶ WeightedUF: height of subtree of size k is at most $\lg k$
- ▶ Path-compression: ultimate improvement of UF (almost $O(1)$, amortized)

Sorting

- ▶ Sorting: putting objects in a certain order
- ▶ MNF130: relation $R \subseteq V \times V$ is a *total order(ing)* if
 1. R is *reflexive*: $\forall x \in V. R(x, x)$
 2. R is *transitive*: $\forall x, y, z \in V. R(x, y) \wedge R(y, z) \rightarrow R(x, z)$
 3. R is *antisymmetric*: $\forall x, y \in V. R(x, y) \wedge R(y, x) \rightarrow x = y$
 4. R is *total*: $\forall x, y \in V. R(x, y) \vee R(y, x)$
- ▶ Natural orderings:
 - ▶ Numbers of any type: ordinary \leq and \geq
 - ▶ Strings: lexicographic
 - ▶ Objects of a Comparable type: `v.compareTo(w) <= 0`

Sorting (ctnd)

- ▶ Bubble sort: `ExampleSort.java`
- ▶ Certification: `assert isSorted(a)` in `main()`
- ▶ No guarantee against modifying the array (but `exch()` is safe)
- ▶ Costmodel 1: number of `exch()`'s and `less()`'s
- ▶ Costmodel 2: number of array accesses
- ▶ Pitfalls: cache misses, expensive `v.compareTo(w) < 0`
- ▶ Why studying sorting? (`java.util.Arrays.sort()`)
- ▶ Comparing sorting algorithms: `SortCompare.java`

Selection Sort

- ▶ Bubble sort: $\sim n^2/2$ compares, $0 \leq \text{exchanges} \leq \sim n^2/2$
- ▶ Selection sort:
 - ▶ Find index of a minimum in $a[0..n-1]$, exchange with $a[0]$
 - ▶ Find index of a minimum in $a[1..n-1]$, exchange with $a[1]$
 - ▶ ... until $n-2$
- ▶ Selection sort: $\sim n^2/2$ compares, $0 \leq \text{exchanges} \leq n-1$ (!)

```
public static void sort(Comparable[] a) {  
    int N = a.length;  
    for (int i=0; i<N-1; i++){  
        int min=i;  
        for (int j=i+1; j<N; j++) if (less(a[j],a[min])) min=j;  
        if (i != min) exch(a,i,min);  
    }  
}
```

Insertion sort

- ▶ Insertion sort:
 - ▶ Insert $a[1]$ on its correct place in (sorted) $a[0..0]$
 - ▶ Insert $a[2]$ on its correct place in (sorted) $a[0..1]$
 - ▶ ... until $a[n-1]$
- ▶ Very good for partially sorted arrays, costs:
 - ▶ Best case: $n-1$ compares and 0 exchanges
 - ▶ Worst case: $\sim n^2/2$ compares and exchanges
 - ▶ Average case: $\sim n^2/4$ compares and exchanges (distinct keys)

```
public static void sort(Comparable[] a) {  
    int N = a.length;  
    for (int i=1; i<N; i++){  
        for (int j=i; j>0 && less(a[j],a[j-1]); j--)  
            exch(a,j,j-1);  
    }  
}
```

Shell sort

- ▶ Insertion sort:
 - ▶ Very good for partially sorted arrays
 - ▶ Slow in transport: step by step `exch(a,j,j-1)`
- ▶ Idea: h-sort, `a[i], a[i+h], a[i+2h], ...` sorted (any `i`)

```
public static void hsort(int h, Comparable[] a) {  
    int N = a.length;  
    for (int i=h; i<N; i++)  
        for (int j=i; j-h>=0 && less(a[j],a[j-h]); j-=h)  
            exch(a,j,j-h);  
}
```

- ▶ Insertion sort: `hsort(1,a)`
- ▶ Shell sort: e.g., `hsort(10,a); hsort(1,a)`

Shell sort (ctnd)

- ▶ `hsort(10,a); hsort(1,a)` faster than just `hsort(1,a)` !
- ▶ Q: How is this possible?
- ▶ A: `hsort(10,a)` transports items in steps of 10, which would be done by `hsort(1,a)` in 10 steps of 1.
- ▶ What about `hsort(100,a); hsort(10,a); hsort(1,a)`?
- ▶ To be expected: depends on the length N of the array
- ▶ Best practice: $h = N/3, N/9, \dots, 364, 121, 40, 13, 4, 1$

Mergesort

- ▶ Top-down (recursive) algorithm:
 - ▶ Mergesort left half, mergesort right half
 - ▶ Merge the results
- ▶ Using an auxiliary array: [TopDownMergeSort.java](#), [Movie](#)
- ▶ Bottom-up algorithm (16 elements):
 - ▶ Merge $a[0], a[1]$, so $a[2], a[3]$, so $a[4], a[5]$, so ...
 - ▶ Merge $a[0..1], a[2..3]$, so $a[4..5], a[6..7]$, so ...
 - ▶ Merge $a[0..3], a[4..7]$, so $a[8..11], a[12..15]$
 - ▶ Merge $a[0..7], a[8..15]$, done!
- ▶ Also using an auxiliary array: [BottomUpMergeSort.java](#)

Run-time and memory use of mergesort

- ▶ Mergesort uses between $\sim (N/2) \lg N$ and $\sim N \lg N$ compares. Proof on bb. Important formula ($N = 2^n$):

$$2C(2^{n-1}) + 2^{n-1} \leq C(2^n) \leq 2C(2^{n-1}) + 2^n$$

- ▶ Mergesort uses at most $\sim 6N \lg N$ array accesses
- ▶ Mergesort uses $\sim 2N$ space (plus some var's)
- ▶ Q: How fast can compare-based sorting of N distinct keys be?
- ▶ A: $\lg N! \sim N \lg N$; Proof in book and on bb. Keywords: binary *compare tree*, inner nodes for each `compare(a[i], a[j])`, permutations in the leaves,
 $N! = \text{number of permutations} \leq \text{number of leaves} \leq 2^{\text{height of tree}}$

Quicksort

- ▶ Top-down (recursive) algorithm:
 - ▶ Choose a (pivot value) v in the array
 - ▶ Partition the array in non-empty parts $\leq v$ and $\geq v$
 - ▶ Quicksort the two parts
- ▶ Pros: in-place, average computation time $O(n \log n)$
- ▶ Cons: stack space for the recursion, worst-case $O(n^2)$
- ▶ Implementation: **QuickSort.java**

Quicksort, details

- ▶ Subtleties in `partition`:
 - ▶ Invariants $l \leq h$ in the two inner loops
 - ▶ Postcondition after the two inner loops
 - ▶ Invariant of the `for(;;)` loop
 - ▶ Termination of the `for(;;)` loop
- ▶ Termination of recursive `quicksort`

Quicksort, performance

- ▶ Compare Quicksort to other sorts ($n = 10^2, 10^3, \dots$)
- ▶ Quicksort: time $O(n^2)$ if pivot is always smallest (or largest)
- ▶ Randomization: choose pivot randomly, or shuffle array
- ▶ If all keys are distinct and randomization is perfect, then quicksort uses on average $\sim 2n \ln n$ compares (proof on bb)
- ▶ Similar result for exchanges holds (proof is complicated)
- ▶ Relevant improvements:
 - ▶ Cutoff to insertion sort for sizes $\leq M$
 - ▶ Median-of-three pivot
 - ▶ Taking advantage of duplicate keys (3-way partitioning)
- ▶ Quicksort is generally very good
- ▶ In special situations other sorts are better (e.g., countsort)

Priority Queues

- ▶ Assume collecting and processing items having keys
- ▶ Examples of keys: time-stamp, price-tag, priority-tag
- ▶ Assume: keys can be ordered
- ▶ Reasonable: processing currently highest (or lowest)
- ▶ Seen this before? Yes, when items are time-stamped when added:
 - ▶ Queue: dequeue currently oldest (lowest time-stamp)
 - ▶ Stack: pop currently newest (highest time-stamp)
- ▶ Priority queue generalizes this
- ▶ Examples: highest priority, largest transaction, lowest price
- ▶ Distinction between 'item' and 'key' inessential

Priority Queues

- ▶ Good info: [Wikipedia](#); API (the essentials):

```
public class  ArrayListPQ<Key extends Comparable<Key>>

void          insert(Key v) // insert a key
Key           delMax() // delete the largest key, if any
boolean       isEmpty()
int           size()
```

Heaps

- ▶ MNF130: A binary tree is complete if all levels are filled. So, a complete binary tree of depth d has $2^d - 1$ nodes (picture).
- ▶ INF102: A binary tree is (left-)complete if all levels $< h$ are filled, the level h may be partially be empty on the right. So, a (left-)complete binary tree of n nodes has height $\lfloor \lg n \rfloor$.
- ▶ A binary tree is heap-ordered if the key in each node is \geq the keys in its children (if any). So, the root has a maximal key.
- ▶ Array representation of heap-ordered binary tree: picture
- ▶ The methods `swim` and `sink`

Purpose of Sorting

- ▶ Sorting makes the following easier and more efficient:
 - ▶ Searching (binary search, example: `ThreeSumOptimized`)
 - ▶ Searching and looking up, e.g., the `pagenumber` in an index
 - ▶ Removing duplicates
 - ▶ Finding the median, quartiles etc.
- ▶ Our sorting algorithms are generic: `sort(Comparable[] a)`, for any user-defined data type with a `compareTo()` method
- ▶ We do *pointer sorting*, manipulating refs to objects.
 - ▶ Pro: not moving full objects
 - ▶ Cons: pointer dereferencing, no `sort(int[] a)`
- ▶ More flexibility: pass a `Comparator` object to `sort()`

Comparator object

- ▶ API: `public static void sort(Object[] a, Comparator c)`
- ▶ Call: `Insertion.sort(a, new Transaction.WhenOrder())`
- ▶ Call: `Insertion.sort(a, new Transaction.SizeOrder())`
- ▶ Obs: `import java.util.Comparator`
- ▶ Obs: `public static boolean less(Object o1, Object o2, Comparator c)`
- ▶ Priority queues also with Comparator

More

- ▶ Stability: relative order of equal keys is preserved
- ▶ Important in multi-key applications (e.g., timestamp and size)
- ▶ Which sorting algorithm to use?
 - ▶ Quicksort is a good general purpose choice
 - ▶ Don't forget: `java.util.Arrays.sort()`
 - ▶ Special care: sorting arrays of primitive type
 - ▶ Special care: many duplicate keys
- ▶ Consider sorting first to make other problems easier

Applications of Sorting

- ▶ Commercial computing
- ▶ Search for information
- ▶ Job scheduling heuristic: longest processing time first
- ▶ Combinatorial search in AI
- ▶ To come: Prim's and Dijkstra's algorithms
- ▶ Data compressions
- ▶ Cryptology and genomics (e.g., longest repeating substring)

Symbol Tables

- ▶ Symbol table associates *keys* with *values*: *key-value pairs*
- ▶ Examples: keyword-page number, ID number-personal data
- ▶ Important operations:
 - ▶ Insert a key-value pair in the symbol table
 - ▶ Search the value for a given key (if any)
- ▶ Important conventions:
 - ▶ Inserting key-value for existing key: overwriting the value
 - ▶ No duplicate keys, no null keys
 - ▶ Value null: no value for this key
 - ▶ Lazy deletion: insert key-null; Eager: really delete key
- ▶ Other operations: contains(key), isEmpty(), size()
- ▶ Aim: all operations in time $\sim c(\lg n)$ with small constant c

ST Basics

- ▶ Archetypical ST-client: frequency counter (code: later)
- ▶ Cost model: number of compares
- ▶ Naive ST: unordered linked list, linear search
 - ▶ Search miss: $\sim n$ compares
 - ▶ Search hit: between 1 and $\sim n$ compares
 - ▶ Random search hit: $(1 + \dots + n)/n \sim n/2$ compares
 - ▶ Inserting n distinct keys: $(1 + \dots + n) \sim n^2/2$ compares
- ▶ algs4-data/leipzig1M.txt: 21M words, 500K distinct
- ▶ Naive ST impracticable for genomics, internet
- ▶ Scale: G-T keys, M-G distinct (Kilo,Mega,Giga,Tera)
- ▶ Better: ordered ArrayList, binary search, **ArrayListST.java**
- ▶ Binary search: $O(\lg n)$; ArrayList: insert amortized $O(1)$

Binary Search Trees

- ▶ Binary *search* tree: for every node, all keys to the left of this node are smaller, and all keys to the right are larger
- ▶ Search time: length of the path to the node where the key 'should' be
- ▶ Balanced binary tree with n keys has $\lg n$ height
- ▶ Unbalanced binary trees can have height n (so long paths)

ToC and topics of general interest

- ▶ Table of Contents on next slide (all items clickable)
- ▶ Practical stuff: slide 2

Introduction

Ch.1.3 Bags, Queues and Stacks

Ch.1.4 Analysis of Algorithms

Ch.1.5 Case Study: Union-Find

Ch.2.1 Elementary Sorts

Ch.2.2 Mergesort

Ch.2.3 Quicksort

Ch.2.4 Priority Queues

Ch.2.5 Applications

Ch.3.1 Symbol Tables

Ch.3.2 Binary Search Trees

Ch.3.3 Balanced Search Trees

Ch.3.4 Hash Tables

Ch.3.5 Applicatios

Ch.4.1 Undirected Graphs

Ch.4.2 Directed Graphs

Ch.4.3 Minimum Spanning
Trees

Ch.4.4 Shortest Paths

Table of Contents