INF102
Algorithms, Data Structures and Programming

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University of Bergen

Fall 2015

INF102, practical stuff

- Lecturer: Marc Bezem; Team: see homepage

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- Textbook: Algorithms, 4th edition

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- Table of Contents of these slides

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- Review of exercises on Friday morning

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- Example: (1 + ((2 + 3) * (4 * 5)))

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- Run some experiments: `1Kints.txt, 2Kints.txt, ...`

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  - Use $O(f(n))$ and $f(n) \sim g(n)$

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- Cost models are abstractions! (NB cache)

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- Double logarithm: $\log(\log n)$ (not used in INF102)

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- Special case of resizing array that is only growing: 1(2)2(4)34(8)5678(16)9 ... 16(32) ..., with (n) the new size. Risizing to (n) costs $2n$ array accesses, so in total (1+4)+(1+8)+(2+16)+(4+32)+(8+64) ..., so 9 p/push.

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- WeightedUF: log depth of tree (Proposition X)

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- Bubble sort: $\sim n^2/2$ compares, $0 \ldots \sim n^2/2$ exchanges

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public  static void sort(Comparable[] a) {
 int N = a.length;
 for (int i=0; i<N-1; i++){
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   exch(a,j,j-1);
 }
}
```

Insertion sort

- Insertion sort:
  - Insert `a[2]` on its correct place in (sorted) `a[1..1]`
  - Insert `a[3]` on its correct place in (sorted) `a[1..2]`
  - ... until `a[n]`
- Very good for partially sorted arrays, costs:
  - Best case: $n-1$ compares and 0 exchanges
  - Worst case: $\sim n^2/2$ compares and exchanges
  - Average case: $\sim n^2/4$ compares and exchanges (distinct keys)

```
public  static void sort(Comparable[] a) {
 int N = a.length;
 for (int i=1; i<N; i++){
  for (int j=i; j>0 && less(a[j],a[j-1]); j--)
   exch(a,j,j-1);
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Shell sort

- Insertion sort:

```
public static void hsort(int h, Comparable[] a) {
 int N = a.length;
 for (int i=h; i<N; i++)
  for (int j=i; j-h>=0 && less(a[j],a[j-h]); j-=h)
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- Shell sort: e.g., `hsort(10,a); hsort(1,a)`

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- Also using an auxiliary array: BottomUpMergeSort.java

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- Implementation: QuickSort.java

ToC and topics of general interest

- Table of Contents on next slide (all items clickable)

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- Table of Contents on next slide (all items clickable)
- Practical stuff: slide 2