

INF102

Algorithms, Data Structures and Programming

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INF102, practical stuff

- ▶ Lecturer: Marc Bezem; Team: see homepage
- ▶ Homepage: [INF102](#) (hyperlinks in red)
- ▶ Also: [GitHub](#) (recommended); Dropbox: [slides](#), [schedule](#)
- ▶ Textbook: [Algorithms, 4th edition](#)
- ▶ Prerequisites: INF100 + 101 (\approx Ch. 1.1 + 1.2)
- ▶ Syllabus (pensum): Ch. 1.3 – 1.5, Ch. 2 – 4
- ▶ Exam: three compulsory exercises and a [written exam](#)
- ▶ Old exams: [2004–2013](#), [2014](#)
- ▶ [Table of Contents of these slides](#)

Resources

- ▶ Good textbook, USA-style: many pages, exercises etc.
- ▶ Average speed must be ca 50 pages p/w
- ▶ Lectures (ca 24) focus on the essentials
- ▶ Slides (ca 120, dense!) summarize the lectures
- ▶ Prepare yourself by reading in advance
- ▶ Workshops: selected exercises
- ▶ Test yourself by trying some exercises in advance
- ▶ If you can do the exercises (incl. compulsory), you are fine
- ▶ Review of exercises on Friday morning

Generic Bags, Queues and Stacks

- ▶ Generic programming in Java, example: **PolyPair.java**
- ▶ Bag, Queue and Stack are generic, iterable collections
- ▶ Queue and Stack: Ch. 9 in textbook INF100/1
- ▶ APIs include: `boolean isEmpty()` and `int size()`
- ▶ All three support adding an element
- ▶ Queue and Stack support removing an element (if any)
- ▶ FIFO Queue (en/dequeue), LIFO Stack (push/pop)
- ▶ Dijkstra's Two-Stack Expression Evaluation **Movie**
- ▶ Example: $(1 + ((2 + 3) * (4 * 5)))$

Implementations

- ▶ `ResizingArray_Stack.java`
- ▶ Arrays give direct access, but have fixed size
- ▶ Resizing takes time and space proportional to size
- ▶ `LinkedList_Stack.java`
- ▶ No fixed size, but indirect access
- ▶ Pointers take space and dereferencing takes time
- ▶ Programming with pointers: make a picture
- ▶ `LinkedList_Queue.java`

Computation time and memory space

- ▶ Two central questions:
 - ▶ How long will my program take?
 - ▶ Will there be enough memory?
- ▶ Example: **ThreeSum.java**
- ▶ Inner loop (here $a[i] + a[j] + a[k] == 0$) is important
- ▶ Sorting helps: **ThreeSumOptimized.java**
- ▶ Run some experiments: `1Kints.txt`, `2Kints.txt`, ...

Methods of Analysis

- ▶ Empirical:
 - ▶ Run program with randomized inputs, measuring time & space
 - ▶ Run program repeatedly, doubling the input size
 - ▶ Measuring time: **StopWatch**
 - ▶ Plot, or log-log plot and **linear regression**
- ▶ Theoretical:
 - ▶ Define a cost model by abstraction (e.g., array accesses, comparisons, operations)
 - ▶ Try to count/estimate/average this cost as function of the input (size)
 - ▶ Use $O(f(n))$ and $f(n) \sim g(n)$

ThreeSum, empirically

- ▶ Input sizes 1K, 2K, 4K, 8K take time 0.1, 0.8, 6.4 ,51.1 sec
- ▶ The log's are 3, 3.3, 3.6, 3.9 and -1, -0.1, 0.8, 1.71
- ▶ Basis of the logarithm should be the same for both
- ▶ Linear regression gives $y \approx 3x - 10$
- ▶ $\log(f(n)) = 3 \log(n) - 10$ iff

$$f(n) = 10^{\log(f(n))} = 10^{3 \log(n) - 10} = n^3 * 10^{-10}$$

- ▶ Conclusion: cubic in the input size, with constant $\approx 10^{-10}$
- ▶ Strong dependence on input can be a problem
- ▶ Constant 10^{-10} depends on computer, exponent 3 does not

ThreeSum, theoretically

- ▶ Number of different picks of triples: $g(n) = n(n-1)(n-2)/6$
- ▶ Inner loop $a[i] + a[j] + a[k] == 0$ executed $g(n)$ times
- ▶ $g(n) = n^3/6 - n^2/2 + n/3$
- ▶ Cubic term $n^3/6$ wins for large n
- ▶ Computational model # array accesses: $3 * n^3/6 = n^3/2$
- ▶ Cost array access t sec: time $t * n^3/2$ sec
- ▶ Cost models are abstractions! (NB cache)

Big Oh, and \sim

- ▶ Q: 'wins for large n ' uhh???
- ▶ A: Big Oh, and \sim will clear this up
- ▶ Costs are positive quantities, so $f, g, \dots : \mathbb{N} \rightarrow \mathbb{R}^+$
- ▶ MNF130: $f(n)$ is $O(g(n))$ if there exist $c \in \mathbb{R}^+$, $N \in \mathbb{N}$ such that $f(n) \leq cg(n)$ for all $n \geq N$ (that is,, for n large enough)
- ▶ Example: n^2 and even $99n^3$ are $O(n^3)$, but n^3 is not $O(n^{2.9})$
- ▶ INF102: $f(n) \sim g(n)$ if $1 = \lim f(n)/g(n)$
- ▶ If $f(n) \sim g(n)$, then $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$
- ▶ Big Oh and \sim aim to capture 'order of growth'
- ▶ Big Oh abstracts from constant factors, \sim does not
- ▶ Large constant factors are important!

Important orders of growth

- ▶ constant: c , $f(n) = c$ for all n
- ▶ linear: n (compare all for $n = 20$ sec)
- ▶ linearithmetic: $n \log n$
- ▶ quadratic: n^2
- ▶ cubic: n^3
- ▶ exponential: 2^n
- ▶ general form: $an^b(\log n)^c$

Logarithms and Exponents

- ▶ Definition: $\log_x z = y$ iff $x^y = z$ for $x > 0$
- ▶ Inverses: $x^{\log_x y} = y$ and $\log_x x^y = y$
- ▶ Exponent: $x^{(y+z)} = x^y x^z$, $x^{(yz)} = (x^y)^z$
- ▶ Logarithm: $\log_x(yz) = \log_x y + \log_x z$, $\log_x z = \log_x y \log_y z$
- ▶ Base of logarithm: the x in \log_x
- ▶ Various bases: $\log_2 = \lg$, $\log_e = \ln$, $\log_{10} = \log$
- ▶ Double exponent: e.g. $2^{(2^n)}$ (not used in INF102)
- ▶ Double logarithm: $\log(\log n)$ (not used in INF102)

Examples

- ▶ Worst case: guaranteed, independent of input
- ▶ Average case: not guaranteed, dependent of input *distribution*
- ▶ Linked list implementations of Stack, Queue and Bag: all operations take constant time in the worst case
- ▶ Resizing array implementations of Stack, Queue and Bag: adding and deleting take linear time in the worst case (easy)
- ▶ Amortized: worst-case cost *per operation*. E.g., each 10-th operation has cost ≤ 21 , all others cost 1, amortized ≤ 3 p/o.
- ▶ Resizing arrays: adding and deleting take constant time *per operation* in the worst case (proof is difficult)
- ▶ Special case of resizing array that is only growing:
 $1(2)2(4)3(8)4(16)5(32)6(64)7(128)8(256)9 \dots 16(32) \dots$, with (n) the new size.
 Resizing to (n) costs $2n$ array accesses, so in total
 $(1+4)+(1+8)+(2+16)+(4+32)+(8+64) \dots$, so 9 p/push.

Staying Connected

- ▶ MNF130: relation $E \subseteq V \times V$ is an *equivalence* if
 - ▶ E is *reflexive*: $\forall x \in V. E(x, x)$
 - ▶ E is *symmetric*: $\forall x, y \in V. E(x, y) \rightarrow E(y, x)$
 - ▶ E is *transitive*: $\forall x, y, z \in V. E(x, y) \wedge E(y, z) \rightarrow E(x, z)$
- ▶ We assume connectedness to be an equivalence
- ▶ Dynamic connectivity means that R can grow and shrink
- ▶ Example: if the 'Bergensbanen' is broken, Oslo and Bergen are no longer connected by rail
- ▶ We want efficient algorithms and datastructures for testing whether two objects are connected
- ▶ Clear relationship with paths in graphs, (connected) components (MNF130)
- ▶ We take $V = \{0, \dots, N - 1\}$.

Union Find

- ▶ UF, idea: every component has an identifier ('hub'), which has edges ('spokes') to the elements of its component
- ▶ API: **UF**
- ▶ Implementations with `int[] id` containing the identifiers
 - ▶ **SlowUF.java**
 - ▶ **FastUF.java**
 - ▶ **WeightedUF.java**
- ▶ WeightedUF: log depth of tree (Proposition X)

Sorting

- ▶ Sorting: putting objects in a certain order
- ▶ MNF130: relation $R \subseteq V \times V$ is a *total order(ing)* if
 1. R is *reflexive*: $\forall x \in V. R(x, x)$
 2. R is *transitive*: $\forall x, y, z \in V. R(x, y) \wedge R(y, z) \rightarrow R(x, z)$
 3. R is *antisymmetric*: $\forall x, y \in V. R(x, y) \wedge R(y, x) \rightarrow x = y$
 4. R is *total*: $\forall x, y \in V. R(x, y) \vee R(y, x)$
- ▶ Natural orderings:
 - ▶ Numbers of any type: ordinary \leq and \geq
 - ▶ Strings: lexicographic
 - ▶ Objects of a Comparable type: $v.compareTo(w) < 0$

Sorting (ctnd)

- ▶ Bubble sort: `ExampleSort.java`
- ▶ Certification: `assert isSorted(a)` in `main()`
- ▶ No guarantee against modifying the array (but `exch()` is safe)
- ▶ Costmodel 1: number of `exch()`'s and `less()`'s
- ▶ Costmodel 2: number of array accesses
- ▶ Pitfalls: cache misses, expensive `v.compareTo(w) < 0`
- ▶ Why studying sorting? (`java.util.Arrays.sort()`)
- ▶ Comparing sorting algorithms: `CompareSort.java`

Selection Sort

- ▶ Bubble sort: $\sim n^2/2$ compares, 0 . . $\sim n^2/2$ exchanges
- ▶ Selection sort:
 - ▶ Find index of a minimal value $a[1..n]$, exchange with $a[1]$
 - ▶ Find index of a minimal value $a[2..n]$, exchange with $a[2]$
 - ▶ ... until $n-1$
- ▶ Selection sort: $\sim n^2/2$ compares, $n-1$ exchanges

```
public static void sort(Comparable[] a) {  
    int N = a.length;  
    for (int i=0; i<N-1; i++){  
        int min=i;  
        for (int j=i+1; j<N; j++) if (less(a[j],a[min])) min=j;  
        exch(a,i,min);  
    }  
}
```

Insertion sort

- ▶ Insertion sort:
 - ▶ Insert $a[2]$ on its correct place in (sorted) $a[1..1]$
 - ▶ Insert $a[3]$ on its correct place in (sorted) $a[1..2]$
 - ▶ ... until $a[n]$
- ▶ Very good for partially sorted arrays, costs:
 - ▶ Best case: $n-1$ compares and 0 exchanges
 - ▶ Worst case: $\sim n^2/2$ compares and exchanges
 - ▶ Average case: $\sim n^2/4$ compares and exchanges (distinct keys)

```
public static void sort(Comparable[] a) {  
    int N = a.length;  
    for (int i=1; i<N; i++){  
        for (int j=i; j>0 && less(a[j],a[j-1]); j--)  
            exch(a,j,j-1);  
    }  
}
```

Shell sort

- ▶ Insertion sort:
 - ▶ Very good for partially sorted arrays
 - ▶ Slow in transport: step by step `exch(a,j,j-1)`
- ▶ Idea: h-sort, `a[i], a[i+h], a[i+2h], ...` sorted (any `i`)

```
public static void hsort(int h, Comparable[] a) {  
    int N = a.length;  
    for (int i=h; i<N; i++)  
        for (int j=i; j-h>=0 && less(a[j],a[j-h]); j-=h)  
            exch(a,j,j-h);  
}
```

- ▶ Insertion sort: `hsort(1,a)`
- ▶ Shell sort: e.g., `hsort(10,a); hsort(1,a)`

Shell sort (ctnd)

- ▶ `hsort(10,a); hsort(1,a)` faster than just `hsort(1,a)` !
- ▶ Q: How is this possible?
- ▶ A: `hsort(10,a)` transports items in steps of 10, which would be done by `hsort(1,a)` in 10 steps of 1
- ▶ What about `hsort(100,a); hsort(10,a); hsort(1,a)`?
- ▶ To be expected: depends on the length N of the array
- ▶ Book:

Mergesort

- ▶ Top-down (recursive) algorithm:
 - ▶ Mergesort left half, mergesort right half
 - ▶ Merge the results
- ▶ Using an auxiliary array: [TopDownMergeSort.java](#), [Movie](#)
- ▶ Bottom-up algorithm:
 - ▶ Merge $a[0], a[1], a[2], a[3], a[4], a[5], \dots$
 - ▶ Merge $a[0..1], a[2..3], a[4..5], a[6..7], \dots$
 - ▶ Merge $a[0..3], a[4..7], a[8..11], a[12..15], \dots$
- ▶ Also using an auxiliary array: [BottomUpMergeSort.java](#)

The complexity of sorting

- ▶ Mergesort uses between $\sim (n/2) \lg n$ and $\sim n \lg n$ compares
- ▶ Mergesort uses between $\sim 6n \lg n$ array accesses
- ▶ Mergesort uses $\sim 2n$ space (plus some var's)
- ▶ Q: How fast can compare-based sorting be?
- ▶ Book:

Quicksort

- ▶ Top-down (recursive) algorithm:
 - ▶ Choose a (pivot value) v in the array
 - ▶ Partition the array in non-empty parts $\leq v$ and $\geq v$
 - ▶ Quicksort the two parts
- ▶ Pros: in-place, average computation time $O(n \log n)$
- ▶ Cons: stack space for the recursion, worst-case $O(n^2)$
- ▶ Implementation: **QuickSort.java**

Quicksort, details

- ▶ Subtleties in `partition`:
 - ▶ Invariants $l \leq h$ in the two inner loops
 - ▶ Postcondition after the two inner loops
 - ▶ Invariant of the `for(;;)` loop
 - ▶ Termination of the `for(;;)` loop
- ▶ Termination of recursive `quicksort`

Quicksort, performance

- ▶ Compare Quicksort to other sorting methods ($n = 10^2, 10^3, \dots$)
- ▶ Quicksort runs in quadratic time if pivot is always smallest (largest)
- ▶ Randomization is important (choose pivot randomly, or shuffle array)
- ▶ If all keys are distinct and randomization is perfect, then quicksort uses on average $\sim 2n \ln n$ compares (proof on blackboard)
- ▶ Similar result for exchanges holds (proof is complicated)
- ▶ Relevant improvements:
 - ▶ Cutoff to insertion sort for sizes $\leq M$
 - ▶ Median-of-three pivot
 - ▶ Taking advantage of duplicate keys (3-way partitioning)
- ▶ Quicksort is generally very good, ... bucketsort

Priority Queues

- ▶ Assume collecting and processing items having keys
- ▶ Examples of keys: time-stamp, price-tag, priority-tag
- ▶ Assume: keys can be ordered
- ▶ Reasonable: processing currently highest (or lowest)
- ▶ Seen this before? Yes, when items are time-stamped when added:
 - ▶ Queue: dequeue currently oldest (lowest time-stamp)
 - ▶ Stack: pop currently newest (highest time-stamp)
- ▶ Priority queue generalizes this
- ▶ Examples: highest priority, largest transaction, lowest price
- ▶ Distinction between 'item' and 'key' inessential

Priority Queues

- ▶ Good info: [Wikipedia](#); API (the essentials):

```
public class  ArrayListPQ<Key extends Comparable<Key>>

void          insert(Key v) // insert a key
Key           delMax() // delete the largest key, if any
boolean       isEmpty()
int           size()
```

Heaps

- ▶ MNF130: A binary tree is complete if all levels are filled. So, a complete binary tree of depth d has $2^d - 1$ nodes (picture).
- ▶ INF102: A binary tree is (left-)complete if all levels $< h$ are filled, the level h may be partially be empty on the right. So, a (left-)complete binary tree of n nodes has height $\lfloor \lg n \rfloor$.
- ▶ A binary tree is heap-ordered if the key in each node is \geq the keys in its children (if any). So, the root has a maximal key.
- ▶ Array representation of heap-ordered binary tree: picture
- ▶ The methods `swim` and `sink`

Purpose of Sorting

- ▶ Sorting makes the following easier and more efficient:
 - ▶ Searching (binary search, example: `ThreeSumOptimized`)
 - ▶ Searching and looking up, e.g., the `pagenumber` in an index
 - ▶ Removing duplicates
 - ▶ Finding the median, quartiles etc.
- ▶ Our sorting algorithms are generic: `sort(Comparable[] a)`, for any user-defined data type with a `compareTo()` method
- ▶ We do *pointer sorting*, manipulating refs to objects.
 - ▶ Pro: not moving full objects
 - ▶ Cons: pointer dereferencing, no `sort(int[] a)`
- ▶ More flexibility: pass a `Comparator` object to `sort()`

Comparator object

- ▶ API: `public static void sort(Object[] a, Comparator c)`
- ▶ Call: `Insertion.sort(a, new Transaction.WhenOrder())`
- ▶ Call: `Insertion.sort(a, new Transaction.SizeOrder())`
- ▶ Obs: `import java.util.Comparator`
- ▶ Obs: `public static boolean less(Object o1, Object o2, Comparator c)`
- ▶ Priority queues also with Comparator

More

- ▶ Stability: relative order of equal keys is preserved
- ▶ Important in multi-key applications (e.g., timestamp and size)
- ▶ Which sorting algorithm to use?
 - ▶ Quicksort is a good general purpose choice
 - ▶ Don't forget: `java.util.Arrays.sort()`
 - ▶ Special care: sorting arrays of primitive type
 - ▶ Special care: many duplicate keys
- ▶ Consider sorting first to make other problems easier

Applications of Sorting

- ▶ Commercial computing
- ▶ Search for information
- ▶ Job scheduling heuristic: longest processing time first
- ▶ Combinatorial search in AI
- ▶ To come: Prim's and Dijkstra's algorithms
- ▶ Data compressions
- ▶ Cryptology and genomics (e.g., longest repeating substring)

Symbol Tables

- ▶ Symbol table associates *keys* with *values*: *key-value pairs*
- ▶ Examples: keyword-page number, ID number-personal data
- ▶ Important operations:
 - ▶ Insert a key-value pair in the symbol table
 - ▶ Search the value for a given key (if any)
- ▶ Important conventions:
 - ▶ Inserting key-value for existing key: overwriting the value
 - ▶ No duplicate keys, no null keys
 - ▶ Value null: no value for this key
 - ▶ Lazy deletion: insert key-null; Eager: really delete key
- ▶ Other operations: contains(key), isEmpty(), size()
- ▶ Aim: all operations in time $\sim c(\lg n)$ with small constant c

ToC and topics of general interest

- ▶ Table of Contents on next slide (all items clickable)
- ▶ Practical stuff: slide 2

Introduction

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