INF102
Algorithms, Data Structures and Programming

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INF102, practical stuff

- Lecturer: Marc Bezem; Team: see homepage
- Homepage: INF102 (hyperlinks in red)
- Also: GitHub (recommended); Dropbox: slides, schedule
- Textbook: Algorithms, 4th edition
- Prerequisites: INF100 + 101 ($\approx$ Ch. 1.1 + 1.2)
- Syllabus (pensum): Ch. 1.3 – 1.5, Ch. 2 – 4
- Exam: three compulsory exercises and a written exam
- Old exams: 2004–2013, 2014
- Table of Contents of these slides

Resources

- Good textbook, USA-style: many pages, exercises etc.
- Average speed must be ca 50 pages p/w
- Lectures (ca 24) focus on the essentials
- Slides (ca 120, dense!) summarize the lectures
- Prepare yourself by reading in advance
- Workshops: selected exercises
- Test yourself by trying some exercises in advance
- If you can do the exercises (incl. compulsory), you are fine
- Review of exercises on Friday morning

Generic Bags, Queues and Stacks

- Generic programming in Java, example: PolyPair.java
- Bag, Queue and Stack are generic, iterable collections
- Queue and Stack: Ch. 9 in textbook INF100/1
- APIs include: `boolean isEmpty()` and `int size()`
- All three support adding an element
- Queue and Stack support removing an element (if any)
- FIFO Queue (en/dequeue), LIFO Stack (push/pop)
- Dijkstra's Two-Stack Expression Evaluation Movie
- Example: (1 + ((2 + 3) * (4 * 5)))

Implementations

- ResizingArray_Stack.java
- Arrays give direct access, but have fixed size
- Resizing takes time and space proportional to size
- LinkedList_Stack.java
- No fixed size, but indirect access
- Pointers take space and dereferencing takes time
- Programming with pointers: make a picture
- LinkedList_Queue.java

Computation time and memory space

- Two central questions:
  - How long will my program take?
  - Will there be enough memory?
- Example: ThreeSum.java
- Inner loop (here `a[i]+a[j]+a[k]==0`) is important
- Sorting helps: ThreeSumOptimized.java
- Run some experiments: `1Kints.txt, 2Kints.txt, ...`

Methods of Analysis

- Empirical:
  - Run program with randomized inputs, measuring time & space
  - Run program repeatedly, doubling the input size
  - Measuring time: StopWatch
  - Plot, or log-log plot and linear regression
- Theoretical:
  - Define a cost model by abstraction (e.g., array accesses, comparisons, operations)
  - Try to count/estimate/average this cost as function of the input (size)
  - Use $O(f(n))$ and $f(n) \sim g(n)$

ThreeSum, empirically

- Input sizes 1K, 2K, 4K, 8K take time 0.1, 0.8, 6.4 ,51.1 sec
- The log's are 3, 3.3, 3.6, 3.9 and -1, -0.1, 0.8, 1.71
- Basis of the logarithm should be the same for both
- Linear regression gives $y \approx 3x - 10$
- $\log(f(n)) = 3\log(n) - 10$ iff

$$f(n) = 10^{\log(f(n))} = 10^{3\log(n)-10} = n^3 * 10^{-10}$$

- Conclusion: cubic in the input size, with constant $\approx 10^{-10}$
- Strong dependence on input can be a problem
- Constant 10^{-10} depends on computer, exponent 3 does not

ThreeSum, theoretically

- Number of different picks of triples: $g(n) = n(n-1)(n-2)/6$
- Inner loop `a[i]+a[j]+a[k]==0` executed $g(n)$ times
- $g(n) = n^3/6 - n^2/2 + n/3$
- Cubic term $n^3/6$ wins for large n
- Computational model # array accesses: $3 * n^3/6 = n^3/2$
- Cost array access t sec: time $t * n^3/2$ sec
- Cost models are abstractions! (NB cache)

Big Oh, and $\sim$

- Q: 'wins for large n' uhh???
- A: Big Oh, and $\sim$ will clear this up
- Costs are positive quantities, so $f, g, \ldots : \mathbb{N} \to \mathbb{R}^+$
- MNF130: $f(n)$ is $O(g(n))$ if there exist $c \in \mathbb{R}^+,\ N \in \mathbb{N}$ such that $f(n) \leq cg(n)$ for all $n \geq N$ (that is,, for n large enough)
- Example: n^2 and even $99n^3$ are $O(n^3)$, but n^3 is not $O(n^{2.9})$
- INF102: $f(n) \sim g(n))$ if $1 = \lim f(n)/g(n)$
- If $f(n) \sim g(n))$, then $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$
- Big Oh and $\sim$ aim to capture 'order of growth'
- Big Oh abstracts from constant factors, $\sim$ does not
- Large constant factors are important!

Important orders of growth

- constant: c, $f(n) = c$ for all n
- linear: n (compare all for $n = 20$ sec)
- linearithmetic: $n \log n$
- quadratic: n^2
- cubic: n^3
- exponential: 2^n
- general form: $an^b(\log n)^c$

Logarithms and Exponents

- Definition: $\log_x z = y$ iff $x^y = z$ for $x > 0$
- Inverses: $x^{\log_x y} = y$ and $\log_x x^y = y$
- Exponent: $x^{(y+z)} = x^y x^z$, $x^{(yz)} = (x^y)^z$
- Logarithm: $\log_x(yz) = \log_x y + \log_x z$, $\log_x z = \log_x y \log_y z$
- Base of logarithm: the x in $\log_x$
- Various bases: $\log_2 = \lg$, $\log_e = \ln$, $\log_{10} = \log$
- Double exponent: e.g. $2^{(2^n)}$ (not used in INF102)
- Double logarithm: $\log(\log n)$ (not used in INF102)

Examples

- Worst case: guaranteed, independent of input
- Average case: not guaranteed, dependent of input *distribution*
- Linked list implementations of Stack, Queue and Bag: all operations take constant time in the worst case
- Resizing array implementations of Stack, Queue and Bag: adding and deleting take linear time in the worst case (easy)
- Amortized: worst-case cost *per operation*. E.g., each 10-th operation has cost ≤ 21, all others cost 1, amortized ≤ 3 p/o.
- Resizing arrays: adding and deleting take constant time *per operation* in the worst case (proof is difficult)
- Special case of resizing array that is only growing: 1(2)2(4)34(8)5678(16)9 ... 16(32) ..., with (n) the new size. Risizing to (n) costs $2n$ array accesses, so in total (1+4)+(1+8)+(2+16)+(4+32)+(8+64) ..., so 9 p/push.

Staying Connected

- MNF130: relation $E \subseteq V \times V$ is an *equivalence* if
  - E is *reflexive*: $\forall x \in V.\ E(x, x)$
  - E is *symmetic*: $\forall x, y \in V.\ E(x, y) \rightarrow E(y, x)$
  - E is *transitive*: $\forall x, y, z \in V.\ E(x, y) \wedge E(y, z) \rightarrow E(x, z)$
- We assume connectedness to be an equivalence
- Dynamic connectivity means that R can grow and shrink
- Example: if the 'Bergensbanen' is broken, Oslo and Bergen are no longer connected by rail
- We want efficient algorithms and datastructures for testing whether two objects are connected
- Clear relationship with paths in graphs, (connected) components (MNF130)
- We take $V = \{0, \ldots, N - 1\}$.

Union Find

- UF, idea: every component has an identifier ('hub'), which has edges ('spokes') to the elements of its component
- API: UF
- Implementations with `int[] id` containing the identifiers
  - SlowUF.java
  - FastUF.java
  - WeightedUF.java
- WeightedUF: log depth of tree (Proposition X)

Sorting

- Sorting: putting objects in a certain order
- MNF130: relation $R \subseteq V \times V$ is a *total order(ing)* if
  1. R is *reflexive*: $\forall x \in V.\ R(x, x)$
  2. R is *transitive*: $\forall x, y, z \in V.\ R(x, y) \wedge R(y, z) \to R(x, z)$
  3. R is *antisymmetic*: $\forall x, y \in V.\ R(x, y) \wedge R(y, x) \to x = y$
  4. R is *total*: $\forall x, y \in V.\ R(x, y) \vee R(y, x)$
- Natural orderings:
  - Numbers of any type: ordinary $\leq$ and $\geq$
  - Strings: lexicographic
  - Objects of a Comparable type: `v.compareTo(w) < 0`

Sorting (ctnd)

- Bubble sort: ExampleSort.java
- Certification: `assert isSorted(a)` in `main()`
- No guarantee against modifying the array (but `exch()` is safe)
- Costmodel 1: number of `exch()`'s and `less()`'s
- Costmodel 2: number of array accesses
- Pitfalls: cache misses, expensive `v.compareTo(w) < 0`
- Why studying sorting? (`java.util.Arrays.sort()`)
- Comparing sorting algorithms: CompareSort.java

Selection Sort

- Bubble sort: $\sim n^2/2$ compares, $0 \ldots \sim n^2/2$ exchanges
- Selection sort:
  - Find index of a minimal value `a[1..n]`, exchange with `a[1]`
  - Find index of a minimal value `a[2..n]`, exchange with `a[2]`
  - ... until `n-1`
- Selection sort: $\sim n^2/2$ compares, $n-1$ exchanges

```
public  static void sort(Comparable[] a) {
 int N = a.length;
 for (int i=0; i<N-1; i++){
   int min=i;
   for (int j=i+1; j<N; j++) if less(a[j],a[min])) min=j;
   exch(a,i,min);
 }
}
```

Insertion sort

- Insertion sort:
  - Insert a[2] on its correct place in (sorted) a[1..1]
  - Insert a[3] on its correct place in (sorted) a[1..2]
  - ... until a[n]
- Very good for partially sorted arrays, costs:
  - Best case: $n-1$ compares and 0 exchanges
  - Worst case: $\sim n^2/2$ compares and exchanges
  - Average case: $\sim n^2/4$ compares and exchanges (distinct keys)

```
public  static void sort(Comparable[] a) {
 int N = a.length;
 for (int i=1; i<N; i++){
  for (int j=i; j>0 && less(a[j],a[j-1]); j--)
   exch(a,j,j-1);
 }
}
```

Shell sort

- Insertion sort:
  - Very good for partially sorted arrays
  - Slow in transport: step by step `exch(a,j,j-1)`
- Idea: h-sort, `a[i],a[i+h],a[i+2h],...` sorted (any `i`)

```
public static void hsort(int h, Comparable[] a) {
 int N = a.length;
 for (int i=h; i<N; i++)
  for (int j=i; j-h>=0 && less(a[j],a[j-h]); j-=h)
    exch(a,j,j-h);
}
```

- Insertion sort: `hsort(1,a)`
- Shell sort: e.g., `hsort(10,a); hsort(1,a)`

Shell sort (ctnd)

- `hsort(10,a); hsort(1,a)` faster than just `hsort(1,a)` !
- Q: How is this possible?
- A: `hsort(10,a)` transports items in steps of 10, which would be done by `hsort(1,a)` in 10 steps of 1
- What about `hsort(100,a); hsort(10,a); hsort(1,a)`?
- To be expected: depends on the length N of the array
- Book:

Mergesort

- Top-down (recursive) algorithm:
  - Mergesort left half, mergesort right half
  - Merge the results
- Using an auxiliary array: TopDownMergeSort.java, Movie
- Bottom-up algorithm:
  - Merge `a[0]`,`a[1]`, `a[2]`,`a[3]`, `a[4]`,`a[5]`, ...
  - Merge`a[0..1]`,`a[2..3]`, `a[4..5]`,`a[6..7]`, ...
  - Merge`a[0..3]`,`a[4..7]`, `a[8..11]`,`a[12..15]`, ...
- Also using an auxiliary array: BottomUpMergeSort.java

The complexity of sorting

- Mergesort uses between $\sim (n/2) \lg n$ and $\sim n \lg n$ compares
- Mergesort uses between $\sim 6n \lg n$ array accesses
- Mergesort uses $\sim 2n$ space (plus some var's)
- Q: How fast can compare-based sorting be?
- Book:

Quicksort

- Top-down (recursive) algorithm:
  - Choose a (pivot value) v in the array
  - Partition the array in non-empty parts $\leq v$ and $\geq v$
  - Quicksort the two parts
- Pros: in-place, average computation time $O(n \log n)$
- Cons: stack space for the recursion, worst-case $O(n^2)$
- Implementation: QuickSort.java

Quicksort, details

- Subtleties in `partition`:
  - Invariants `l<=h` in the two inner loops
  - Postcondition after the two inner loops
  - Invariant of the `for(;;)` loop
  - Termination of the `for(;;)` loop
- Termination of recursive `quicksort`

Quicksort, performance

- Compare Quicksort to other sorting methods ($n = 10^2, 10^3, \ldots$)
- Quicksort runs in quadratic time if pivot is always smallest (largest)
- Randomization is important (choose pivot randomly, or shuffle array)
- If all keys are distinct and randomization is perfect, then quicksort uses on average $\sim 2n \ln n$ compares (proof on blackboard)
- Similar result for exchanges holds (proof is complicated)
- Relevant improvements:
  - Cutoff to insertion sort for sizes $\leq M$
  - Median-of-three pivot
  - Taking advantage of duplicate keys (3-way partitioning)
- Quicksort is generally very good, ... bucketsort

Priority Queues

- Assume collecting and processing items having keys
- Examples of keys: time-stamp, price-tag, priority-tag
- Assume: keys can be ordered
- Reasonable: processing currently highest (or lowest)
- Seen this before? Yes, when items are time-stamped when added:
  - Queue: dequeue currently oldest (lowest time-stamp)
  - Stack: pop currently newest (highest time-stamp)
- Priority queue generalizes this
- Examples: highest priority, largest transaction, lowest price
- Distinction between 'item' and 'key' inessential

Priority Queues

- Good info: Wikipedia; API (the essentials):

```
public class  ArrayListPQ<Key extends Comparable<Key>>

void          insert(Key v) // insert a key
Key           delMax() // delete the largest key, if any
boolean       isEmpty()
int           size()
```

Heaps

- MNF130: A binary tree is complete if all levels are filled. So, a complete binary tree of depth d has $2^d - 1$ nodes (picture).
- INF102: A binary tree is (left-)complete if all levels $< h$ are filled, the level h may be partially be empty on the right. So, a (left-)complete binary tree of n nodes has height $\lfloor \lg n \rfloor$.
- A binary tree is heap-ordered if the key in each node is $\geq$ the keys in its children (if any). So, the root has a maximal key.
- Array representation of heap-ordered binary tree: picture
- The methods `swim` and `sink`

ToC and topics of general interest

- Table of Contents on next slide (all items clickable)
- Practical stuff: slide 2