Lorentz force law

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1 Background

We have tackled two formulation of the fundamental law of electrostatics, seeing it both as *Coulomnb's law* and then as *Gauss' law*. We are now ready for magnetism.

Magnetism is the electric force's slightly trickier cousin. And in fact, you will learn over time that they are strongly related - for now, however, we will just concern ourselves with the governing equations of motion that magnetism generates. In fact, magnetism is tricky enough that we will split up our understanding of it into two parts:

1. The force caused by a magnetic field
2. What causes a magnetic field in the first place

In electrics, we swallowed both of these questions in one go by announcing Coulomb's law, which tells us both what causes the electric field (point charges) and the force that is cause by the electric field (inverse square law). But since magnetism is a bit trickier, this is how we will do it, and we will begin by tackling question 1.

The magnitude of the force on a particle with charge q, moving with speed v through a magnetic field of magnitude B is given by the *Lorentz force law*:

$$F = qvB \sin\theta$$

where θ is the angle formed between the particle and the magnetic field. Some things to note here. Firstly, the force is proportional to the particles velocity. This means that a particle which doesn't move, or whose movement is parallel to the magnetic field, exerts no force. This is a totally different kind of dependence than we ahve seen previously. Secondly, we remember that force is not a scalar property, but a vector. So to determine what the force is (and not just its magnitude), we must know its direction. The direction of the magnetic force is perpendicular to both the velocity of the particle, and the magnetic field. This still leaves two options, and to uniquely determine the direction of the force, we use the *right-hand rule*. Here is how it works:

1. Take your right hand, and point all your fingers along the particle's velocity.
2. Swing (curl) your fingers into your palm so they point toward the magnetic field direction.
3. Your thumb now points the way the magnetic force acts. (If the charge is negative (electron), flip the thumb direction.)

The fact that the magnetic field has these two properties - the force is proportional to the velocity, and points in a perpendicular direction to the velocity - leads to some distinctly different equations of motion than we have seen so far! You will explore these in the problems below.

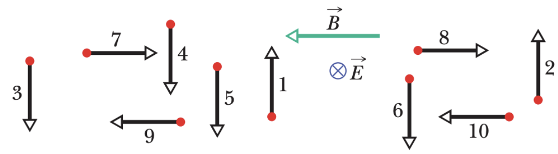

Figure 1: Particles in E and B-fields.

2 Questions

1. A singly ionized helium ion (He^+) from a solar storm shoots due north with speed $v = 4.0 \times 10^5\,\mathrm{m\,s^{-1}}$ as it enters a uniform magnetic field of magnitude $B = 1.2 \times 10^{-3}\,\mathrm{T}$ that points straight upward. Take $q = +e = 1.60 \times 10^{-19}\,\mathrm{C}$ and $m_{\mathrm{He}^+} = 6.64 \times 10^{-27}\,\mathrm{kg}$.

   (a) Using the Lorentz force law $\vec{F} = q\,\vec{v} \times \vec{B}$, find the *magnitude* of the magnetic force on the ion at that instant and state its *direction* relative to the cardinal directions (north/east/up).

   (b) Now suppose the ion's velocity makes an angle 45° with the magnetic field (i.e., $\vec{v}$ has both parallel and perpendicular components to $\vec{B}$). Find the magnitude of the force $|\vec{F}|$.

2. Figure 1 shows crossed uniform electric and magnetic fields E and B and, at a certain instant, the velocity vectors of the 10 charged particles listed in Table 1. (The vectors are not drawn to scale.) The speeds given in the table are either less than or greater than E/B (see Question 5). Which particles will move out of the page toward you after the instant shown in Fig. 1?

Particle	**Mass**	**Charge**	**Speed**
1	$2m$	q	v
2	m	$2q$	v
3	$\frac{m}{2}$	q	$2v$
4	$3m$	$3q$	$3v$
5	$2m$	q	$2v$
6	m	$-q$	$2v$
7	m	$-4q$	v
8	m	$-q$	v
9	$2m$	$-2q$	$3v$
10	m	$-2q$	$8v$
11	$3m$	0	$3v$

Table 1: Particle charges, masses, and speeds

3. An electron traveling to the right with speed v enters an area with an electric field E pointing downwards. There is also a magnetic field B present. The electron continus to move with constant velocity through the area.

   (a) What is the direction of the magnetic field?

   (b) What is the magnitude of the electric field B? The electron has mass m_e and charge $-e$.

4. A proton moving upward with speed v enters an area with a megnatic field B pointing out of the page. Find an expression for the curve that the proton traces out in its motion.

5. A small charged ball suspended on an inextensible thread of length l moves in a uniform, time-independent upward magnetic field of induction B. The mass of the ball is m, the charge is q, and the period of revolution is T. Determine the radius r of the circle in which the ball moves if the thread is always stretched.

6. A uniform electric field E and a uniform magnetic field B meet at an angle ϕ ($0° < \phi < 90°$). A proton of fixed speed v must be launched in a direction that lies *in the plane* containing E and B (no out-of-plane component allowed). At the instant it enters the fields, which launch angle θ (measured from B) maximizes the magnitude of its acceleration? What is that maximum, in terms of q, m, v, E, B, and ϕ?

7. A straight wire segment of length L carries a steady current I. Mobile charges (charge q) drift along the wire with speed u. A uniform magnetic field of magnitude B makes an angle θ with the wire.

   (a) Using only the definition of current as "charge per unit time," find I in terms of the charge per unit length λ and the drift speed u.

   (b) At one instant, consider all the mobile charge inside the length L. What is the total magnetic-force *magnitude* on that charge?

   (c) Express your result entirely in terms of I, L, B, and θ.

8. A particle of mass m and charge q moves in a uniform magnetic field of magnitude B. A dissipative drag force of magnitude $F_{\text{drag}} = \gamma v$ opposes its motion. At $t = 0$ the particle's speed is v_0, directed perpendicular to the field, and the particle has a physical radius a. How many revolutions does the particle complete before it stops?

Answers

1. (a) $F = q\,v\,B = 7.68 \times 10^{-17}$ N (east).

   (b) $F = q\,v\,B\,\sin 45° = 5.43 \times 10^{-17}$ N.

2. —

3. (a) Magnetic field direction: into the page.

   (b) $B = \dfrac{E}{v}$.

4. Trajectory: $x^2 + (y - r)^2 = r^2, \quad r = \dfrac{m\,v}{q\,B}$

5. $r = \sqrt{l^2 - \dfrac{m^2 g^2 T^4}{\left(4\pi^2 m - 2\pi q B T\right)^2}}$

6. $\theta^* = 90°, \qquad a_{\max} = \dfrac{q}{m}\sqrt{E^2 + (vB)^2}$.

7. (a) $I = \lambda\,u$.

   (b) $F = \lambda\,L\,u\,B\,\sin\theta$.

   (c) $F = I\,L\,B\,\sin\theta$.

8. $N = \dfrac{qB}{2\pi\,\gamma}\ln\!\left(\dfrac{m\,v_0}{a\,qB}\right)$ (valid for $v_0 > a\,qB/m$).