

Analyzing melanotaeniidae and osphronemidae population interaction models with computational solutions of ordinary differential equations

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1 Summary

2 Introduction

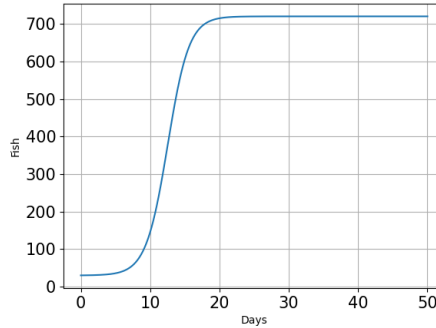
3 Simple early model

We want to analyze how a rainbowfish population shifts inside of an aquarium over time. We want to know if it's feasible to sell 20 fishes per day and still retain a population. This is why we need to find the equilibrium points, where the fish population is stable.

For that we need to construct an Ordinary Differential Equation. The growth rate of the rainbowfish population is 70% with a maximum aquarium capacity of 750 fishes. The death rate is 0.001 times the population, aka one fish lives for 1000 days. But this is negligible and is removed. 20 rainbowfishes are bought every day, so that's included in the model.

$$\frac{dP}{dt} = 0.7P(t)\left(1 - \frac{P(t)}{750}\right) - 20 \quad (1)$$

The model was then solved numerically in Python using Euler's formula with $\Delta t = \frac{1}{16}$ which gave the following results:



As we can see, the amount of rainbowfish approaches somewhere above 700. The exact amount is 720.2, but since you can't have fractional fish we can approximate it to 720. This is a stable equilibrium point as it increases from below and increases from above.

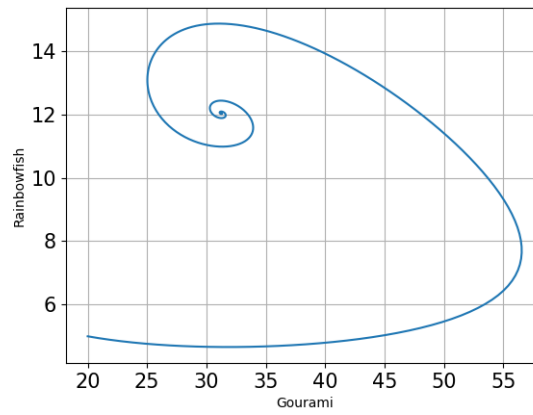
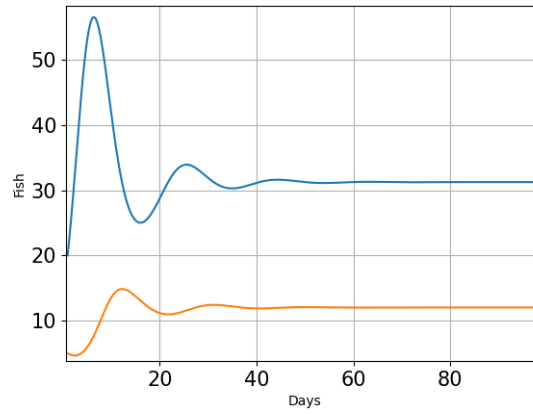
As a conclusion this model does work for one fish, however we want to introduce a second type of fish into the model. The gourami.

4 Model including the second fish

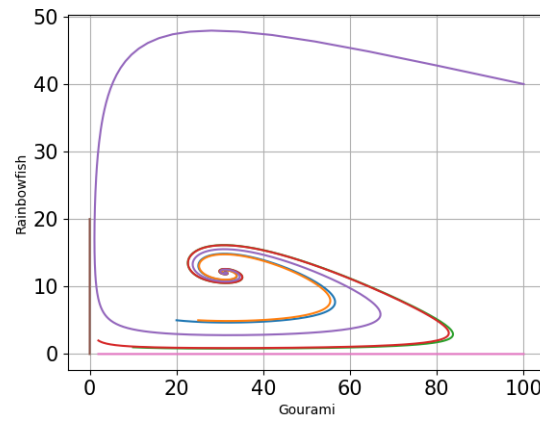
The fishowner desperately wanted two types of competing fish species: rainbowfish and gourami. The budget was for 20 rainbowfish and 5 gourami. To make sure that both fishes both had enough food and didn't manage to kill each other we made a system of differential equations. In this case they interact with each other. There is a 4% chance that a gourami will kill a rainbowfish, thus the $-0.04PG$. Gouramis also don't survive on their own so they slowly die, explaining the $-0.25G$

$$\begin{cases} \frac{dP}{dt} = 0.7P - 0.007P^2 - 0.04PG \\ \frac{dG}{dt} = -0.25G + 0.008PG \end{cases}$$

Now doing the rest was easy. So easy in fact that I wanted to shoot my foot with a rocket launcher. The result was the following:



These models tell us that the fish seem to approach an equilibrium at roughly 30 rainbowfishes and 12 gourami. Modelling a few more scenarios with different starting populations reveal that there is a stable equilibrium point as long as the amount of fish of one species is more than 0.



5 Conclusions

6 Appendix