Analyzing melanotaeniidae and osphronemidae population interaction models with computational solutions of ordinary differential equations

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1 Summary

2 Introduction

The Melings Lake in Fagersta, Västmanland, Sweden has been without fish for a hundred years. Today conservationists has asked us if an reintroduction of fish could boost the ecosystem. We decided on two types of fish and what the potential outcome would be. Would these two fishes go extingt and eat eachother up, or would they live in equelibrium?

Since you can only buy so many fish we need to know how many is feasable and if the population will stabilize at a good point that doesn't lead to bad consequences.

The purpose of this research is to find out how the fish will behave and if they will live in equilibrium with nature so that this effort is worth while.

The limitations of this project is that it cannot simulate all of nature. It also doesn't take into account

Outline for the report

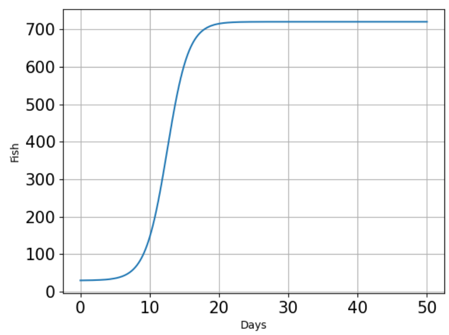
3 Simple early model

We want to analyze how a rainbowfish population shiftes inside of an aquarium over time. We wan't to know if it's feasable to sell 20 fishes per day and still retain a population. This is why we need to find the equalibrium points, were the fish population is stable.

For that we need to construct an Ordinary Differential Equation. The growth rate of the rainbowfish population is 70% with a maximum aquarium capacity of 750 fishes. The death rate is 0.001 times the population, aka one fish lives for 1000 days. But this is negligable and is removed. 20 rainbowdishes are bought every day, so that's included in the model.

$$\frac{dP}{dt} = 0.7P(t)(1 - \frac{P(t)}{750}) - 20 \tag{1}$$

The model was then solved numericaly in Python using eulers formula with $\Delta t = \frac{1}{16}$ which gave the following results:



As we can see, the amount of rainbowfish approaches somewhere above 700. The exact amount is 720.2, but since you can't have fractional fish we can approximate it to 720. This is a stable equilibrium point as it increases from below and increases from above.

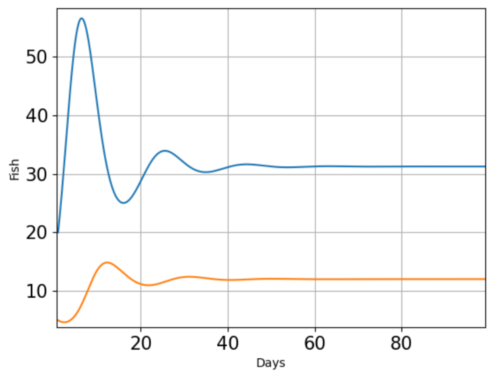
As a conclusion this model does work for one fish, however we want to introduce a second type of fish into the model. The grourami.

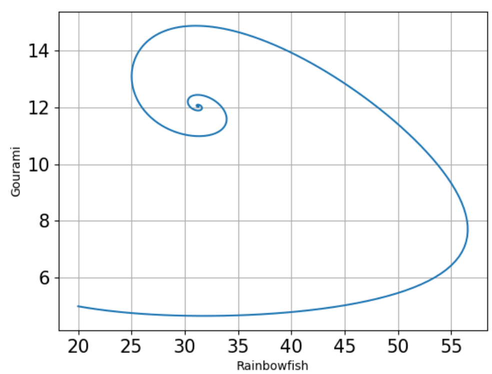
4 Model including the second fish

The fishowner desperately wanted two types of competing fish species: rainbowfish and gourami. The budget was for 20 rainbowfish and 5 gourami. To make sure that both fishes both had enough food and didn't manage to kill eachother we made a system of differential equations. In this case they interact with each other. There is a 4% chance that a gourami will kill a rainbowfish, thus the $-0.04PG$. Gouramis also don't survive on their own so they slowly die, explaining the $-0.25G$

$$\begin{cases} \dfrac{dP}{dt} = 0.7P - 0.007P^2 - 0.04PG \\ \dfrac{dG}{dt} = -0.25G + 0.008PG \end{cases}$$

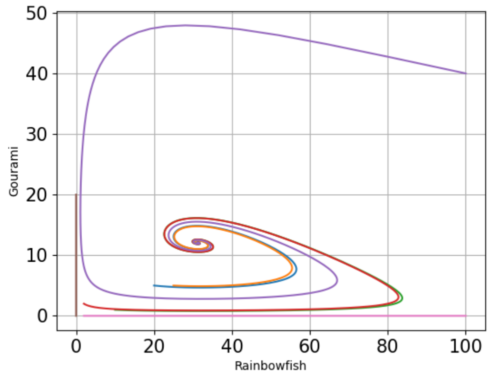
Now doing the rest was easy. So easy in fact that I wanted to shoot my foot with a rocket launcher. The result was the following:





These models tells us that the fish seem to approach an equilibrium at roughtly 30 rainbowfishes and 12 gourami. The exact equilibrium point is when $\frac{dP}{dt} = 0$ and $\frac{dG}{dt} = 0$. This evaluates to the number of rainbowfish being 31.25 and the amount of gourami is 12.03125. Again, since you cannot have fractional fishes the equilibrium is roughly at 31 rainbowfish and 12 gourami.

Modelling a few more scenarios with different starting populations reveal that there is a stable equilibrium point aslong as the amount of fish of one species is more than 0.



5 Conclusions

6 Appendix