Analyzing melanotaeniidae and osphronemidae population interaction models with computational and analytical solutions of ordinary differential equations

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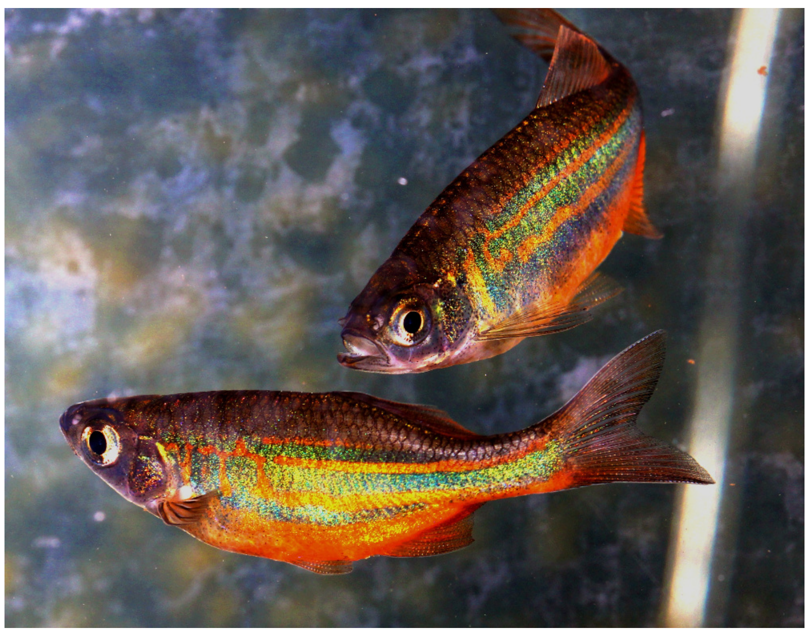
2020-04-30



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Summary

An analysis of a fish population at a popular aquarium for a shop owner was shown to be necessary. The shop owner wanted both rainbowfish and gouramis, despite gouramis appetite for rainbowfish so we needed to build a mathematical model to see if the populations would stabilize at an equilibrium or if they would die off over time.

A system of differential equations were used to model the fish populations in Python using euler's formula. It was then shown that an equilibrium does in fact exist.

The conclusion is that the aquarium owners original idea does in fact work, and that the fishes form a stable equilibrium at roughly 31 rainbowfish and 12 gourami. This shows that having both fishes in the same aquarium is feasible. However the modell cannot control for unexpected occurrences.

List of Variables

t: Time in days

P(t): Population of rainbowfish at time t

G(t): Population of gourami at time t

1 Introduction

An analysis of a fish population at a popular aquarium for a shop owner was shown to be necessary. The shop owner wanted both rainbowfish and gouramis, despite gouramis appetite for rainbowfish so we needed to build a mathematical model to see if the populations would stabilize at an equilibrium or if they would die off over time.

We have a budget constraint so we need to know how many fishes are feasible and if the population will stabilize at some point that doesn't lead to bad consequences such as all rainbowfish being eaten by the gouramis.

The purpose of this research is to find out how the fish population will adapt and evolve over time and if they can live in a stable equilibrium.

The limitations of this project is that it cannot simulate all of nature and the fish-fish interaction. The model can only simulatie the population and how different populations interacts with each other.

In chapter 3 I discuss the early model for just rainbowfish. In chapter 4 I add the gourami fish.

2 First Model

2.1 Deriving the ODE for the rainbowfish population

An Ordinary Differential Equation needs to model the rainbow fish population change. The growth rate of the rainbowfish population is 70% with a maximum aquarium capacity of 750 fishes. The death rate is estimated to be 0.001 because one fish lives for 1000 days, therefore the odds of a fish dying is 0.001, which is then multiplied by the number of fishes. $0.001P(t)$ However this is negligible and is removed since the decrease it makes is really small. However, 20 rainbowfishes are sold every day, so that's included in the model and is not negligible.

$$\frac{dP}{dt} = 0.7P(t)(1 - \frac{P(t)}{750}) - 20 \tag{1}$$

2.2 Results from numerical solution in Python

The model was then solved numerically in Python using euler's formula with timestep $\Delta t = \frac{1}{16}$ which gave the following results:

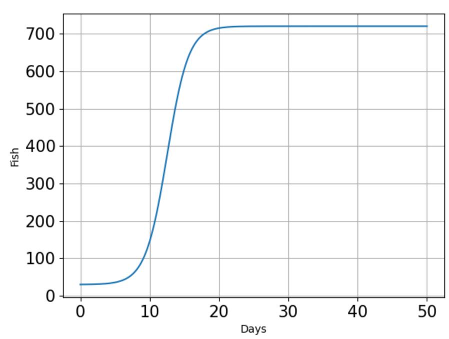

Figure 1: The number of rainbowfish over time with $P(0) = 20$

As we can see, the amount of rainbowfish approaches somewhere above 700. The exact amount is 720.2, but since you can not have fractional fish we can approximate it to 720. This is a stable equilibrium point as the population would decrease if it went up and increase if it was lowered.

As a conclusion this model does work for one fish, however we want to introduce a second type of fish into the model. The gourami.

3 Second model

3.1 Deriving a system of ODE's with gourami fish

The fish owner desperately wanted two types of fish species: rainbowfish and gourami. The current supply and budget allows for 20 rainbowfish and 5 gourami. To make sure that both fishes both had enough food and didn't go extinct a system of differential equations were derived. There is a 4% chance that a gourami will kill a rainbowfish, or in math terms $-0.04PG$. Gouramis also don't survive on their own so they slowly die, in math terms $-0.25G$

$$\begin{cases} \dfrac{dP}{dt} = 0.7P - 0.007P^2 - 0.04PG \\ \qquad \dfrac{dG}{dt} = -0.25G + 0.008PG \end{cases}$$

3.2 Numerically solving the system in Python

Now doing the rest was easy. So easy in fact that I wanted to shoot my foot with a rocket launcher. The results are shown below.

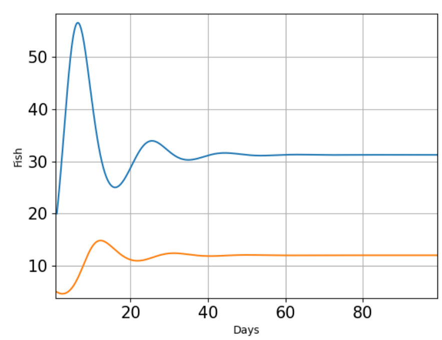

Figure 2: Number of rainbowfish(Blue) and gourami (Orange) over time. Note that the rainbowfish population is shifting slightly "behind" the gourami population

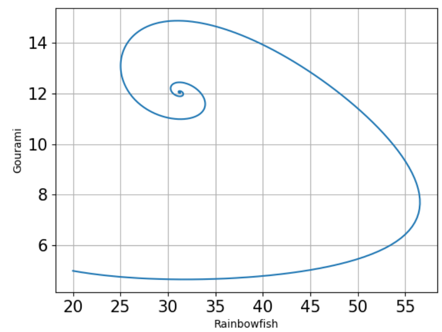

Figure 3: The amount of Gouramis on the y-axis and the amount of rainbowfish on the x-axis.

These models tells us that the fish seem to approach an equilibrium at roughly 30 rainbowfishes and 12 gourami. The exact equilibrium point is when $\frac{dP}{dt} = 0$ and $\frac{dG}{dt} = 0$. This evaluates to the number of rainbowfish being 31.25 and the amount of gourami is 12.03125. Again, since you cannot have fractional fishes the equilibrium is approximated to be at 31 rainbowfish and 12 gourami.

Modelling a few more scenarios with different starting populations reveal that there is a stable equilibrium point as long as the amount of fish of one species is more than 0. The reason for doing this is for extra accuracy. If you plot fish populations against each other rather than over time as shown in figure 4 it creates a "vortex shape" since the populations always approach the equilibrium value.

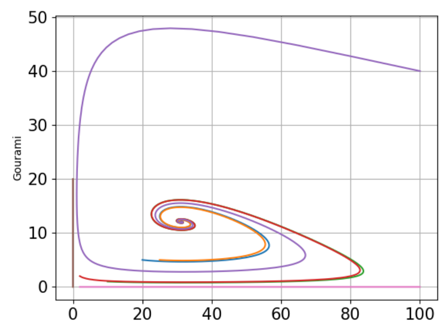

Figure 4: Many different starting populations to illustrate a "vortex shape". Note that if you begin with 0 rainbowfish the gourami always drop to 0 and if you begin with 0 gourami then the rainbowfish will approach 100

4 Conclusions

Could we keep 20 rainbowfish and 5 gouramis? Indeed this is the case. With our desired starting population of 20 rainbowfish and 5 gouramis the fish population reaches a stable equilibrium. And this equilibrium is reached even with a large range of starting populations. This also implies the possibility of buying fewer fishes to save money.

However this model can only explain so much. Birth rates and death rates may be affected by unknown causes. And it cannot model the fact that fishes lay and hatch many eggs at once instead of continuously over time. But despite these drawbacks it's safe to say that the fishes will reach a stable equilibrium point.