Analyzing melanotaeniidae and osphronemidae population interaction models with computational and analytical solutions of ordinary differential equations

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1 Summary

An analyzis of a fish population for a popular aquarium for a shop owner was shown to be neccecary.The shop owner wanted both rainbowfish and gouramis, but gouramis eat rainbowfish so we needed to build a mathematical modell to see if the populations would stabilize at an equilibrium or if they would die off in some way.

A system of differential equations were used to model the fish populations in Python using eulers formula. It then shows that an equilibrium does infact exist.

The conclusion is that this does infact work, and that the fishes form an equilibrium at roughly 31 rainbowfish and 12 gourami. This shows that having both fishes in the same aquarium is feasable. However the modell cannot controll for unexpected occurances.

2 Introduction

The Aquarium we own needs more fish. We decided on two types of fish and what the potential outcome would be. Would these two fishes go extingt and eat eachother up, or would they live in equelibrium?

Since you can only buy so many fish we need to know how many is feasable and if the population will stabilize at a good point that doesn't lead to bad consequences.

The purpose of this research is to find out how the fish will behave and if they will live in equilibrium with nature so that this effort is worth while.

The limitations of this project is that it cannot simulate all of nature and fish behaviour, just the population and how different populations interacts with eachother.

In chapter 3 I discuss the early model for just rainbowfish. In chapter 4 I add a second fish, gourami fish that eats rainbowfish.

3 Simple early model

We want to analyze how a rainbowfish population shiftes inside of an aquarium over time. We wan't to know if it's feasable to sell 20 fishes per day and still retain a population. This is why we need to find the equalibrium points, were the fish population is stable.

For that we need to construct an Ordinary Differential Equation. The growth rate of the rainbowfish population is 70% with a maximum aquarium capacity of 750 fishes. The death rate is estimated to be 0.001 because one fish lives for 1000 days, therefore the odds of a fish dying is 0.001, which is then multiplied by the number of fishes. $0.001P(t)$ However this is negligable and is removed since the decrease it makes is really small. Also, 20 rainbowfishes are sold every day, so that's included in the model and is not negligable.

$$\frac{dP}{dt} = 0.7P(t)(1 - \frac{P(t)}{750}) - 20 \tag{1}$$

The model was then solved numericaly in Python using eulers formula with $\Delta t = \frac{1}{16}$ which gave the following results:

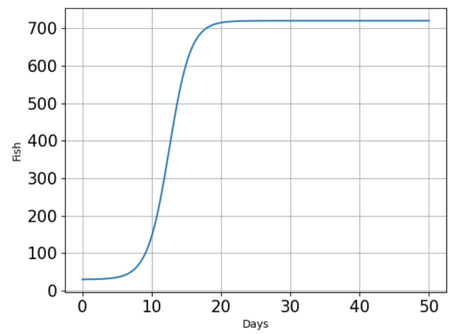

Figure 1: The number of rainbowfish over time with $P(0) = 20$

As we can see, the amount of rainbowfish approaches somewhere above 700. The exact amount is 720.2, but since you can not have fractional fish we can approximate it to 720. This is a stable equilibrium point as it increases from below and increases from above.

As a conclusion this model does work for one fish, however we want to introduce a second type of fish into the model. The grourami.

4 Model including the second fish

The fishowner desperately wanted two types of competing fish species: rainbowfish and gourami. The budget was for 20 rainbowfish and 5 gourami. To make sure that both fishes both had enough food and didn't manage to kill eachother we made a system of differential equations. In this case they interact with each other. There is a 4% chance that a gourami will kill a rainbowfish, thus the $-0.04PG$. Gouramis also don't survive on their own so they slowly die, explaining the $-0.25G$

$$\begin{cases} \dfrac{dP}{dt} = 0.7P - 0.007P^2 - 0.04PG \\ \dfrac{dG}{dt} = -0.25G + 0.008PG \end{cases}$$

Now doing the rest was easy. So easy in fact that I wanted to shoot my foot with a rocket launcher. The result are shown below.

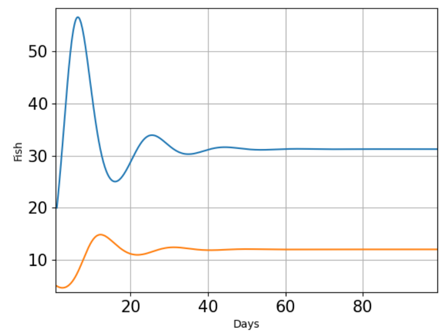

Figure 2: Number of rainbowfish(Blue) and gourami (orange) over time. Note that the rainbowfish population is slightly "behind" the gourami population

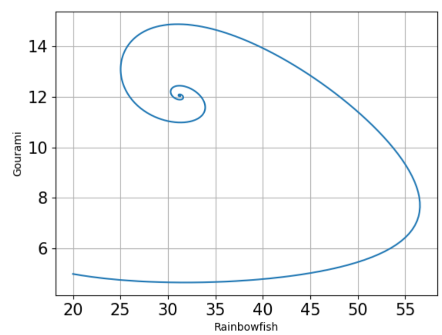

Figure 3: The amount of Gouramis on the y-axis and the amount of rainbowfish on the x-axis.

These models tells us that the fish seem to approach an equilibrium at roughly 30 rainbowfishes and 12 gourami. The exact equilibrium point is when $\frac{dP}{dt} = 0$ and $\frac{dG}{dt} = 0$. This evaluates to the number of rainbowfish being 31.25 and the amount of gourami is 12.03125. Again, since you cannot have fractional fishes the equilibrium is roughly at 31 rainbowfish and 12 gourami.

Modelling a few more scenarios with different starting populations reveal that there is a stable equilibrium point aslong as the amount of fish of one species is more than 0. It creates a "vortex shape" since the populations always approach the equilibrium value.

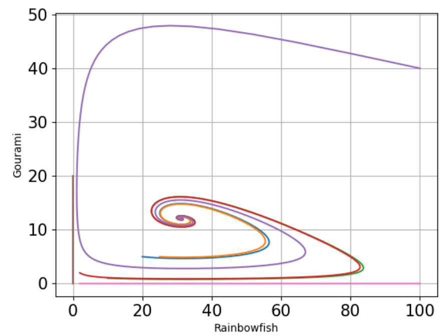

Figure 4: Many different starting populations to illustrate a "vortex shape". Note that if you begin with 0 rainbowfish the gourami always drop to 0 and if you begin with 0 gourami then the rainbowfish will approach 100

5 Conclusions

So in conclusion, could we keep 30 rainbowfish and 5 gourami's? Indeed this is the case. With our desired starting population of 30 rainbowfish and 5 gouramis the fish population reaches a stable equilibrium. And this equilibrium is reached even with a large range of starting populations. This implies that we can buy fewer fishes to save money.

However this model can only explain so much. Birth rates and death rates may be affected by unknown causes. And it cannot model the fact that fishes lay and hatch many eggs at once instead of continiously over time. But despite these drawbacks it's safe to say that the fishes will reach an equilibrium point.

6 Appendix