MULTIPLICADOR DE LAGRANGE

$$\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)$$
$$g(x_0, y_0, z_0) = k$$

Determine los extremos de la función $f(x, y) = x^3 + 3xy^2$ restringida para la circunferencia $x^2 + y^2 = 4$

$$f(x, y) = x^3 + 3xy^2$$
$$^{s}/_{a} = x^2 + y^2 = 4$$
$$g(x, y) = x^2 + y^2 - 4 = 0$$
$$\nabla f(x, y) = \lambda \nabla g(x, y)$$
$$(fx, fy) = \lambda(gx, gy)$$
$$(fx, fy) = (\lambda gx, \lambda gy)$$

$$(fx, fy) = (\lambda gx, \lambda gy)$$
$$(3x^2 + 3y^2, 6xy) = (\lambda(2x), \lambda(2y))$$
$$\iota)\ 3x^2 + 3y^2 = \lambda(2x)$$
$$\iota\iota)\ 6xy = \lambda(2y)$$
$$\iota\iota\iota) x^2 + y^2 - 4 = 0$$

$$\frac{3x^2}{2x} + \frac{3y^2}{2x} = \frac{\lambda(2x)}{2x}$$
$$\frac{3x}{2} + \frac{3y^2}{2x} = \lambda$$

$$\frac{6xy}{2y} = \frac{\lambda(2y)}{2y}$$

$$3x = \lambda$$

Igualar λ

$$\frac{3x}{2}+\frac{3y^2}{2x}=3x$$

Multiplicar por 2

$$\frac{6x}{2}+\frac{6y^2}{2x}=6x$$

$$3x+\frac{3y^2}{x}=6x$$

Multiplicar por x

$$3x^2+\frac{3xy^2}{x}=6x^2$$

$$3x^2+3y^2=6x^2$$

División por 3

$$\frac{3x^2}{3}+\frac{3y^2}{3}=\frac{6x^2}{3}$$

$$x^2+y^2=2x^2$$

$$y^2=2x^2-x^2$$

$$y^2=x^2$$

$$x^2+y^2-4=0$$

$$x^2+x^2-4=0$$

$$2x^2-4=0$$

$$x^2=\frac{4}{2}$$

$$x^2=2$$

$$\sqrt{x^2}=\sqrt{2}$$

$$x=\pm\sqrt{2}$$

$$x=\sqrt{2}$$

$$x=-\sqrt{2}$$

$$y^2=\left(\sqrt{2}\right)^2$$

$$y^2=2$$

$$\sqrt{y^2}=\sqrt{2}$$

$$y=\pm\sqrt{2}$$

$$y^2=\left(-\sqrt{2}\right)^2$$

$$y^2=-2$$

$$\sqrt{y^2}=-\sqrt{2}$$

$$y=\pm\sqrt{2}$$

$$P_1\left(\sqrt{2},\sqrt{2}\right)$$

$$P_2\left(\sqrt{2},-\sqrt{2}\right)$$

$$P_3\left(-\sqrt{2},\sqrt{2}\right)$$

$$P_4\left(-\sqrt{2},-\sqrt{2}\right)$$

$$f\left(\sqrt{2},\sqrt{2}\right)=x^3+3xy^2=8\sqrt{2}$$

$$f\left(\sqrt{2},-\sqrt{2}\right)=x^3+3xy^2=8\sqrt{2}$$

Máximo Absoluto

$$f\left(-\sqrt{2},\sqrt{2}\right)=x^3+3xy^2=-8\sqrt{2}$$

$$f\left(-\sqrt{2},-\sqrt{2}\right)=x^3+3xy^2=-8\sqrt{2}$$

Mínimo Absoluto

Determine los extremos de la función $f(x,y) = y^2 - 4x$ restringida para la circunferencia $x^2 + y^2 = 9$

$$f(x,y) = y^2 - 4x$$
$$^{S}/_{a} = x^2 + y^2 = 9$$
$$g(x,y) = x^2 + y^2 - 9 = 0$$
$$\nabla f(x,y) = \lambda \nabla g(x,y)$$
$$(fx, fy) = \lambda(gx, gy)$$
$$(fx, fy) = (\lambda gx, \lambda gy)$$

$$(fx, fy) = (\lambda gx, \lambda gy)$$
$$(-4, 2y) = (\lambda(2x), \lambda(2y))$$
$$\iota)\ -4 = \lambda(2x)$$
$$\iota\iota)\ 2y = \lambda(2y)$$
$$\iota\iota\iota) x^2 + y^2 - 9 = 0$$

$$-\frac{4}{2} = \frac{\lambda(2x)}{2}$$
$$-2 = \lambda\, x$$
$$\frac{2y}{2} = \frac{\lambda(2y)}{2}$$
$$y - \lambda(y) = 0$$
$$y(1-\lambda) = 0$$
$$1 = \lambda$$
$$y = 0$$
$$-2 = (1)\, x$$
$$-2 = x$$

$$x^2 + y^2 - 9 = 0$$
$$(2)^2 + y^2 - 9 = 0$$
$$y^2 = 9 - 4$$
$$y^2 = 5$$
$$y = \pm\sqrt{5}$$

$$x^2 + 0^2 - 9 = 0$$
$$x^2 = 9$$
$$x = \sqrt{9}$$
$$x = \pm 3$$

$P_1\ (3,0\ )$
$P_2\ (-3,0)$

$P_3\ (-2, \sqrt{5})$
$P_4\ (-2, -\sqrt{5})$

$$f(x,y) = y^2 - 4x$$

$P_1\ (3,0\ )$

$P_2\ (-3,0)$

$P_3\ \left(-2, \sqrt{5}\right)$

$P_4\ \left(-2, -\sqrt{5}\right)$

$f(3,0) = y^2 - 4x = -12$ Mínimo Absoluto

$f(-3,0) = y^2 - 4x = 12$

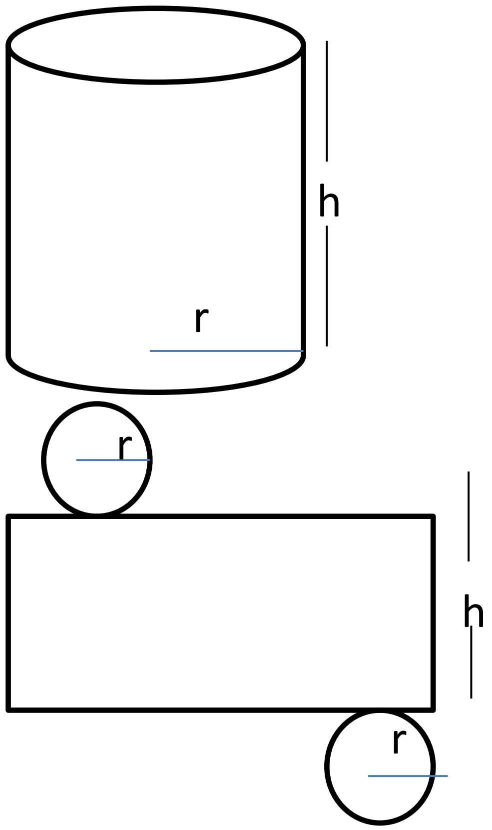
$f\left(-2, \sqrt{5}\right) = y^2 - 4x = 13$

Máximo Absoluto

$f\left(-2, -\sqrt{5}\right) = y^2 - 4x = 13$

Un cilindro circular recto cerrado tendrá un volumen de $1000\ pies^3$ la parte superior y el fondo de cilindro se construirá con metal de 2 dólares por $pies^2$. El costado se formará con metal que cuesta $2{,}50\ dolares\ por\ pies^2$. Determine el costo mínimo de fabricación

$v = 1000\ pies^3$ $\quad$ $c/costado = 2{,}5USD/\ ft^2$ $\quad$ $c/tapa = 2USD/\ ft^2$



Función costo $C(r,h) = 2(2\pi r^2) + 2{,}5(2\pi rh)$

Cuesta Tapa

Área Circulo

2 Tapa

$C(r,h) = (4\pi r^2) + (5\pi rh)$ Función costo

$v = \pi r^2 h = 1000$

$v(r,h) = \pi r^2 h - 1000 = 0$ Función restricción

$$\iota)\ 8\pi r + 5\pi h = \lambda(2r\pi h) = 8r + 5h = \lambda(2rh)$$
$$\iota\iota)\ 5\pi r = \lambda(\pi r^2) = 5 = \lambda r$$
$$\iota\iota\iota)\pi r^2 h - 1000 = 0$$

$p = \pi D = (2\pi r)$

$$8r + 5h = \lambda(2rh)$$

$$5 = \lambda r$$

$$r = \frac{5}{\lambda}$$

$$8\left(\frac{5}{\lambda}\right) + 5h = \lambda\left(2\left(\frac{5}{\lambda}\right)h\right)$$

$$\frac{40}{\lambda} + 5h = 10h$$

$$\frac{40}{\lambda} = 10h - 5h$$

$$\frac{40}{\lambda} = 5h$$

$$\frac{40}{5\lambda} = h$$

$$\frac{8}{\lambda} = h$$

$$\frac{8}{h} = \lambda$$

$$\lambda = \frac{5}{r}$$

$$\frac{5}{r} = \frac{8}{h}$$

$$5h = 8r$$

$$h = \frac{8r}{5}$$

$$\pi r^2 h - 1000 = 0$$

$$\pi r^2\left(\frac{8r}{5}\right) - 1000 = 0$$

$$\frac{8\pi r^3}{5} = 1000$$

$$r^3 = \frac{1000 \times 5}{8\pi}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\frac{1000 \times 5}{8\pi}}$$

$$r = 5{,}837$$

$$h = \frac{8(5{,}837)}{5}$$

$$h = 9{,}3392$$

$$C(5{,}837{,}9{,}3392) = (4\pi(5{,}837)^2) + \left(5\pi(5{,}837)(9{,}3392)\right)$$

$$C(5{,}837{,}9{,}3392) = 1284{,}5$$