Introduction to Computational Methods

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Comp. methods can help you with:

- Simulation of economic variables
  - Visualize economic variables when we know the process (pseudo-randomness)
- Solution of an economic model
  - Equations characterize relationship between variables implicitly, not always useful ($C_t + K_{t+1} \leq z_t K_t$ or worse $C_t^{-\sigma} = \beta C_{t+1}^{-\sigma}(1 + R_t)$)
  - Instead, solve for policy function – choice variables (e.g. C_t or K_{t+1} as known functions of states, K_t, z_t etc.)

Once we have an economic model with qualitatively plausible relationships:

- Quantifying economic models:
  - estimation – choose parameters based on data series (e.g. find $\{z_t\}$ using series for $\{Y_t, K_t\}$)
  - calibration – chooses parameters based on micro-estimates, past studies etc. ($\sigma = 2$)

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Once this is done, conduct computational experiments:

- How do economic outcomes vary with parameters (σ, δ etc.) ?
- What happens if the government pursues expansionary fiscal policy?
  - Construct counterfactual economy

Modern Macro and Computational Methods

This course will have a strong macro flavour:

- Macro models are *dynamic*, so often need to be studied with computational techniques
- These models are usually highly non-linear and analytically intractable

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- The model may or may not have uncertainty

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- Discretization of stochastic processes
  - Simulate Markov processes

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From these, you should be able to build up and support your own research! eg. Krusell Smith $\rightarrow$ heterogeneous firms

Logistics

- Software: Matlab
  - Download, install, do online tutorial if needed (Onramp)
  - Lecture notes are mostly self-contained but supplementary textbooks useful
  - Problem sets are *critically* important

1. Numerical Methods in Economics (The MIT Press), by Kenneth Judd

- ∗ Reference book, brilliant for theorems.

2. Dynamic General Equilibrium Modelling: Computational Methods and Applications, by Burkhard Heer and Alfred Maussner

- ∗ Macro models with Matlab implementation

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See also www.wouterdenhaan.com/notes.htm

Lecture plan

1. Tools: Functional Approximation, Non-Linear Equations
   - polynomial theorems, Newton method, discretization etc.
2. Numerical Integration and Stochastic Processes
3. Value Function Iteration
   - One continuous time example (labour market matching) *
4. Local linear methods
   - Theory of perturbation, Dynare and other toolboxes, implementation outside Dynare
   - Linear time iteration (Pontus Rendahl) *
5. Global Solution Methods (policy function iteration)
6. Heterogeneous Agent models *

Material acknowledgements

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