Computational Methods for Economics, UC Davis

Problem Set 2

Value Function Iteration

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1. **RBC Model with Irreversible Investment**
Consider the following RBC model with irreversible investment:

$$\max_{c_t, l_t, k_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to:

$$\begin{aligned} c_t + k_{t+1} &= z_t F(k_t, h_t) + (1-\delta) k_t \\ k_{t+1} &\geq (1-\delta) k_t \\ 1 &= h_t + l_t \\ \ln(z_{t+1}) &= (1-\rho)\ln(z^*) + \rho \ln(z_t) + \sigma \varepsilon_t \end{aligned}$$

where k is capital, c consumption, h hours worked, l leisure and z a technology shock.

Assume $F(k_t, h_t) = k_t^{\alpha} h_t^{1-\alpha}$, $u(c_t, l_t) = \ln c_t + \mu \ln l_t$, $\beta = 0.99$, $\alpha = 0.36$, $\delta = 0.025$, $\rho = 0.98$, $\sigma = 0.002$ and $z^* = 1$. Choose μ such that the hours worked h in steady state is 1/3.

(a) Solve the model above using VFI and *discretization*:
   i. begin with a deterministic model, no labour and no occasionally binding constraint
   ii. add labour leisure choice
   iii. add stochasticity
   iv. add occasionally binding constraint

(b) Calculate and report the first and second moments of consumption, hours, capital, investment and output.

(c) Change σ such that $k_{t+1} \geq (1-\delta)k_t$ binds on average 5% of the time.Repeat (b) and compare moments.

(d) Check the accuracy of your approximation in (c) by calculating the Euler Equation Errors.

2. (*This question is optional, but strongly recommended.*)

   (a) **Improvement algorithms**

      i. exploit the concavity of the value function (hint: only need to change the maximization part of the code)

      ii. iterate on the policy function (Howard's improvement algorithm)

   (b) **Finite element methods**

      i. Instead of discretizing the value function, use a polynomial to obtain a continuous approximation

   You can use the commands *tic* and *toc* to measure speed gains and Euler errors to measure accuracy gains.

Reading

Den Haan, W. J., *Teaching Notes*, `www.wouterdenhaan.com/notes.htm`

Heer, B. and A. Maussner (2009), *Dynamic General Equilibrium Modeling: Computational Methods and Applications*, 2nd Edition, Springer, Chapters 3-4, 8-9.

Judd, K. L. (1998), *Numerical Methods in Economics*, The MIT Press, Chapter 17.