Computational Methods for Economics, UC Davis

Problem Set 1
Function Approximation and Local Solution Methods

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1 Solving Equations on MATLAB

Consider the function:

$$f(x) = x^{-5} - x^{-3} - c$$

1. Set $c = 1$ and plot f on $[0.6, 10]$. Find x^* such that $f(x^*) = 0$ using (i) bisection (ii) Newton's method.
2. Now construct an equidistant grid containing 10 nodes between 1 and 8, and for each value of c on the grid find x^*.
3. Construct a new equidistant grid containing 1000 nodes between 1 and 10, and plot your solution for each value on this grid using a spline.
4. Using the MATLAB function `fzero`, find x^*.
5. Relabel your solution x^* from part (a) $x(c)$. Find the inverse function $c(x)$, and plot it on an equidistant grid of 1000 nodes on [0.6,10] using a spline approximation.
6. Now find the solution to:

$$0 = c(x) + x$$

2 Approximation Methods: Chebyshev Collocation and Regression

(a) Consider the function $f(z) = e^z$ on the domain $z \in [0, 1]$ i. Approximate $f(z)$ by Chebyshev collocation of order $n = 2$ with $m = 3$ grid points. Plot the actual function along with your approximation.
ii. Approximate $f(z)$ by Chebyshev regression of order $n = 2$ with $m = 10$ grid points. Plot this new approximation along with both the actual function and your approximation from (i).
iii. For both (i) and (ii), calculate the maximum approximation error between e^x and the approximation. Comment on the differences between the results.
iv. What happens to the maximum approximation error if you approximate the function $f(z)$ to order $n = 5$ with $m = 10$ grid points? Comment on the value-added of higher- order polynominal terms for this example.

(b) Consider the function:

$$g(z) = \begin{cases} 0 & \text{if } z < 1 \\ (z-1) & \text{if } z \geq 1 \end{cases}$$

on the domain $z \in [0, 2]$ by:

i. Approximate $g(z)$ by Chebyshev collocation of order $n = 5$ with $m = 6$ grid points. Plot the actual function along with your approximation.

ii. Approximate $g(z)$ by Chebyshev regression of order $n = 5$ with $m = 30$ grid points. Plot this new approximation along with both the actual function and your approximation from (i).

iii. Approximate $g(z)$ by Chebyshev collocation of order $n = 2$ with $m = 3$ grid points. Plot this new approximation along with both the actual function and your approximations from (i) and (ii).

iv. Compare your results from (i), (ii) and (iii). Comment on the likely reasons for the differences in results. What do the results from part (iii), in particular, imply for the approximation of kinked functions in general?

v. In the light of your answer to (iv), now approximate $g(z)$ by linear interpolation, choosing $n = 3$ nodes for your approximation. What motivated your choice of nodes?

3 An Edgeworth Box Application

Consider a two-country (or agent model) where consumers choose their consumption bundle (c_1, c_2). The Home country aggregate consumption is given by:

$$C = \left[\alpha^{1/\rho} c_1^{\frac{\rho-1}{\rho}} + (1-\alpha)^{1/\rho} c_2^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

where $\alpha > 0.5$ is the preference for consuming good 1 and ρ is the intratemporal elasticity between the two goods. Households face the following budget constraint:

$$c_1 + pc_2 \leq y_1 + py_2$$

where p is the relative price of good 2 and y_1, y_2 are endowments of each good. The foreign country faces an analogous problem but Foreign aggregate consumption is given by:

$$C^* = \left[(1-\alpha)^{1/\rho} c_1^{*\frac{\rho-1}{\rho}} + \alpha^{1/\rho} c_2^{*\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

Total aggregate endowments are $y_1 + y_1^*$ and $y_2 + y_2^*$. Moreover, the law of one price holds ($p = p^*$). Choose $\rho = 2, \alpha = 0.6, y_1 = 0.8, y_2 = 1 - y_1, y_1^* = 1 - y_1, y_2^* = 1 - y_2^*$.

(a) Show that the demands for good 1 and good 2 for each country i.e. $c_1(p), c_2(p), c_1^*(p), c_2^*(p)$ are given by: $c_1 = \frac{y_1 + py_2}{1 + \frac{1-\alpha}{\alpha} p^{1-\rho}}$, $c_2 = \frac{y_1 + py_2}{\frac{\alpha}{1-\alpha} p^{\rho} + p}$, $c_1^* = \frac{y_1^* + py_2^*}{1 + \frac{\alpha}{1-\alpha} p^{1-\rho}}$, $c_2^* = \frac{y_1^* + py_2^*}{\frac{1-\alpha}{\alpha} p^{\rho} + p}$

(b) Using a grid for p, plot excess demand for either good.

(c) Using a grid for p, generate values using $c_1(p), c_2(p), c_1^*(p), c_2^*(p)$. Generate a polynomial approximation of the excess demand function for either good and plot against the true excess demand. What happens as your grid becomes finer? What happens as the order of the approximation increases?

(d) Using the excess demand for good 1, solve for the equilibrium level of $\hat{p}$. Do you need to find the root for the excess demand of good 2? Why? Now that you have $\hat{p}$, generate $c_1(\hat{p}), c_2(\hat{p}), c_1^*(\hat{p}), c_2^*(\hat{p})$ as well as $C(\hat{p})$ and $C^*(\hat{p})$.

(e) Repeat the above exercise while varying y_1, while keeping other endowments constant. How does p vary?

(f) Approximate a polynomial for $c_1(C)$ and $c_2(C)$. Plot in $< c_2, c_1 >$ space alongside the actual data of (c_1, c_2). Describe two salient *economic* features of your results.

4 Stochastic Processes

(a) Write a program which approximates an AR(1) process

$$y_t = \mu(1-\rho) + \rho y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N\left(0, \sigma^2\right)$ with a Markov chain, based on the Tauchen method (with equal intervals). Write the function so that it allows the user to determine the number of grid points, as well as the parameter r which determines the first and last grid points. (b) Use this programme to generate and plot $T = 200$ realisations from a Markov chain approximation of the AR(1) process

$$y_t = 0.8 y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, 0.01)$. To generate the realisations, use as initial state of the chain the one that best approximates $y_0 = 0$, and use $r = 3$. Do the following experiments (remember to always use the same seed):

i. Start by generating the series using $N = 3$ grid points for the approximation. What do you observe? Why?

ii. Next, use $N = 7$ and $N = 101$ and compare how the results differ in terms of quality of approximation.

5 Numerical Integration

Write a program that generates the approximate Chebyshev quadrature of a univariate function $f(x)$ in the interval $[a, b]$, for a given polynomial order n. Use it to calculate

$$\int_{-5}^{5} \frac{1}{1+x^2} dx$$

for $n = 20, 100$ and 500 . Compare the results with the exact solution.