

# PROBLEM SET 3

## PROJECTION METHODS

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### 1. RBC Model with Irreversible Investment

Consider the following RBC model with irreversible investment:

$$\max_{c_t, l_t, k_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to:

$$\begin{aligned} c_t + k_{t+1} &= z_t F(k_t, h_t) + (1 - \delta)k_t \\ k_{t+1} &\geq (1 - \delta)k_t \\ 1 &= h_t + l_t \\ \ln(z_{t+1}) &= (1 - \rho) \ln(z^*) + \rho \ln(z_t) + \sigma \varepsilon_t \end{aligned}$$

where  $k$  is capital,  $c$  consumption,  $h$  hours worked,  $l$  leisure and  $z$  a technology shock.

Assume  $F(k_t, h_t) = k_t^\alpha h_t^{1-\alpha}$ ,  $u(c_t, l_t) = \ln c_t + \mu \ln l_t$ ,  $\beta = 0.99$ ,  $\alpha = 0.36$ ,  $\delta = 0.025$ ,  $\rho = 0.98$ ,  $\sigma = 0.002$  and  $z^* = 1$ . Choose  $\mu$  such that the hours worked  $h$  in steady state is  $1/3$ .

- (a) Solve the model above using projection methods. Specifically approximate  $k_{t+1}$  using a polynomial function (e.g. Chebyshev).
  - i. begin with a deterministic model, no labour and no occasionally binding constraint
  - ii. add labour leisure choice
  - iii. add stochasticity
  - iv. add occasionally binding constraint
- (b) Calculate and report the first and second moments of consumption, hours, capital, investment and output.
- (c) Compare your results with the model solved by Value Function Iteration.

2. Improvement algorithms (*This question is optional, but strongly recommended.*)

- i. Compare fixed point iteration and time iteration
- i. Use the endogenous grid method and then combine it with time iteration.

You can use the commands *tic* and *toc* to measure speed gains and Euler errors to measure accuracy gains.

### Reading

DEN HAAN, W. J., *Teaching Notes*, [www.wouterdenhaan.com/notes.htm](http://www.wouterdenhaan.com/notes.htm)

HEER, B. AND A. MAUSSNER (2009), *Dynamic General Equilibrium Modeling: Computational Methods and Applications*, 2nd Edition, Springer, Chapters 3-4, 8-9.

JUDD, K. L. (1998), *Numerical Methods in Economics*, The MIT Press, Chapter 17.