

Solving the Hubble tension at intermediate redshifts with dynamical dark energy

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The current expansion rate of the Universe, the Hubble constant H_0 , is an important cosmological quantity. However, two different ways to measure its value do not agree – building a low-redshift distance ladder leads to a higher value of H_0 than inferring it from high-redshift observations in a Λ CDM cosmology. Most approaches to solve this tension either act at very low redshift by modifying the local distance ladder, or at high redshift by introducing new physics that changes the normalization of the inverse distance ladder. Here we discuss an attempt to instead addressing the Hubble tension at intermediate redshifts. By keeping the low- and high-redshift normalizations unchanged, we find a violation of the distance duality in the redshift range where luminosity and angular diameter distances overlap. We ‘solve’ this problem by introducing a redshift-dependent systematic effect that brings the luminosity distance into agreement with the angular diameter distance. The resulting expansion history is no longer compatible with Λ CDM, but this can be fixed with a dynamical dark energy component. In this way, we are able to address the Hubble tension at intermediate redshifts.

I. INTRODUCTION

Since the discovery of the accelerated expansion 25 years ago [1, 2], the cosmological standard model has been the Lambda-Cold-Dark-Matter (Λ CDM) model. Λ CDM has weathered many precision tests like the measurements of the anisotropies in the cosmic microwave background (CMB) by the Planck satellite [3] or measurements of the distance-redshift relation using type-Ia supernovae (SN-Ia, e.g. [4]) and baryonic acoustic oscillations (BAO, e.g. [5]).

With advancements in experimental sensitivity, it is reasonable to expect tensions between different data sets to emerge, either due to unknown systematic effects in the data, or because we have reached a level of precision where we are able to uncover new physics beyond the standard model. Because of the latter, it is important to investigate and understand the source of such disagreements. The currently most significant tension between data sets comes from the determination of the Hubble constant H_0 that describes the late-time expansion rate of the Universe, see e.g. [6]: Local measurements with Cepheids and SN-Ia find $H_0 = 73.04 \pm 1.04$ km/s/Mpc [7] while the Planck data in combination with the Λ CDM model predicts $H_0 = 67.7 \pm 0.42$ km/s/Mpc

[8], a number that is in agreement with the value obtained from BAO data combined with constraints on the baryon density ω_b from big-bang nucleosynthesis, without CMB anisotropies [5]. The difference of more than five standard deviations between these measurements has led to a great interest in the community, as this might be the first sign of a breakdown of the Λ CDM model. Unfortunately, so far no systematic effects have been identified that can explain the tension fully, but also no model has been proposed that provides a convincing explanation for the observed discrepancy.

In this paper we focus particularly on intermediate redshifts where we have both luminosity distances from SN-Ia and angular diameter distances from BAO. As we will show below, these measurements are not compatible at equal redshifts. This is a puzzle, since they should simply be related by the distance duality in any metric theory of gravity if photons are conserved [9, 10]. Since SN-Ia are anchored at low redshifts and BAO at high redshifts, most proposals aim to change one of these anchors, either through systematic effects affecting the SN-Ia normalisation or through extensions of the standard model, like early dark energy, that affect the sound horizon at high z . Here we will instead assume that a systematic effect at intermediate redshifts changes the luminosity distance so that it agrees with the angular diameter distance. This is motivated by earlier studies that have considered a possible redshift dependence of the intrinsic SN-Ia luminosity. One example is the luminosity dependence on the local star formation rate. Although some analyses do not de-

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tect a significant dependence of the luminosity on this quantity, see e.g. [11], others do find a significant local environmental dependence of the intrinsic luminosity, see e.g. [12] and references therein. Given the redshift dependence of the star formation rate, this effect would lead to a redshift-dependent intrinsic SN-Ia luminosity. Another example is the dependence on the metallicity of the host galaxy. Metallicity evolves as a function of redshift, therefore, any luminosity dependence on the metallicity would introduce a redshift dependence, see e.g. [13, 14]. Allowing for a redshift-dependent intrinsic SN-Ia luminosity can have a large impact on the cosmological conclusions derived from the observations, see e.g. [15, 16]. As we will see, the “deformation” of the luminosity distance required to match the BAO distances then requires additionally late-time dark energy.

II. COSMOLOGY AND DATA SETS

In this article, we will use distance data to constrain the expansion rate of the Universe. We will only consider flat models and choose the radiation density parameter Ω_r in agreement with the CMB temperature. The expansion rate as a function of redshift z is then

$$H(z)^2 = H_0^2 (\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{DE}(z)) . \quad (1)$$

For the Λ CDM standard model, the dark energy density parameter is given by

$$\Omega_{DE}(z) = (1 - \Omega_r - \Omega_m) , \quad (2)$$

while for a general dark energy component with time-evolving equation of state parameter $w(z) = \bar{p}(z)/\bar{\rho}(z)$ it becomes

$$\Omega_{DE}(z) = (1 - \Omega_r - \Omega_m) \exp \left\{ 3 \int_0^z \frac{1 + w(s)}{1 + s} ds \right\} . \quad (3)$$

A. Type Ia supernovae

Type Ia supernovae are standardizable candles, i.e. they measure the luminosity distance

$$D_L(z) = (1+z) \int_0^z \frac{ds}{H(s)} . \quad (4)$$

The luminosity distance is usually expressed in terms of the distance modulus

$$\mu(z) = 5 \log_{10} \left(\frac{D_L(z)}{10 \text{ pc}} \right) . \quad (5)$$

[Isaac, please check – what do we actually do/use/vary?] Here we use the Pantheon+ data [4], for which the observed distance modulus is defined as

$$\mu_{\text{obs}} = m_B - M + \alpha x_1 - \beta c + \delta . \quad (6)$$

Here m_B is the observed magnitude from a light-curve fit. The fit also provides the stretch parameter x_1 and the light-curve colour c , and α and β are the associated global nuisance parameters. M is the absolute magnitude of a SN-Ia, which is calibrated with the help of Cepheid distances, and δ denotes any bias corrections.

B. Baryon Acoustic Oscillations

Transverse baryon acoustic oscillations measure the co-moving angular diameter distance,

$$D_M(z) = D_L(z)/(1+z) . \quad (7)$$

This relation is also called the distance duality and holds in any metric theory of gravity (see e.g. [10]).

Like the SN-Ia distances, the BAO distances are also relative to a normalization, in this case the sound horizon scale at the drag epoch. This normalization can be affected by various mechanisms that act at high redshift, like early dark energy [17]. Here we assume that the physics at high redshift is not modified, so that we can use the standard values for that normalization.

In practice, we consider the legacy results of eBOSS [5], which contain the transverse BAO measurements from BOSS galaxies [18], eBOSS LRGs [19, 20], eBOSS ELGs [21, 22], eBOSS quasars [23, 24], and the BOSS+eBOSS Lyman- α auto-correlation and cross-correlation [25]. In addition to the transverse measurements, we also include the radial BAO measurements from these data sets, as well as the angle-averaged measurement from the main galaxy sample of SDSS [26, 27]. We follow [5] and use the covariance matrix for the BOSS galaxies measurements, as well as the covariance for the eBOSS LRGs and eBOSS quasars measurements. For the eBOSS LRGs and the Lyman- α forest measurements we employ instead the full likelihood provided by the collaboration, given its non-Gaussianity. The different data sets are considered to be independent from one another.

C. Cosmic Microwave Background

The temperature fluctuations observed with CMB anisotropies are not a simple distance measurement, and predicting them requires the use of a Boltzmann code. However, most of the information contained in the CMB can be compressed into a few numbers [28]:

- the CMB shift parameter $R = \sqrt{\Omega_m H_0^2} D_M(z_*)$ where z_* is the redshift for which the optical depth becomes unity;
- the angular scale of the sound horizon at last scattering (equivalent to θ_*), $\ell_a = \pi D_M(z_*)/r_s(z_*)$ where r_s is the coming size of the sound horizon;
- as well as the baryon density ω_b and the scalar spectral index n_s . In this work, we keep ω_b but

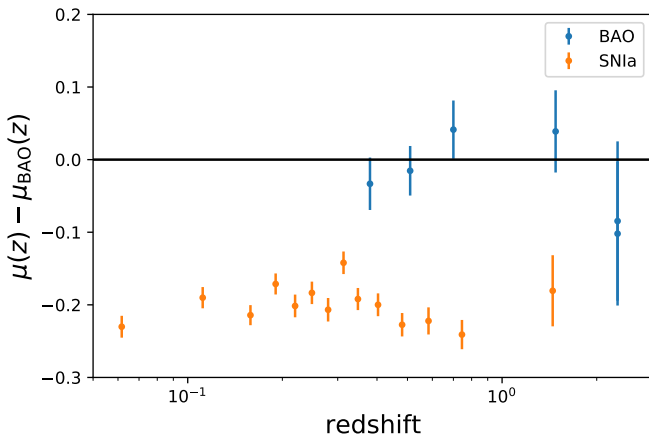


FIG. 1. Residuals of the distance moduli for SN-Ia and transverse BAO with respect to the BAO best-fit model. The discrepancy between the two data sets illustrates the Hubble tension.

marginalize over n_s since our other probes do not depend on it.

These numbers are effectively observables for the CMB. Their distribution is Gaussian to a good approximation, and is also valid for smooth dark energy components (or non-zero curvature) even when derived in a Λ CDM model. For the 2015 Planck data release, the mean values and covariance matrix was computed in [29]. However, we derive an updated likelihood based on the Planck 2018 Λ CDM chains [8] – as shown in [28], the compressed likelihood does not depend on the model used.

Specifically we use the following mean values for the reduced parameters, based on the 2018 plik TT+TEEE+lowl+lowE Planck likelihood,

$$(\ell_a, R, \omega_b) = (301.205, 1.7478, 0.02236), \quad (8)$$

together with the covariance

$$\text{cov} = 10^{-5} \times \begin{pmatrix} 712.9489 & -1.7156 & 0.2656 \\ -1.7156 & 2.1569 & -0.0459 \\ 0.2656 & -0.0459 & 0.0023 \end{pmatrix}. \quad (9)$$

D. Compatibility of the distances

In Fig. 1 we show the redshift-binned luminosity distance moduli of SN-Ia, together with the BAO distance moduli. The latter have been obtained from the BAO distances multiplied by the factor $(1+z)^2$ [is it squared? BAO are comoving?] that should transform them to luminosity distances according to the distance duality. It is easy to see that the distances do not agree in the shared redshift range. We note that such an inconsistency has already been identified in previous analyses [30–32], where the authors reconstructed the dark energy component that could better fit both data sets at the

same time. While dark energy can improve the overall fit, it cannot solve the discrepancy between the distance measurements, as it affects all distances. For this reason, we instead rely on the distance duality and invoke the presence of systematic uncertainties to reconcile the two data sets without modifying their anchors. We also note that the use of the cosmic distance duality has been considered recently to constrain a possible redshift evolution of the intrinsic SN-Ia luminosity [33]. The authors of this work combined SN-Ia and strong lensing measurements, but the distance measurements of these two data sets are compatible and led to no detection of a redshift evolution of the intrinsic SN-Ia luminosity.

III. SCHEMATIC ANALYSIS

Here we concoct a simple, schematic example of how to address the H_0 discrepancy at intermediate redshifts. Although the two distances do not agree, each distance agrees separately with a Λ CDM model, for different values of H_0 . We assume that there is a systematic effect that affects the observed luminosity distances so that they transition from distances corresponding to one value of H_0 to another. More precisely, we use a transition function $s(z)$ that changes from $s = 0$ to $s = 1$,

$$s(z) = \frac{1}{1 + e^{-(z-d)c}}, \quad (10)$$

where c, d are parameters that describe the transition: d the location in z and c the rapidity. We then assume that there are two Λ CDM expansion rates given at late times by

$$H^{[i]}(z) = H_0^{[i]} (\Omega_m(1+z)^3 + (1 - \Omega_m))^{1/2} \quad (11)$$

for two different values of $H_0^{[i]}$, $i = 1, 2$. The effective expansion rate for the luminosity distance is then

$$H_{\text{SN-Ia}}(z) = (1 - s(z))H^{[1]}(z) + s(z)H^{[2]}(z). \quad (12)$$

We compute the luminosity distance and the distance modulus from this expansion rate as well as from $H^{[2]}(z)$, and consider the difference of the distance moduli $\Delta\mu(z)$ as the necessary systematic effect that can reconcile the data sets. We show an example for $\Delta\mu(z)$ that is able to fit the data well in Fig. 2.

The resulting luminosity distance is able to reconcile SN-Ia and BAO distances, see Fig. 3. However, it does not correspond to the expansion rate of a Λ CDM model, because of the transition. But for any expansion rate $H(z)$ and a given Ω_m , there is a dark energy equation of state $w(z)$ that reproduces $H(z)$ [34]. Since we have an explicit form for $H(z)$ from Eq. (12), we can easily derive the corresponding equation of state parameter. By adjusting Ω_m , we find that we can bring it into a form where $w(z)$ is flat at small and large redshifts, and has a bump around the transition redshift $z = d$. This suggests

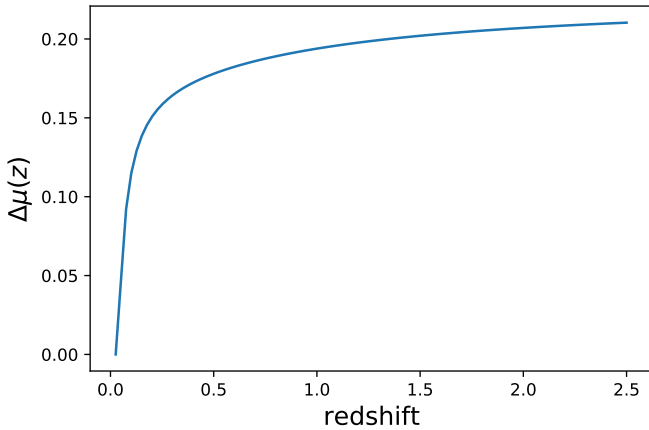


FIG. 2. The change of SN-Ia distance moduli, $\Delta\mu(z)$, necessary to reconcile BAO and SN-Ia distances.

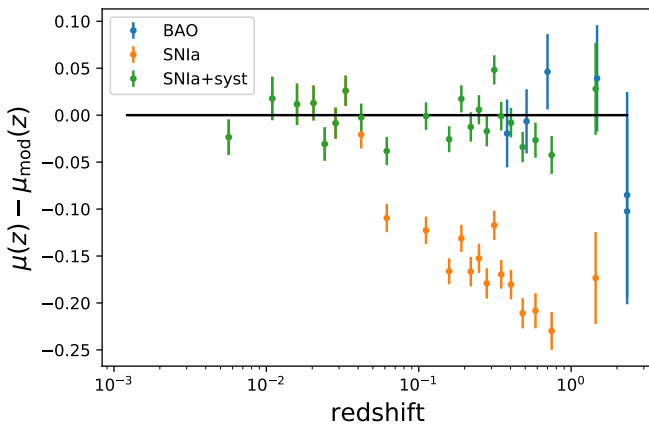


FIG. 3. A comparison of distance moduli for SN-Ia and BAO distances, relative to the best fit for the BAO points (black line, and blue BAO points). The yellow points show the SN-Ia distance moduli from the Pantheon+ data for $H_0 = ??$. The green points show the result of applying the correction shown in Fig. 2. Clearly the green points are in much better agreement with the blue BAO points than the yellow points.

that we should be able to model such a shape with two asymptotic values, w_1 and w_2 , and a Gaussian peak,

$$w(z) = w_1(1 - s(z)) + w_2s(z) + ue^{-\frac{1}{2}(z-d)^2/v^2}. \quad (13)$$

Here u is the amplitude of the bump and v controls the width, while the location is given by d .

Maximizing the goodness of fit (including varying Ω_m), we obtain the shape shown in Fig. 4, where w remains close to -1 down $z \approx 0.1$ and then exhibits a ‘phantom bump’ around $z \sim 0.05$.

[we should give the (cosmological) parameters that we end up with?]

Overall our scenario improves the goodness of fit of the data sets by $\Delta\chi^2 \approx 40$ and brings SN-Ia, BAO and CMB data into agreement, at the price of introducing two new ingredients at low redshifts, a systematic effect

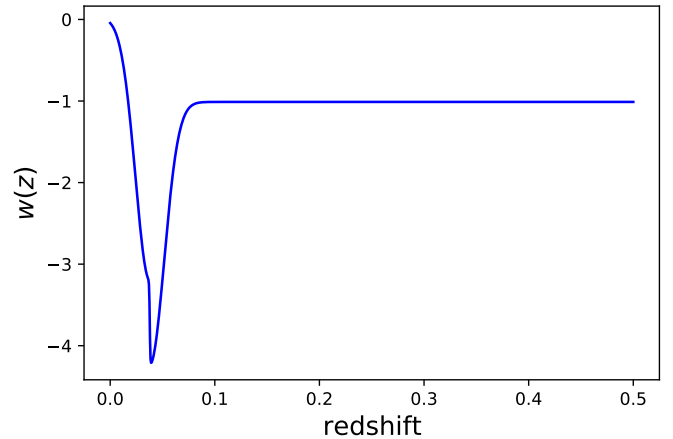


FIG. 4. The equation of state parameter $w(z)$ required to fit the full Hubble diagram that corresponds to the green and blue points in Fig. 3.

affecting the SN-Ia distances and a late-time dark energy. While requiring two ingredients may not appear compelling, each one of them is not very exotic per se, and a key point of this analysis is to show that the combination of the two provides an alternative scenario that provides a good fit to the data.

IV. CONCLUSION

In this paper we show that it is possible in principle to fit simultaneously the normalized supernova data, the BAO data and the main features of the CMB with late-time dark energy, if there is a systematic effect affecting the supernova luminosities, even if that effect does not change the late-time H_0 . If we neither want to change the normalisation of the SN-Ia, nor introduce a deviation from Λ CDM at high redshifts, then the presence of such a systematic effect is effectively unavoidable as the distance duality requires luminosity and angular diameter distances to agree.

This systematic effect needs to reach about 0.2 magnitudes at high redshift, and needs to reach about 80% of this change by redshift $z \approx 0.3$ in order to bring the SN-Ia distances into agreement with the BAO distances. One future way to test for such a systematic effect could be a comparison between the gravitational wave (GW) inferred luminosity distance and the SN-Ia luminosity distance as soon as we are able to observe a GW event in a galaxy that also hosted a SN-Ia [35]. This would be a crucial test whether the H_0 tension can be explained by a mechanism that affects neither the distance anchors nor the primordial universe.

The systematic effect changes the true distance-redshift relation at low z away from Λ CDM, requiring now the existence of a late-time evolving dark energy component. The best-fit $w(z)$ from our schematic analysis shows a relatively large excursion to $w \approx -4$ below

$z \approx 0.1$. This might be visible in other data sets, but it is challenging to observe as it happens at very low redshifts.

[add some discussion on recent related papers like <https://arxiv.org/pdf/2309.17163.pdf>, <https://arxiv.org/pdf/2309.07795.pdf>, <https://arxiv.org/pdf/2301.09695.pdf>]

Another potential tension concerns the amplitude of fluctuations on scales of $8h^{-1}$ Mpc, σ_8 . The value of σ_8 inferred from the CMB in a Λ CDM model is higher than

the value found by weak lensing measurements. [add some references] We computed the value of σ_8 in our scenario where w varies, and it turns out to be about 5% higher than in the corresponding Λ CDM case. However, our preferred matter density is also somewhat lower than in the standard model, so that our value of σ_8 is still compatible with the weak lensing observations at about 2σ .

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