① $r = 2, \theta = \frac{11\pi}{6}$
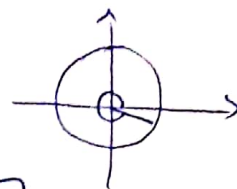
$z = 2\cos\frac{11\pi}{6} + i\, 2\sin\frac{11\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} + i\, 2 \cdot (-\frac{1}{2})$

$\underline{\underline{= \sqrt{3} - i}}$

$\boxed{E}$

② $z = -2\sqrt{2} - 2i\sqrt{2}$

$r = \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2} = \sqrt{8+8} = \underline{\underline{4}}$

$\left.\begin{array}{l} \cos\theta = -\frac{2\sqrt{2}}{4} = -\frac{\sqrt{2}}{2} \\ \sin\theta = -\frac{2\sqrt{2}}{4} = -\frac{\sqrt{2}}{2} \end{array}\right\} \quad \underline{\underline{\theta = \frac{5\pi}{4}}}$

$\boxed{C}$

③ $z = 3e^{i\frac{5\pi}{12}}, \quad w = 2e^{i\frac{13\pi}{12}}$

$zw = 3 \cdot 2 e^{i\left(\frac{5\pi}{12} + \frac{13\pi}{12}\right)} = 6e^{i\frac{3\pi}{2}} = 6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = \underline{\underline{-6i}} \quad \boxed{A}$

④ $\lim_{n\to\infty} \frac{7n^2 + 3\sqrt{n}}{4\sqrt{n} - 2n^2} = \lim_{n\to\infty} \frac{\frac{7n^2}{n^2} + \frac{3\sqrt{n}}{n^2}}{\frac{4\sqrt{n}}{n^2} - \frac{2n^2}{n^2}} = \lim_{n\to\infty} \frac{7 + \overbrace{\left(\frac{3}{n^{3/2}}\right)}^{0}}{\underbrace{\left(\frac{4}{n^{3/2}}\right)}_{0} - 2} = \underline{\underline{-\frac{7}{2}}} \quad \boxed{D}$

⑫ $h(x) = f(x)^{g(x)}$, $f(x) > 0$, f, g derivierbar

$\ln h(x) = g(x) \ln f(x)$

$\left(\ln h(x)\right)' = \left(g(x)\ln f(x)\right)'$

$\frac{1}{h(x)} h'(x) = g(x) \cdot \frac{1}{f(x)} f'(x) + g'(x)\ln f(x)$

$\boxed{C}$

$\underline{\underline{h'(x) = h(x)\left(\frac{g(x)f'(x)}{f(x)} + g'(x)\ln f(x)\right)}}$

(13) $\lim_{x \to 0} (1 + \sin x)^{\frac{3}{x}} \overset{1^{\infty}}{=} \lim_{x \to 0} e^{\ln(1+\sin x)^{\frac{3}{x}}}$

$= \lim_{x \to 0} e^{\frac{3}{x} \ln(1+\sin x)} = e^{\lim_{x \to 0} \frac{3}{x} \ln(1+\sin x)} = e^3$ 3 $\boxed{E}$

$\lim_{x \to 0} \frac{3}{x} \ln(1+\sin x) \overset{\infty \cdot 0}{=} \lim_{x \to 0} \frac{3\ln(1+\sin x)}{x} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x \to 0} \frac{\frac{3}{1+\sin x} \cdot \cos x}{1} = 3$

(14) $P(z) = z^3 + az^2 + bz + c$, $a,b,c,d \in \mathbb{R}$

$P(3) = 0$, $P(1-i) = 0$. Da vet vi at $P(1+i) = 0$

(den konjugerte til $1-i$ er også
en rot siden P er et reelt polynom)

$P(z) = (z-3)(z-(1-i))(z-(1+i))$

$= (z-3)(z^2 - z(1-i) - z(1+i) + (1-i)(1+i))$

$= (z-3)(z^2 - 2z + 2)$

$= z^3 - 3z^2 - 2z^2 + 6z + 2z - 6$

$= z^3 - 5z^2 + 8z - 6$ $\boxed{C}$

(18) $f, g : [a,b] \to \mathbb{R}$, kont. på $[a,b]$, deriverbare på (a,b)

$f(a) = g(a)$, $f(b) = g(b)$

Cauchys MVS sier at det
fins $c \in (a,b)$ slik at
$\underbrace{(f(b) - f(a))}_{=} g'(c) = \underbrace{(g(b) - g(a))}_{} f'(c)$

$g'(c) = f'(c)$ $\boxed{B}$

(opg. (19) neste forelesning)