Syntax.

Term variables	$x, y \in \mathit{Var}$	Type variables	$t, u \in \mathit{TVar}$
Type constructors	$T^\kappa \in \mathcal{T}^\kappa$ where $\{\multimap^\bullet, \multimap^\circ, \rightarrowtriangle^\bullet, \rightarrowtriangle^\circ\} \subseteq \mathcal{T}^{\star\to\star\to\star}$		

Kinds	$\kappa \ ::= \star \mid \kappa \to \kappa$
Types	$\tau^\kappa \ ::= t \mid T^\kappa \mid \tau^{\kappa' \to \kappa}\, \tau^{\kappa'}$
Predicates	$\pi ::= \mathsf{Un}\ \tau \mid \mathsf{SeFun}\ \tau \mid \mathsf{ShFun}\ \tau \mid \tau \geq \upsilon$
Qualified types	$\rho ::= \tau^\star \mid \pi \Rightarrow \rho$
Type schemes	$\sigma ::= \rho \mid \forall t.\sigma$
Environments	$H ::= \varepsilon \mid x : \sigma \mid H, H \mid H; H$
Environment contexts	$\mathcal{H} ::= \varepsilon \mid x : \sigma \mid \square \mid \mathcal{H}, H \mid H, \mathcal{H} \mid H; H$
Expressions	$M, N \ ::= x \mid \lambda{*}x.M \mid \lambda\&x.M \mid M\ N \mid \underline{\mathtt{let}}\ x = M\ \underline{\mathtt{in}}\ N$

Figure 1: Types and terms.

$$\boxed{P \mid H \vdash M : \sigma}$$

$$(\textsc{var})\ \frac{}{P \mid x : \sigma \vdash x : \sigma} \qquad (\textsc{let})\ \frac{P \mid H \vdash M : \sigma \quad P \mid H', x : \sigma \vdash N : \tau}{P \mid H, H' \vdash \underline{\mathtt{let}}\ x = M\ \underline{\mathtt{in}}\ N : \tau}$$

$$(\textsc{ctr-un})\ \frac{P \mid \mathcal{H}(H', H') \vdash M : \sigma \quad P \vdash H'\ \mathsf{un}}{P \mid \mathcal{H}(H') \vdash M : \sigma} \qquad (\textsc{wkn-un})\ \frac{P \mid \mathcal{H}(\varepsilon) \vdash M : \sigma \quad P \vdash H'\ \mathsf{un}}{P \mid \mathcal{H}(H') \vdash M : \sigma}$$

$$(\textsc{ctr-sh})\ \frac{P \mid \mathcal{H}(H'; H') \vdash M : \sigma}{P \mid \mathcal{H}(H') \vdash M : \sigma} \qquad (\textsc{wkn-sh})\ \frac{P \mid \mathcal{H}(H') \vdash M : \sigma}{P \mid \mathcal{H}(H'; H'') \vdash M : \sigma}$$

$$(-\!\!*\,\mathrm{I})\ \frac{\begin{array}{c} P \mid H, x : \tau \vdash M : \upsilon \\ P \Rightarrow \mathsf{SeFun}\ \phi \quad P \vdash H \geq \phi \end{array}}{P \mid H \vdash \lambda{*}x.M : \phi\,\tau\,\upsilon} \qquad (-\!\!*\,\mathrm{E})\ \frac{\begin{array}{c} P \mid H \vdash M : \phi\,\tau\,\upsilon \\ P \mid H' \vdash N : \tau \quad P \Rightarrow \mathsf{SeFun}\ \phi \end{array}}{P \mid H, H' \vdash M\,N : \upsilon}$$

$$(-\!\!\triangleright\,\mathrm{I})\ \frac{\begin{array}{c} P \mid H; x : \tau \vdash M : \upsilon \\ P \Rightarrow \mathsf{ShFun}\ \phi \quad P \vdash H \geq \phi \end{array}}{P \mid H \vdash \lambda\&x.M : \phi\,\tau\,\upsilon} \qquad (-\!\!\triangleright\,\mathrm{E})\ \frac{\begin{array}{c} P \mid H \vdash M : \phi\,\tau\,\upsilon \\ P \mid H' \vdash N : \tau \quad P \Rightarrow \mathsf{ShFun}\ \phi \end{array}}{P \mid H; H' \vdash M\,N : \upsilon}$$

$$(\Rightarrow \mathrm{I})\ \frac{P, \pi \mid H \vdash M : \rho}{P \mid H \vdash M : \pi \Rightarrow \rho} \qquad (\Rightarrow \mathrm{E})\ \frac{P \mid H \vdash M : \pi \Rightarrow \rho \quad P \Rightarrow \pi}{P \mid H \vdash M : \rho}$$

$$(\forall\,\mathrm{I})\ \frac{P \mid H \vdash M : \sigma \quad t \notin \mathit{ftv}(P, H)}{P \mid H \vdash M : \forall t.\sigma} \qquad (\forall\,\mathrm{E})\ \frac{P \mid H \vdash M : \forall t.\sigma}{P \mid H \vdash M : [\tau/t]\sigma}$$

$$\boxed{P \vdash \cdot\ \mathsf{un}} \qquad \boxed{P \vdash \cdot \geq \phi}$$

$$(\textsc{un}\text{-}\tau)\ \frac{P \Rightarrow \mathsf{Un}\ \tau}{P \vdash \tau\ \mathsf{un}} \qquad (\textsc{un}\text{-}\rho)\ \frac{P, \pi \vdash \rho\ \mathsf{un}}{P \vdash \pi \Rightarrow \rho\ \mathsf{un}} \qquad (\geq\text{-}\tau)\ \frac{P \Rightarrow \tau \geq \phi}{P \vdash \tau \geq \phi} \qquad (\geq\text{-}\rho)\ \frac{P, \pi \vdash \rho \geq \phi}{P \vdash (\pi \Rightarrow \rho) \geq \phi}$$

$$(\textsc{un}\text{-}\sigma)\ \frac{P, \mathsf{Un}\ t \vdash \sigma\ \mathsf{un}}{P \vdash \forall t.\sigma\ \mathsf{un}} \qquad (\textsc{un}\text{-}H)\ \frac{\bigwedge_{x:\sigma \in H} P \vdash \sigma\ \mathsf{un}}{P \vdash H\ \mathsf{un}} \qquad (\geq\text{-}\sigma)\ \frac{P, \mathsf{Un}\ t \vdash \sigma \geq \phi}{P \vdash (\forall t.\sigma) \geq \phi} \qquad (\geq\text{-}H)\ \frac{\bigwedge_{x:\sigma \in H} P \vdash \sigma \geq \phi}{P \vdash H \geq \phi}$$

Figure 2: Typing rules.

$$(*\,\mathrm{I})\ \frac{P \mid H \vdash M : \tau \quad P \mid H' \vdash N : \tau'}{P \mid H, H' \vdash (M, N) : \tau * \tau'} \qquad (*\,\mathrm{E})\ \frac{P \mid H \vdash M : \tau * \tau' \quad P \mid H', x : \tau, x' : \tau' \vdash N : \upsilon}{P \mid H, H' \vdash \underline{\mathtt{let}}\ (x, x') = M\ \underline{\mathtt{in}}\ N : \upsilon}$$

$$(\&\,\mathrm{I})\ \frac{P \mid H \vdash M : \tau \quad P \mid H' \vdash N : \tau'}{P \mid H; H' \vdash (M; N) : \tau \,\&\, \tau'} \qquad (\&\,\mathrm{E}_1)\ \frac{P \mid H \vdash M : \tau \,\&\, \tau'}{P \mid H \vdash \mathsf{fst}\ M : \tau} \qquad (\&\,\mathrm{E}_2)\ \frac{P \mid H \vdash M : \tau \,\&\, \tau'}{P \mid H \vdash \mathsf{snd}\ M : \tau'}$$

Figure 3: Derivable typing rules

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  \dfrac{
    \dfrac{
      \dfrac{\color{red}{\text{oops}}}{(x:t, y:u); z:v \vdash (x,z) : t * v}
    }{x:t, y:u \vdash \lambda\& z.(x,z) : v \mathrel{-\!\triangleright} t * v}\;(\mathrel{-\!\triangleright}\mathrm{I})
  }{x:t \vdash \lambda y \,\&\, z.(x,z) : u \mathrel{-\!\!*} v \mathrel{-\!\triangleright} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
}{\vdash \lambda x * y \,\&\, z.(x,z) : t \mathrel{-\!\!*} u \mathrel{-\!\!*} v \mathrel{-\!\triangleright} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
$$

$$
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  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{\dfrac{}{x:t \vdash x:t}\;(\textsc{var})}{x:t; y:u \vdash x:t}\;(\textsc{ctr-sh})
        \qquad
        \dfrac{}{z:v \vdash z:v}\;(\textsc{var})
      }{(x:t; y:u), z:v \vdash (x,z) : t * v}\;(*\,\mathrm{I})
    }{x:t; y:u \vdash \lambda{*}z.(x,z) : v \mathrel{-\!\!*} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
  }{x:t \vdash \lambda\& y.\lambda{*}z.(x,z) : u \mathrel{-\!\triangleright} v \mathrel{-\!\!*} t * v}\;(\mathrel{-\!\triangleright}\mathrm{I})
}{\vdash \lambda x \,\&\, y * z.(x,z) : t \mathrel{-\!\!*} u \mathrel{-\!\triangleright} v \mathrel{-\!\!*} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
$$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{}{\mathsf{Un}\ u \mid x:t \vdash x:t}\;(\textsc{var})
        \qquad
        \dfrac{\dfrac{}{\mathsf{Un}\ u \mid z:v \vdash z:v}\;(\textsc{var})}{\mathsf{Un}\ u \mid y:u, z:v \vdash z:v}\;(\textsc{ctr-un})
      }{\mathsf{Un}\ u \mid x:t, y:u, z:v \vdash (x,z) : t * v}\;(*\,\mathrm{I})
    }{\mathsf{Un}\ u \mid x:t, y:u \vdash \lambda z.(x,z) : v \mathrel{-\!\!*} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
  }{\mathsf{Un}\ u \mid x:t \vdash \lambda y * z.(x,z) : u \mathrel{-\!\!*} v \mathrel{-\!\!*} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
}{\mathsf{Un}\ u \mid \varepsilon \vdash \lambda x * y * z.(x,z) : t \mathrel{-\!\!*} u \mathrel{-\!\!*} v \mathrel{-\!\!*} t * v}\;(\mathrel{-\!\!*}\mathrm{I})
$$

Figure 4: Sample derivations