

Assignment2

2024-09-22

Uploading Data

```
file_name <- "/Users/biancaschaardt/Downloads/PolandLifeTable.txt"

life_table <- read.delim(file_name, header = TRUE, stringsAsFactors = FALSE, skip = 2,
                          sep = "")
str(life_table)
```

```
## 'data.frame': 6882 obs. of 10 variables:
## $ Year: int 1958 1958 1958 1958 1958 1958 1958 1958 1958 1958 ...
## $ Age : chr "0" "1" "2" "3" ...
## $ mx : num 0.07489 0.00489 0.00189 0.00133 0.00097 ...
## $ qx : num 0.07116 0.00488 0.00189 0.00133 0.00096 ...
## $ ax : num 0.3 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 ...
## $ lx : int 100000 92884 92431 92257 92135 92046 91969 91903 91841 91790 ...
## $ dx : int 7116 453 174 123 89 76 67 61 52 48 ...
## $ Lx : int 95020 92658 92344 92196 92090 92008 91936 91872 91816 91766 ...
## $ Tx : int 6591932 6496913 6404255 6311910 6219715 6127624 6035617 5943681 5851809 5759993 ...
## $ ex : num 65.9 70 69.3 68.4 67.5 ...
```

```
head(life_table)
```

```
##   Year Age      mx      qx ax      lx      dx      Lx      Tx      ex
## 1 1958   0 0.07489 0.07116 0.3 100000 7116 95020 6591932 65.92
## 2 1958   1 0.00489 0.00488 0.5 92884 453 92658 6496913 69.95
## 3 1958   2 0.00189 0.00189 0.5 92431 174 92344 6404255 69.29
## 4 1958   3 0.00133 0.00133 0.5 92257 123 92196 6311910 68.42
## 5 1958   4 0.00097 0.00096 0.5 92135 89 92090 6219715 67.51
## 6 1958   5 0.00083 0.00083 0.5 92046 76 92008 6127624 66.57
```

```
year <- life_table$Year
age <- life_table$Age
qx <- life_table$qx
ex <- life_table$ex
px <- (1 - qx)
```

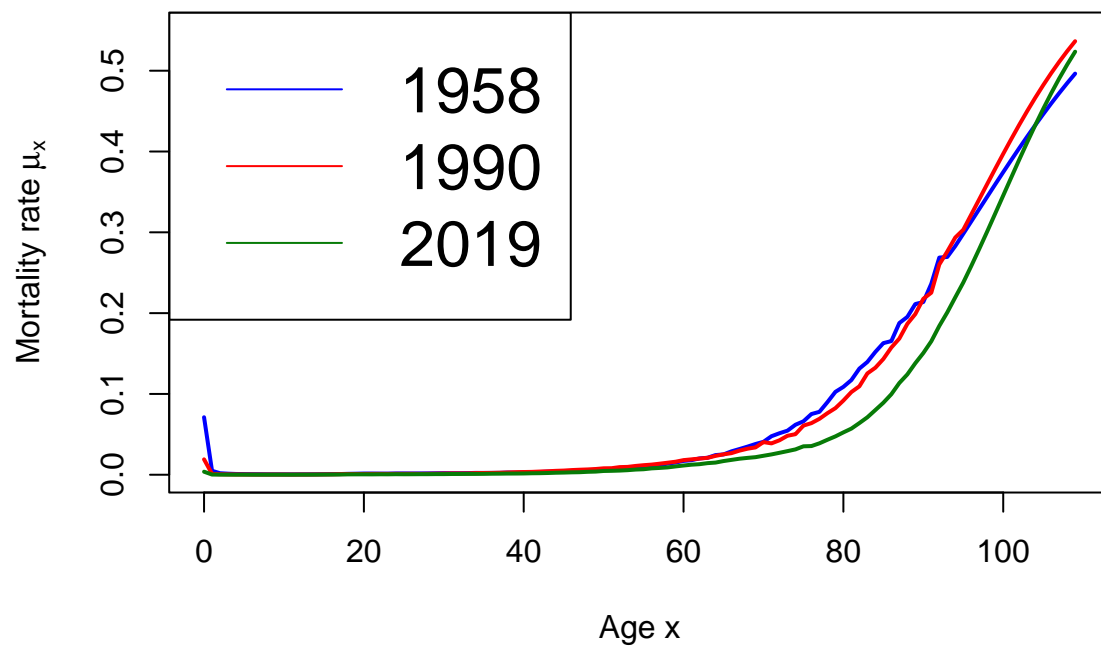
Question 2a

Approximate $\mu_{x,t} = q_{x,t}$, using Taylor approximation and UDD stating that mortality rate is constant between two integer years

```
mux1958 <- qx[year == 1958 & age != "110+"]
agesx <- c(0:109)
mux1990 <- qx[year == 1990 & age != "110+"]
mux2019 <- qx[year == 2019 & age != "110+"]

par(mar = c(5, 5, 4, 2) + 0.5)
plot(agesx, mux1958,
     main = "Mortality rates for each age (Poland, 1958, 1990 and 2019)",
     xlab = "Age x",
     ylab = expression(paste("Mortality rate ", mu[x])),
     type = "l",
     col = "blue",
     ylim = c(0, 0.55),
     lwd = 2)
lines(agesx, mux1990, col = "red", lwd = 2)
lines(agesx, mux2019, col = "#077b07", lwd = 2)
legend("topleft", legend = c("1958", "1990", "2019"),
      col = c("blue", "red", "#077b07"), lty = 1, cex = 2)
```

Mortality rates for each age (Poland, 1958, 1990 and 2019)

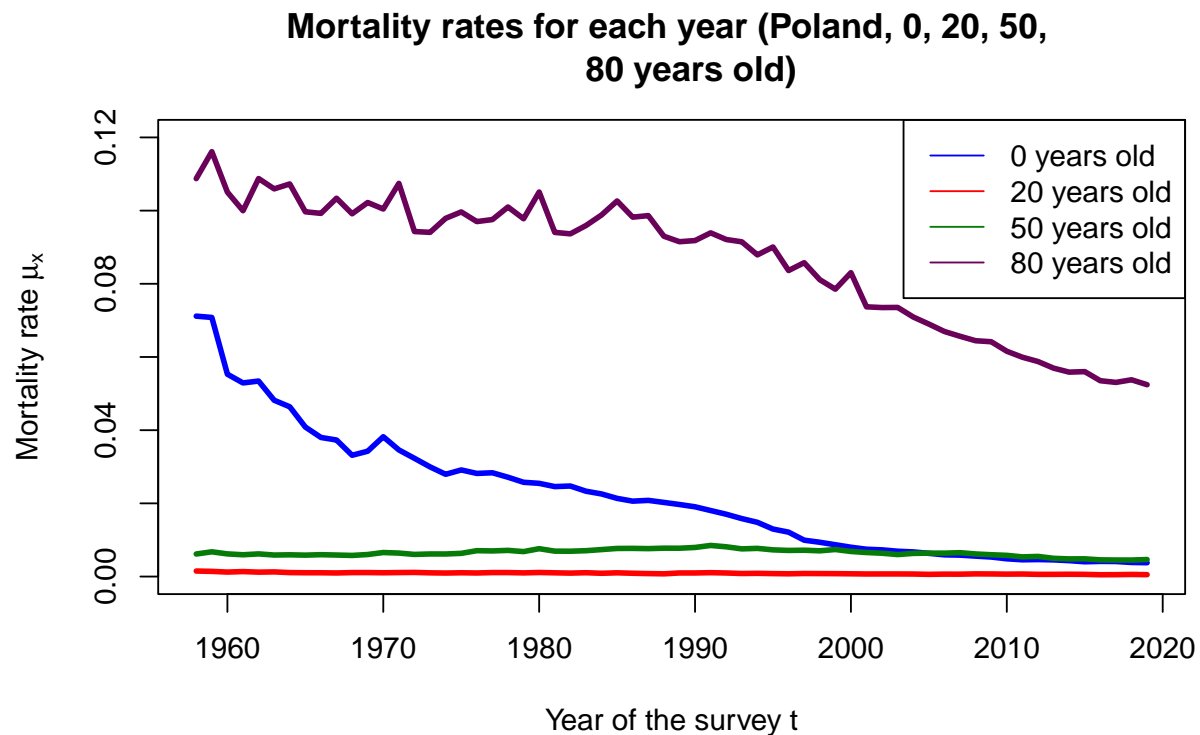


Question 2b

Again, we approximate $\mu_{x,t} = q_{x,t}$

```
yearst <- c(1958:2019)
mux0 <- qx[age == 0]
mux20 <- qx[age == 20]
mux50 <- qx[age == 50]
mux80 <- qx[age == 80]

plot(yearst, mux0,
     main = "Mortality rates for each year (Poland, 0, 20, 50,
           80 years old)",
     xlab = "Year of the survey t",
     ylab = expression(paste("Mortality rate ", mu[x])),
     type = "l",
     col = "blue",
     ylim = c(0, 0.12),
     lwd = 3)
lines(yearst, mux20, col = "red", lwd = 3)
lines(yearst, mux50, col = "#077b07", lwd = 3)
lines(yearst, mux80, col = "#6a0259", lwd = 3)
legend("topright", legend = c("0 years old", "20 years old", "50 years old", "80 years old"),
     col = c("blue", "red", "#077b07", "#6a0259"), lty = 1,)
```



Question 2c

To calculate that, we need to use multiplication formula to express each $S_0(x)=x_{p_0}$ as $p_0p_1p_2\dots p_{(x-1)}$. For that, we use `cumprod` and add 1 in the beginning as $0_{p_0} = 1$.

```
px_1958 <- px[year == 1958 & age != "110+"]
xp0_1958 <- c(1, cumprod(px_1958[(1):(110)]))

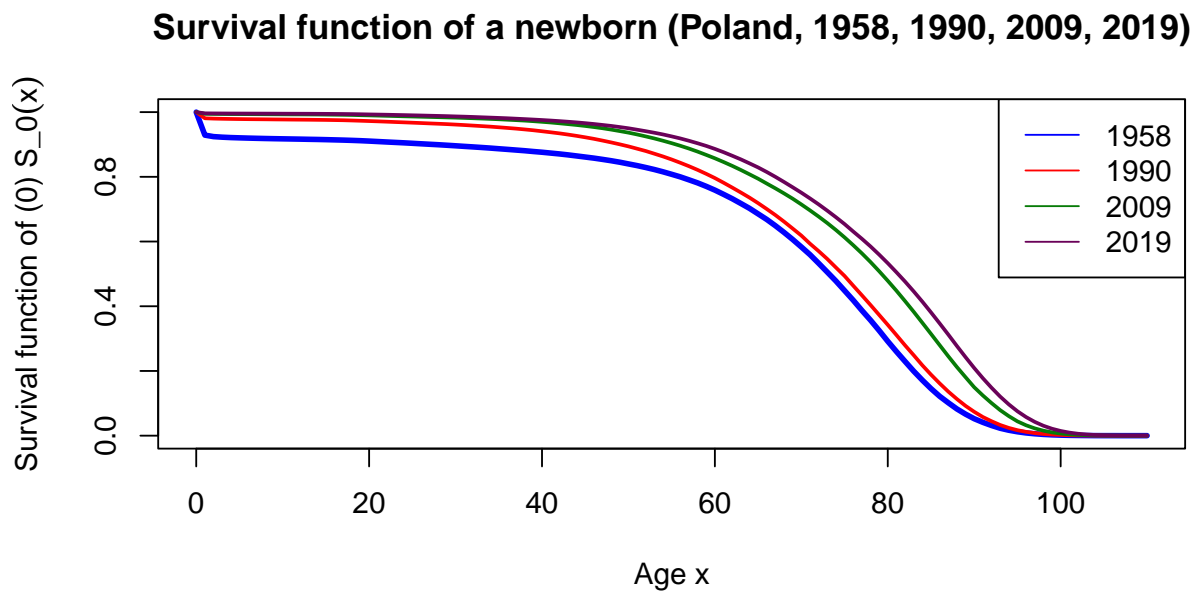
px_1990 <- px[year == 1990 & age != "110+"]
xp0_1990 <- c(1, cumprod(px_1990[(1):(110)]))

px_2009 <- px[year == 2009 & age != "110+"]
xp0_2009 <- c(1, cumprod(px_2009[(1):(110)]))

px_2019 <- px[year == 2019 & age != "110+"]
xp0_2019 <- c(1, cumprod(px_2019[(1):(110)]))

ages_x2 <- (0:110)

plot(ages_x2, xp0_1958,
     main = "Survival function of a newborn (Poland, 1958, 1990, 2009, 2019)",
     xlab = "Age x",
     ylab = expression(paste("Survival function of (0) ", S_0(x))),
     type = "l",
     col = "blue",
     ylim = c(0, 1),
     lwd = 3)
lines(ages_x2, xp0_1990, col = "red", lwd = 2)
lines(ages_x2, xp0_2009, col = "#077b07", lwd = 2)
lines(ages_x2, xp0_2019, col = "#6a0259", lwd = 2)
legend("topright", legend = c("1958", "1990", "2009", "2019"),
      col = c("blue", "red", "#077b07", "#6a0259"), lty = 1)
```



Question 2d

Future life expectancies for a person that is aged x at time t

```
e_0_t <- ex[age == 0]
e_20_t <- ex[age == 20]
e_40_t <- ex[age == 40]
e_70_t <- ex[age == 70]
e_90_t <- ex[age == 90]
yearst <- c(1958:2019)

par(mfrow = c(1, 2), mar = c(8, 4, 4, 4)) # Adjust margins as needed

plot(yearst, e_0_t,
     main = "Future life expectancies,
           Person aged x at time t
           Poland, Age: 0, 10, 20, 40, 70, 90",
     xlab = "Time t",
     ylab = "Future life expectancy  $E_x(t)$  ",
     type = "l",
     col = "blue",
     ylim = c(0, 110),
     lwd = 3)
lines(yearst, e_20_t, col = "red", lwd = 3)
lines(yearst, e_40_t, col = "#077b07", lwd = 3)
lines(yearst, e_70_t, col = "#6a0259", lwd = 3)
lines(yearst, e_90_t, col = "#fda605", lwd = 3)

plot.new()
legend("center", legend = c("0 years old", "20 years old", "40 years old",
                           "70 years old", "90 years old"),
      col = c("blue", "red", "#077b07", "#6a0259", "#fda605"), lty = 1)
```

