Modelling Report: Safety Zone around Playground Swings

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Specify the Purpose

This report aims to determine the minimum area required for a safety zone around a playground swing. The goal is to ensure that if a child or an object falls from the swing while it is in motion, the impact and the resulting injuries are minimized.

Create the Model

Describe Features Investigated and Outline Mathematics Used

This model is being used to predict the area required for a safety zone around a set of swings. We will be using a two step approach to determine the length of the zone, width will not be investigated. Firstly, we model the swing as a pendulum using simple harmonic motion, and find its velocity through conservation of energy. Secondly, we will apply projectile motion principles to determine the distance a child or object travels upon being released from the swing.

State Assumptions

The model is based on the following assumptions:

1. The child or object is modeled as a particle
2. Motion occurs in two dimensions
3. The swing is modeled as a rod length l
4. The swing sits at height h above the ground when at rest, such that the distance between the ground and the swing pivot is $(l{+}h)$
5. The width of the safety zone will the same as the width of the swing supports
6. The ground is level and perpendicular to the plane of travel
7. Air resistance and friction are neglected
8. There are no external forces acting on the swing
9. The initial position of the swing will be at $90°$ to the vertical, with the initial velocity $v_0 = 0\ ms^{-1}$

Variables and Parameters

Below is a list of all the variables and parameters used in the report, with further clarifications in Fig. 1

Symbol	Physical Quantity	Dimensions
A	Starting position of swing	(x, y)
B	Final position of swing fall point	(x, y)
C	Landing point of child/object	(x, y)
θ_0	Initial angle of the swing when at starting point A	Degrees
θ	Angle when swing is in motion (release point)	Degrees
w	Width of the swing safety landing zone	m
H_A	Height from the ground to the top of the swing pivot and swing at point A	m
H_B	Height from the ground to the swing at point B	m
l	Length of the swing rod, from seat to pivot	m
h	Height from ground to swing seat when vertical and at rest	m
D	Total distance from swing centre point, O, to where the child/object hits the ground	m
d_s	Horizontal distance from swing centre O to release point, B	m
d_f	Horizontal distance travelled during fall, from release point B to landing point C	m
s	Arbitrary height	m
t	Time of flight	s
g	The magnitude of the acceleration due to gravity	ms^{-2}
m	Mass of the child/object	kg
v	Magnitude of the velocity of the child/object	ms^{-1}
v_A	Magnitude of the velocity of the child/object at point A	ms^{-1}
v_B	Magnitude of the velocity of the child/object at point B	ms^{-1}
E	Total mechanical energy of the system	J
U	Potential energy of the system	J
T	Kinetic energy of the system	J

Table 1: Variable and Parameter list

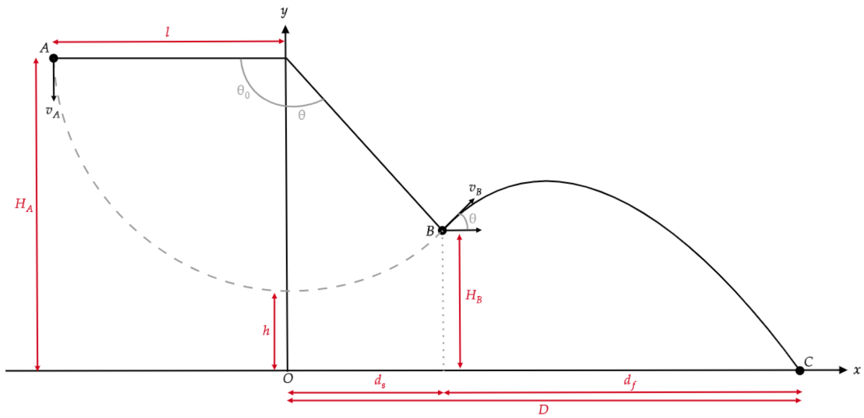

Figure 1: Complete system

Formulate Mathematical Relationships

Part 1: Pendulum

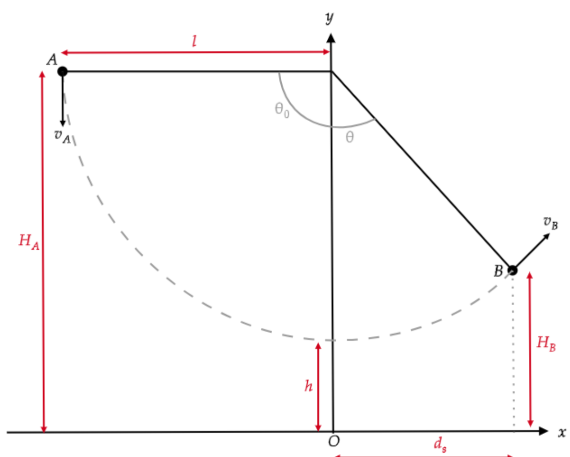

Figure 2: Pendulum

We first use conservation of energy to compare the Total Mechanical Energy from our start point, A, to the point at which the child/object falls from the swing, B[1]. We will use this relationship to find the velocity at the point the child/object leaves the swing.

[1] As per points 8, 9 and 11 on Page 67 of the Handbook

By using Assumptions $(1), (2), (3), (4), (6), (7), (8)$ we know that the total mechanical energy, E, is conserved.

We can therefore use the following:

$$E = U + T \tag{1}$$

where the Potential Energy U for some arbitrary height s can be expressed as

$$U = mgs$$

and Kinetic energy can be found using

$$T = \frac{1}{2}mv^2$$

Using conservation of energy allows us to formulate the following expression:

$$\begin{aligned} E_A &= E_B \\ \Rightarrow mgH_A + \frac{mv_A^2}{2} &= mgH_B + \frac{mv_B^2}{2} \end{aligned} \tag{2}$$

simplifying further by dividing through by m gives us

$$gH_A + \frac{v_A^2}{2} = gH_B + \frac{v_B^2}{2}$$

As per assumptions $(7), (8)$ and (9), we take $v_A = 0$, therefore giving us

$$gH_A = gH_B + \frac{v_B^2}{2}$$

which can be simplified even further to retrieve an expression for v_B

$$\begin{aligned} gH_A - gH_B &= \frac{v_B^2}{2} \\ \Rightarrow 2(gH_A - gH_B) &= v_B^2 \\ \Rightarrow 2g(H_A - H_B) &= v_B^2 \end{aligned} \tag{3}$$

We can now express H_A and H_B in terms of the swing rod length l and the height from the ground to the swing h, as per assumptions $(3), (4), (6)$

$$H_A = l + h$$
$$H_B = l + h - lcos(\theta)$$

Therefore, the expression for $H_A - H_B$ becomes

$$H_A - H_B = l + h - (l + h - lcos(\theta)$$
$$\Rightarrow H_A - H_B = lcos(\theta) \tag{4}$$

We can now substitute this into our expression for v_b^2:

$$v_B^2 = 2g(H_A - H_B)$$
$$\Rightarrow v_B^2 = 2glcos(\theta)$$

Our final expression for v_B will therefore be

$$v_B = \pm\sqrt{2glcos(\theta)} \tag{5}$$

where only the positive value will be considered.

We have now found our value for the magnitude of velocity at the point in the swing where the child/object falls. As per assumption (8) we can state that due to there not being any external forces acting on the swing, the child/object will leave the swing with the same velocity as v_B, allowing us to move onto Part 2 of our model.

Part 2: Projectile Motion

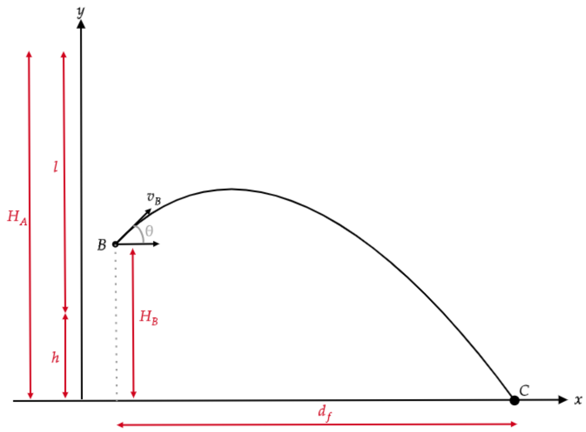

Figure 3: Projectile motion

Having concluded the initial speed of our projectile will be will be the same as that of the child/object at point B on the swing, we are now able to find an expression for the total distance travelled, D

As shown in Figure 1, we can express D as:

$$D = d_s + d_f \tag{6}$$

d_s can be found through simple trigonometric calculations based on the length of the swing rod as illustrated in Figure 2

$$d_s = lsin(\theta) \tag{7}$$

we then find d_f through the equations of motion[2], where v_{Bx} is the horizontal component of the magnitude of velocity at B and t is the time taken for the child/object to hit the ground. It is important to note that our acceleration due to gravity, g, is negative here, as we are following the downwards motion of the particle.

$$d_f = v_{Bx}t \tag{8}$$

[2] As per point 7 on Page 37 of the Handbook

and t is found using

$$y = H_B + v_{By}t - \frac{gt^2}{2} \tag{9}$$

Once an expression for t is obtained, we will substitute back to fine D

$$D = lsin(\theta) + v_{Bx}t$$

It is important to note that this is only the length travelled from the centre of the swing, O, to the point at which the child/object hits the ground, C. As the child or object can fall off the swing in either direction, the safety zone must have a length of $2D$, a distance of D either side of the swing.

Having found this length, we are able to propose an area for our safety zone. By assumption (5), the width of our safety zone is w, so our total area will be

$$Area = 2Dw \tag{10}$$

Do the Mathematics

Derive a first model

To accurately provide an expression for the area of our swing safety zone, we must find an expression for the distance travelled from the midpoint of the swing, O, to the point at which the child or object makes contact with the ground at C. We will then duplicate this to find the total length $2D$.

As per equations (6), (7), and (8), this distance can be found by

$$D = d_s + d_f$$

where

$$d_s = lsin(\theta)$$

$$d_f = v_{Bx}t$$

We first need to find the time taken for the child/object to fall to the ground (Point C), so using equation (9) and taking $y = 0$ will give us

$$0 = H_B + v_{By}t - \frac{gt^2}{2}$$

$$\Rightarrow \frac{gt^2}{2} - v_{By}t - H_B = 0$$

this expression can now be solved for t using the quadratic formula

$$t = \frac{v_{By} \pm \sqrt{v_{By}^2 + 2gH_B}}{g}$$

where we will take the positive value as $t > 0$.

This gives us a final value for t

$$t = \frac{1}{g}(v_{By} + \sqrt{v_{By}^2 + 2gH_B}) \tag{11}$$

Our next step will be to find a final expression for d_f from which we can then derive our total distance.

$$d_f = v_{Bx}t$$
$$\Rightarrow d_f = \frac{1}{g}v_{Bx}(v_{By} + \sqrt{v_{By}^2 + 2gH_B})$$

To simplify this expression we evaluate v_B into its components using trigonometry, as shown in Figure 3.

$$v_{Bx} = \sqrt{2glcos(\theta)}cos(\theta)$$
$$v_{By} = \sqrt{2glcos(\theta)}sin(\theta)$$

Using these expressions, alongside the value previously found for H_B, we can now substitute everything into equation (8)

$$d_f = \frac{1}{g}\sqrt{2glcos(\theta)}cos(\theta)(\sqrt{2glcos(\theta)}sin(\theta)+\sqrt{(\sqrt{2glcos(\theta)}sin(\theta))^2 + 2g(l + h - lcos(\theta))})$$

This can then be expanded and simplified as follows

$$\begin{aligned}
d_f &= \frac{1}{g}\sqrt{2glcos(\theta)}cos(\theta)(\sqrt{2glcos(\theta)}sin(\theta) + \sqrt{2glcos(\theta)sin^2(\theta) + 2g(l + h - lcos(\theta))}) \\
&= \frac{1}{g}\sqrt{2glcos(\theta)}cos(\theta)(\sqrt{2glcos(\theta)}sin(\theta) + \sqrt{2glcos(\theta)(sin^2(\theta) - 1) + 2gl + 2gh}) \\
&= \frac{1}{g}\sqrt{2glcos(\theta)}cos(\theta)(\sqrt{2glcos(\theta)}sin(\theta) + \sqrt{-2glcos^3(\theta) + 2gl + 2gh}) \\
&= \frac{1}{g}cos(\theta)(2glcos(\theta)sin(\theta) + \sqrt{-4g^2l^2cos^4(\theta) + 4g^2l^2cos(\theta) + 4g^2hlcos(\theta)}) \\
&= \frac{1}{g}cos(\theta)(2glcos(\theta)sin(\theta) + 2g\sqrt{-l^2cos^4(\theta) + l^2cos(\theta) + hlcos(\theta)}) \\
&= 2lcos^2(\theta)sin(\theta) + 2cos(\theta)\sqrt{hlcos(\theta) + l^2cos(\theta) - l^2cos^4(\theta)}
\end{aligned}$$

Having now found an expression for both d_s and d_f, our total distance from the mid point of the swing can now be found, from equation (6)

$$D = lsin(\theta) + 2lcos^2(\theta)sin(\theta) + 2cos(\theta)\sqrt{hlcos(\theta) + l^2cos(\theta) - l^2cos^4(\theta)} \quad (12)$$

Draw graphs showing typical relationships

Taking l and h as constants, we can plot a graph of D against θ to evaluate the relationship between them. As shown in Figure 4, the function has a single maximum, illustrated by the red line to be in the range of $30° < \theta < 40°$.

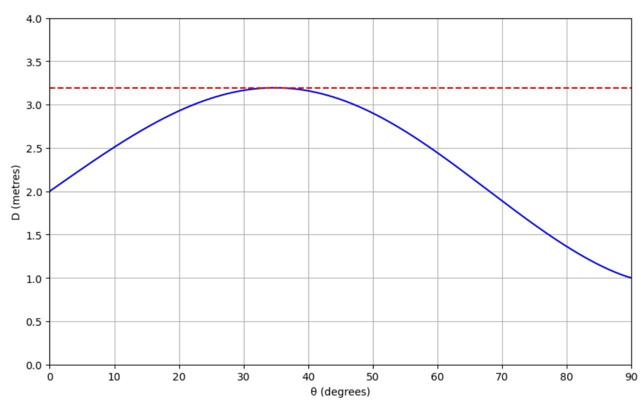

Figure 4: Graph of D against θ

We can therefore imply the value for D will be at its maximum when the child or object leaves the swing when the angle is within this range.

Check your model using dimensional analysis

We must now check the dimensions of our final formula for the length of the safety zone, equation (12), which is as follows:

$$D = lsin(\theta) + 2lcos^2(\theta)sin(\theta) + 2cos(\theta)\sqrt{hlcos(\theta) + l^2cos(\theta) - l^2cos^4(\theta)}$$

From Table 1, we take the units of l and h to both be metres/m, whereas θ is dimensionless. This allows us to find the dimensions of D to be

$$\begin{aligned} D &= [m] + [m] + \sqrt{[m][m] + [m]^2 + [m]^2} \\ &= [m] + [m] + \sqrt{[m]^2} \\ &= [m] + [m] + [m] \\ &= [m] \end{aligned}$$

The dimensions of D correctly simplify to be metres, as required.

Interpret the Results

Collect relevant data for parameter values

From equation (12), our equation for D, our parameters are l, h and θ

The British and European Standard for playground equipment and surfacing is BS EN 1176.

From this, we will assume that $h = 0.35$[3], which will change equation (12) to

$$D = lsin(\theta) + 2lcos^2(\theta)sin(\theta) + 2cos(\theta)\sqrt{0.35lcos(\theta) + l^2cos(\theta) - l^2cos^4(\theta)} \quad (13)$$

We can now calculate our maximum vales for D and θ by substituting different l values into our equation.

[3]As stated in EN 1176, the minimum seat ground clearance in the rest position must be $0.35m$

Describe the mathematical solution

In this model, I will consider values of l ranging from $2.0 - 4.0m$, all expressed in Table 2.

To calculate these, I went through the following steps:

Take $l = 2.0m$, equation (13) will now become

$$
\begin{aligned}
D &= 2sin(\theta) + 4cos^2(\theta)sin(\theta) + 2cos(\theta)\sqrt{0.70cos(\theta) + 4cos(\theta) - 4cos^4(\theta)} \\
&= 2sin(\theta) + 4cos^2(\theta)sin(\theta) + 2cos(\theta)\sqrt{4.70cos(\theta) - 4cos^4(\theta)}
\end{aligned}
$$

To find D_{max}, we must find the critical points of the function and classify them. To do this, we evaluate where $D'_{max}(\theta) = 0$. For $l = 2.0m$, this gives us a value of $\theta_{max} = 39.1°$

We can now substitute our θ_{max} value back into our expression for D to give us our value for D_{max}, which is $5.08m$ for $l = 2.0m$.

Following a similar method, we can find D_{max} and θ_{max} for other values of l. I have included a value for $2D_{max}$, which gives us our value for the total length of our Safety Zone, accounting for a fall from either end of the swing.

l/m	$\theta_{max}/degrees$	D_{max}/m	$2D_{max}$
2.0	39.1°	5.08	10.2
2.5	39.5°	6.28	12.6
3.0	39.7°	7.48	15.0
3.5	39.9°	8.68	17.4
4.0	40.0°	9.87	19.8

Table 2: Maximum Values

Find predictions to compare with reality

According to the guidance mentioned previously, EN 1176, the minimum safety zone length for a set of swings should be as stated

Surfacing requirements: Free Fall height is calculated from the centre of the stationary seat surface at 60° forward and back, where there are different areas for synthetic and loose-fill surfaces in a box or pit:

synthetic: $sin(60°)$ *x length of suspension member + 1.75m*

loose-fill: $sin(60°)$ *x length of suspension member + 2.25m*

Taking the worst case scenario, we will compare our data to the equation

$$sin(60^{\circ})h + 2.25 \tag{14}$$

Evaluate the Model

Collect data to compare with the model

As seen in Figure 5, an EN 1176 compliant swing by The Playground Centre has a total height of $2.20m$ and a safety zone of $7.62m$.

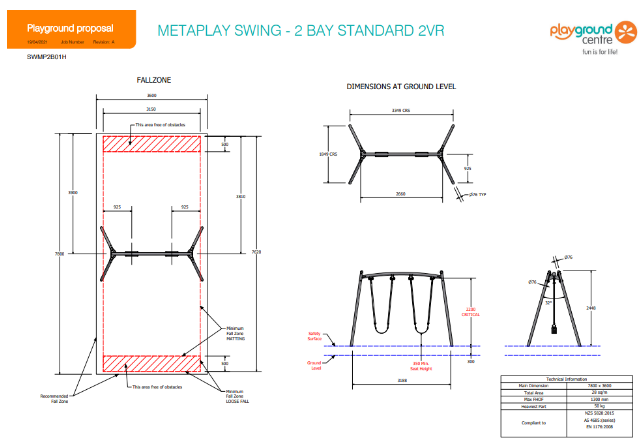

Figure 5: Comparison to real swing

Test your first model

Comparing my model predictions to the data given by the swing manufacturer, the model gives a much longer length than actually required. We have been considering the worst case scenarios for our model, therefore arising greater values than necessary in most cases.

Criticise your first model

My model is calculating lengths greater than required, and is very complicated in terms of the formulas and equations used, as in equation (12). I believe the formulas could be simplified to make seamless calculations easier and faster for different heights of the swing, as well as adjusting certain variables to make the predictions more aligned with real values.

Review your assumptions

I believe the following assumptions need changing:

(1) The dimensions of the child need to be taken into consideration.

(2) Motion will be in more than one direction, due to wind and other influences.

(7), (8) There will be other forces acting on the swing, which all must be considered.

Revise the Model

Decide whether to revise your first model

I believe a revision to the model is necessary, due to the length predicted being incorrect, and new factors needing to be taken into consideration. As seen in the evaluation, the model does not align with real data. By considering factors that reflect reality such as all external fores and and dimensions of the child or object falling off the swing, the maximum distance travelled will be greatly reduced, as the fall will be slowed down a lot faster.

Describe your intended revision

Assumptions (1), (7) and (8) will be changed to the following.

Assumption (1) will be : *The length and width of the child or object will be considered.*

Assumption (7) will be : *Air resistance and Friction will be considered.*

Assumption (8) will be : *All external forces acting on the swing will be taken into consideration.*

Conclusions

Summarise your modelling

Overall, the first model was successful in predicting a safe landing zone for the child/object, but due to no external influences acting on the swing, the distance travelled was much greater than required. This is due to there not being any other factors to affect the projectile motion, leading to a maximised value of D. By revising our model to consider new factors such as air resistance and friction, the child or object falling will slow down and fall much quicker, helping our data align with that of reality and make the model more successful.