External Force Observers for the NAO

Analysis and C++ Implementation of a Testbed for External Force Observers acting on the NAO robot

DIPARTIMENTO DI INGEGNERIA INFORMATICA AUTOMATICA E GESTIONALE ANTONIO RUBERTI

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Our Work

- **Estimation** of an **external force** acting on the center of mass of a NAO robot
  - Important to design a controller that can counteract it
  - Usually medium-sized robots do not possess F/T sensors
- Implementation of three **state-disturbance observers**:
  - *Luenberger* Observer
  - *Kalman Filter based* Observer
  - *Stephens'* Observer
- Implementation of a mutual **framework**
  - Automatic handling of the observer
  - Widget extension for an external setup
- Validation of the observers through **experiments** and analysis of the results

Luenberger Observer

- Inspired by **[2]**
- Relies on the continuous perturbed **LIP** model
- Gain matrix ***G*** found via pole placement
- Asymptotically stable if (***A***-***GC***) is Hurwitz

Luenberger Observer:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}} = & \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{G}(\boldsymbol{y} - \hat{\boldsymbol{y}}) \\ \hat{\boldsymbol{y}} = & \boldsymbol{C}\hat{\boldsymbol{x}} \end{cases}$$

STATE $\boldsymbol{x} = \begin{pmatrix} x_c & \dot{x}_c & x_z & w_x & \dot{w}_x \end{pmatrix}$

OUTPUT $\boldsymbol{y} = \begin{pmatrix} x_c & x_z \end{pmatrix}$

INPUT $\boldsymbol{u} = \dot{x}_z$

System matrices:

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ \eta^2 & 0 & -\eta^2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}^T$$

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \boldsymbol{G} = \begin{pmatrix} 240 & 0 \\ 21254.7 & -29.5 \\ 0 & 70 \\ 820250 & 12.36 \\ 11700000 & 236.15 \end{pmatrix}$$

Kalman Filter based Observer

- Inspired by **[4]**
- Uses Kalman Filter equations to compute estimates of the state
- Relies on the discrete perturbed **LIP** model
- Uses estimates of *z*-axis to compute ***Cx*** and ***Cy***

System for applying KF equations:

$$^{**}\begin{cases} \boldsymbol{\mathcal{X}}_z(k+1) = \boldsymbol{A}\boldsymbol{\mathcal{X}}_z(k) + \boldsymbol{\omega_z}(k) \\ \boldsymbol{Y}_z(k) = \boldsymbol{C}_z\boldsymbol{\mathcal{X}}_z(k) + \boldsymbol{v}_z(k) \end{cases}$$

with $\begin{matrix} \boldsymbol{\omega}_z \sim \mathcal{N}(0, \boldsymbol{Q}) \\ \boldsymbol{v}_z \sim \mathcal{N}(0, \boldsymbol{R}_z) \end{matrix}$ **

STATE $\boldsymbol{\mathcal{X}}_z(k)^{**} = [z_c(kT) \;\; \dot{z}_c(kT) \;\; \ddot{z}_c(kT) \;\; F_z(kT) \;\; \dot{F}_z(kT)]^T$

OUTPUTS $\boldsymbol{Y}_z(k) = [z_c(kT) \;\; \ddot{z}_c(kT) \;\; f_n^o(kT) + M_c g]^T$

$\boldsymbol{Y}_{\boldsymbol{x}}(k)^{*} = [x_c(kT) \;\; \ddot{x}_c(kT) \;\; x_z(kT)]^T$

Dependency for xy-axes:

$$\boldsymbol{p} = \begin{pmatrix} x_c + \frac{M_c z_c}{f_n^o}\ddot{x}_c - \frac{z_c}{f_n^o}F_x \\ y_c + \frac{M_c z_c}{f_n^o}\ddot{y}_c - \frac{z_c}{f_n^o}F_y \end{pmatrix} \Rightarrow {}^{*}\boldsymbol{C}_{\boldsymbol{x}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{M_c \hat{z}_c(kT)}{\hat{f}_n^o(kT)} & -\frac{\hat{z}_c(kT)}{\hat{f}_n^o(kT)} & 0 \end{pmatrix}$$

Dependency for z-axis:

$$f_n^o = -M_c g - M_c \ddot{z}_c + F_z \Rightarrow \boldsymbol{C}_z = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -M_c & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^T$$

System matrices:

$$\boldsymbol{A} = \begin{pmatrix} 1 & T & \frac{T^2}{2} & 0 & 0 \\ 0 & 1 & T & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} \frac{T^3}{6} & 0 \\ \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{pmatrix}$$

$$\boldsymbol{Q}^{**} = \boldsymbol{B}\begin{pmatrix} \sigma^2_{\dddot{z}_c} & 0 \\ 0 & \sigma^2_{\ddot{F}_z} \end{pmatrix}\boldsymbol{B}^T$$

* *holds also for y-axis* ** *holds also for xy-axes*

Stephens' Observer

- Inspired by **[5]**
- Relies on the discrete perturbed **LIP** model
- Explicit use of system inputs
- Uses KF equations to compute estimates of the state
- Independent observer for each dimension

System dynamics:

$$\begin{cases} \boldsymbol{\mathcal{X}}(k+1) = & \boldsymbol{A}\boldsymbol{\mathcal{X}}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{\omega}(k) \\ \boldsymbol{Y}(k) = & \boldsymbol{C}\boldsymbol{\mathcal{X}}(k) + \boldsymbol{v}(k) \end{cases}$$

with
$$\boldsymbol{\omega} \sim \mathcal{N}(0, \boldsymbol{Q}) \qquad \boldsymbol{v} \sim \mathcal{N}(0, \boldsymbol{R})$$

** STATE $\boldsymbol{\mathcal{X}}_x = \begin{pmatrix} x_c & \dot{x}_c & x_p & w_x \end{pmatrix}$

** OUTPUT $\boldsymbol{Y}_x = \begin{pmatrix} x_c & x_p \end{pmatrix}$

** INPUT $u_x = \dot{x}_p$

System matrices:

$$\boldsymbol{A} = \begin{pmatrix} 1 & T & 0 & 0 \\ \eta^2 T & 1 & -\eta^2 T & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ T \\ 0 \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}^T$$

$$\boldsymbol{R} = \begin{pmatrix} 10^{-8} & 0 \\ 0 & 10^{-4} \end{pmatrix} \quad \boldsymbol{Q} = \begin{pmatrix} 10^{-8} & 0 & 0 & 0 \\ 0 & 10^{-4} & 0 & 0 \\ 0 & 0 & 10^{-4} & 0 \\ 0 & 0 & 0 & 10^{-1} \end{pmatrix}$$

*** holds also for yz-axes*

Implementation Details

DART

- Used to simulate NAO behaviour
- Open-source framework with 3D Physical engine
- Support for Kinematics and dynamics and allows application of external forces

IS-MPC

- Based on perturbed LIP Model
- Acts as controller
- Uses first sample of output sequence of a Quadratic Programming Problem s.t.
  - ZMP Constraint
  - Stability Constraint

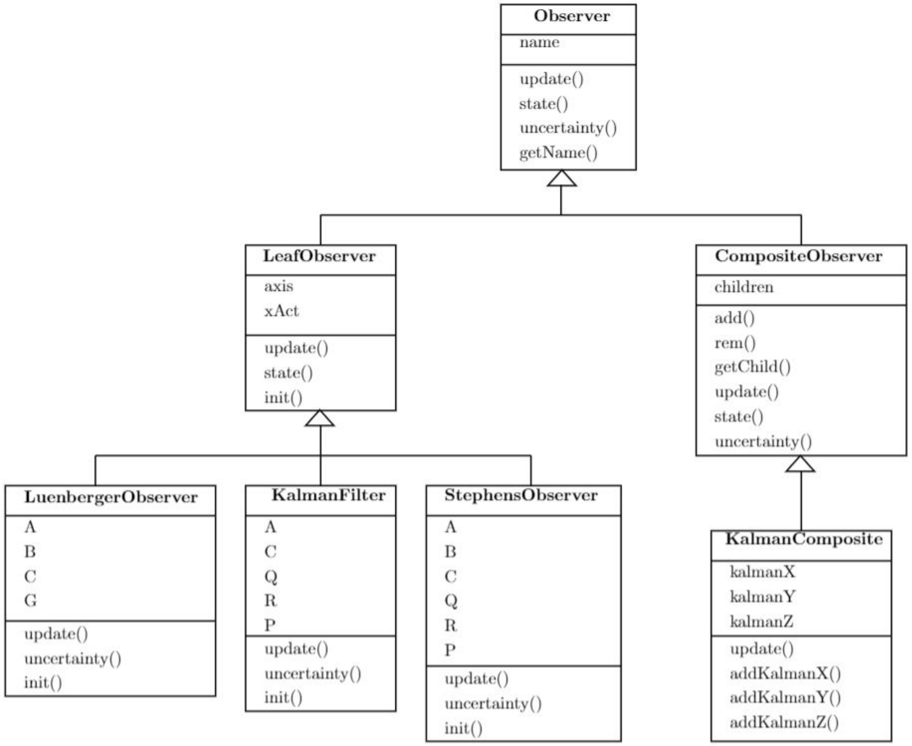
Measurements

- Used DART and custom methods to obtain measurements to feed the observers
- Different methods and settings tested to get most accurate data

Implementation Details (cont’d)

COMPOSITE PATTERN

- Each Observer implements ***Observer*** and ***LeafObserver*** interfaces
- ***CompositeObserver*** is a container class from which the observers can be updated



LUENBERGER OBSERVER

- Implementation of abstract methods of Observer and *LeafObserver*
- G matrix obtained with pole placement after grid-search on poles

KALMAN FILTER BASED OBSERVER

- Implementation of abstract methods of Observer and *LeafObserver*
- Definition of ***KalmanComposite*** to manage the coupling between *z* and *xy*-axes.
- Covariances of inputs and ***R*** found empirically

STEPHENS’ OBSERVER

- Implementation of abstract methods of Observer and LeafObserver
- Covariance matrices ***Q***, ***R*** found empirically

Experimental Setup

- Three different behaviours:
  - Stand without balance
  - Stand with balance
  - Walking
- Two experiments for each behaviour:
  - Constant force: $F_{ext} = (4.1 \;\; 4.7 \;\; 0)$
  - Periodic force: $F_{ext} = \begin{pmatrix} A_x \sin(2\pi f_x t + \phi_x) \\ A_y \sin(2\pi f_y t + \phi_y) \\ 0 \end{pmatrix}^T$

  with $A_x = 4.1$ N, $A_y = 4.7$ N, $f_x = f_y = 0.1$ Hz, $\phi_x = 0$ rad and $\phi_y = \pi/2$ rad.

- Whole state estimated
- Implementation of an observer for **each dimension** for the *Kalman Filter based* observer
- Implementation of an observer for ***x* and *y* axes** only for the *Luenberger* and *Stephens* observers

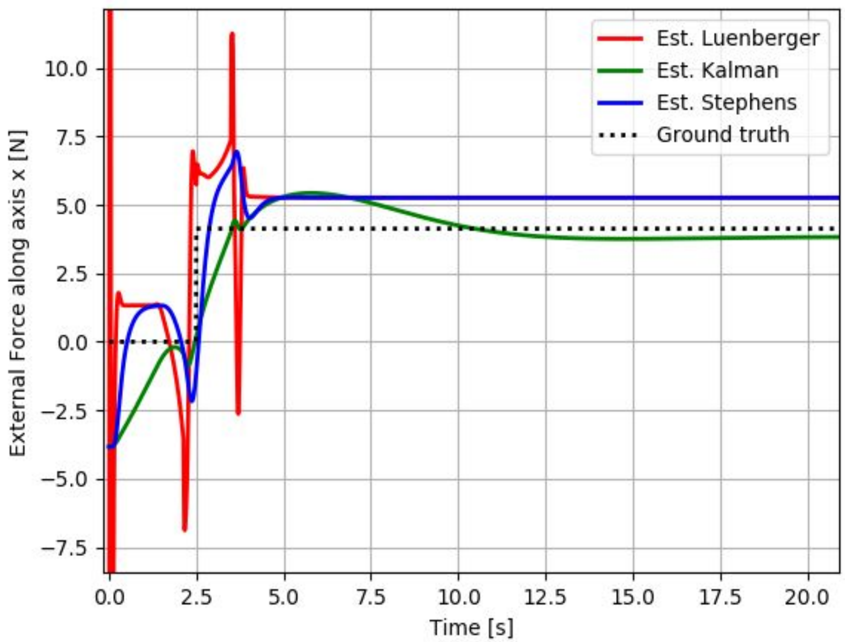
Analysis of the results through

- Qualitative evaluation of the resulting plots
- Quantitative evaluation through selected metrics:
  - **RMSE** (*Root Mean Square Error*)
  - **MAE** (*Mean Absolute Error*)
  - **TTC** (*Time to Convergence*)

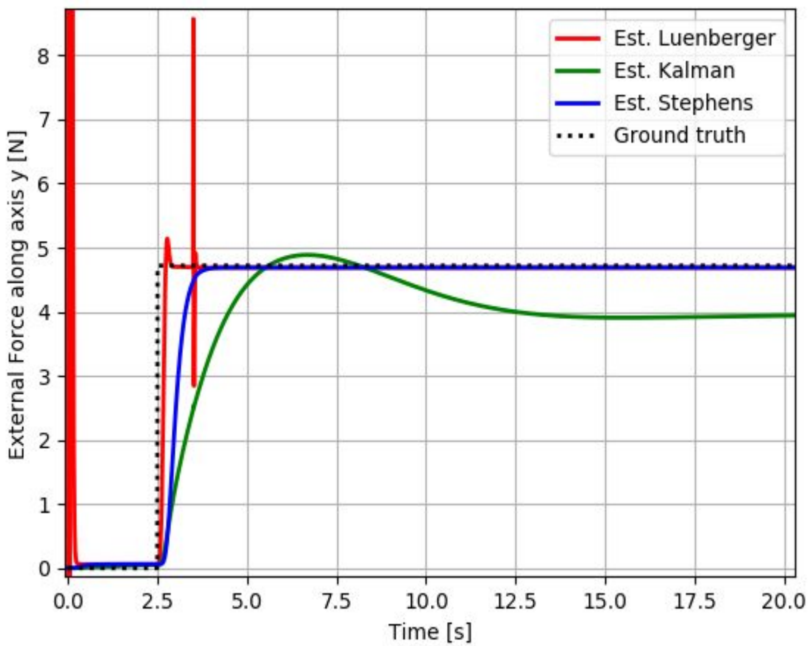
  RMSE computed for the whole signal and at steady-state

Experiment 1 | Stand no Balance: Constant force

X - AXIS



Y - AXIS

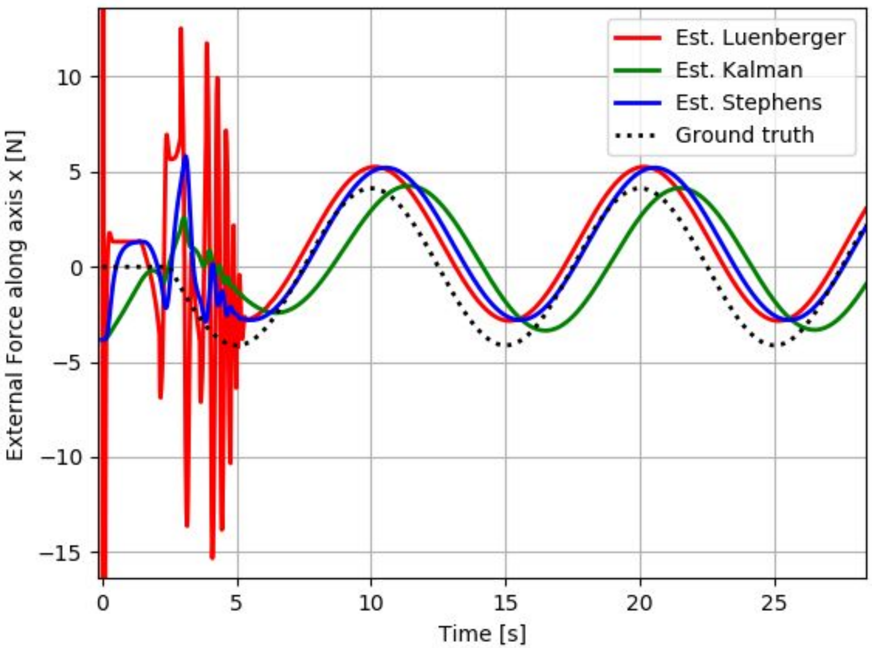


- More precise estimation for ***y*-axis**
- Luenberger was the **quickest**, but had the **highest total RMSE** due to presence of peaks
- KF had **lowest errors** but **longer TTC**
- Stephens' results are similar to KF ones but was **faster**

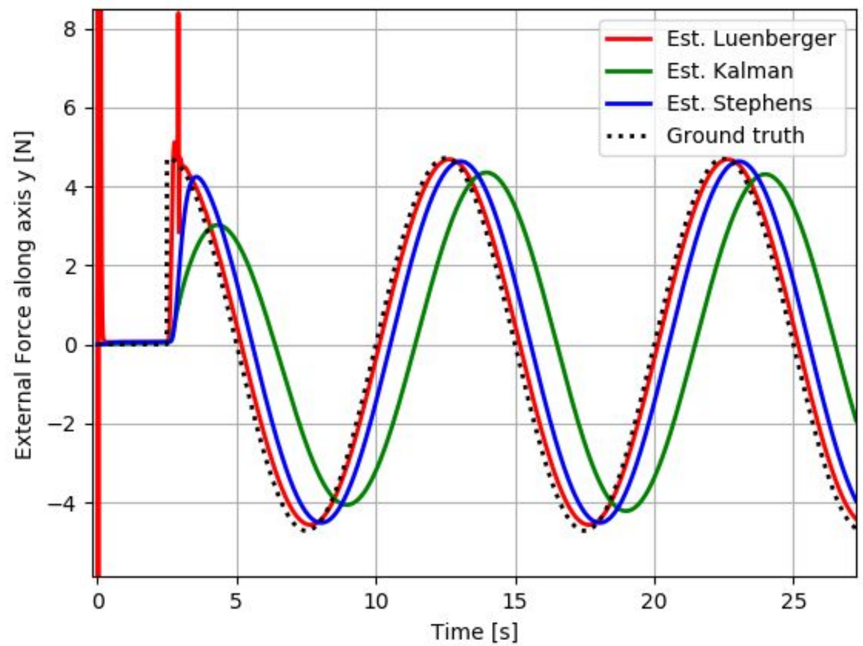
	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	50.25	2.89	1.13	**4.5**
KF based	**1.21**	**0.99**	**0.83**	23.0
Stephens	1.32	1.21	1.13	5.0

Experiment 2 | Stand no Balance: Periodic force

X - AXIS



Y - AXIS

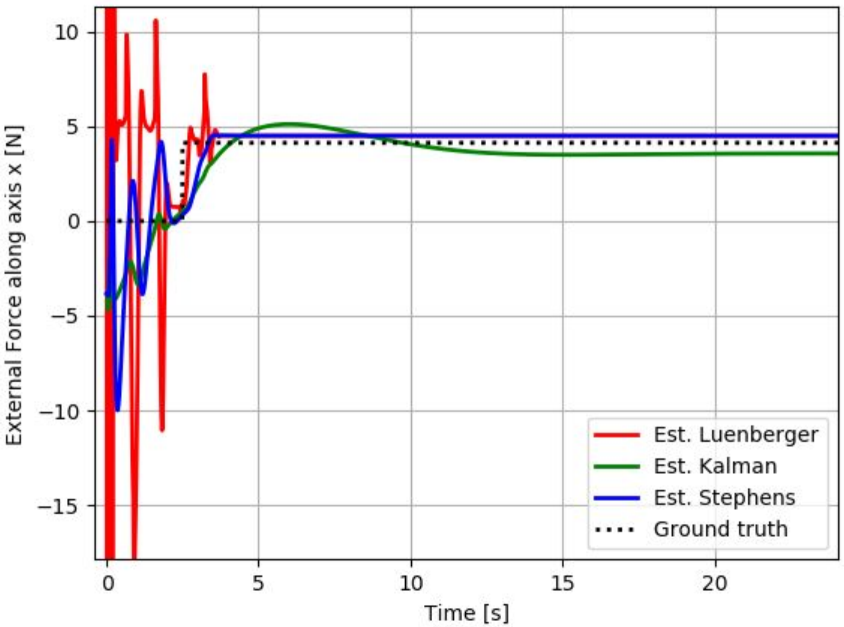


- Better estimation for ***y*-axis,** especially Luenberger
- Luenberger was the **quickest**, but had the **highest total RMSE**
- KF presented the highest delay and scaled down the amplitude o the signal
- Stephens' results are similar to KF ones but it had lowest **total RMSE** and **MAE**

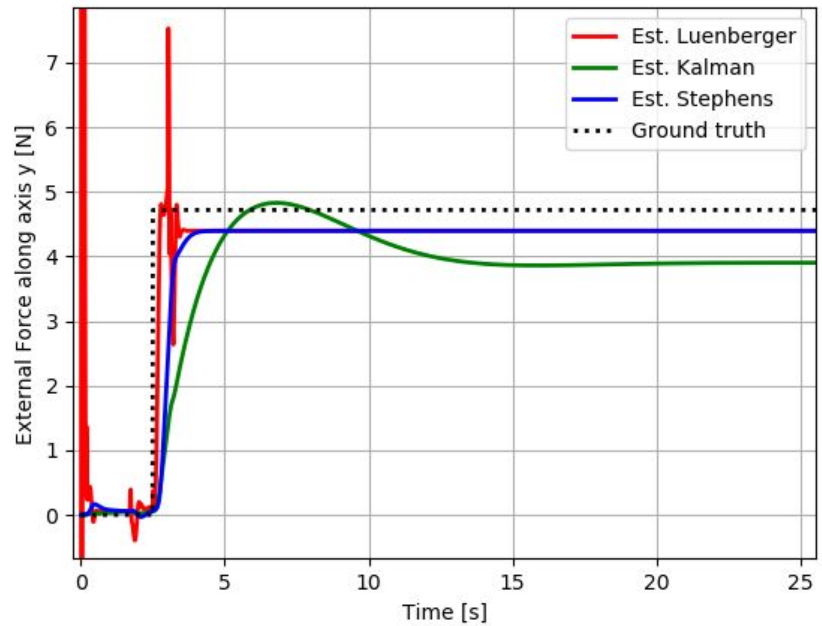
	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	51.50	3.35	**1.28**	**5.5**
KF based	3.68	3.59	3.75	**5.5**
Stephens	**2.09**	**1.83**	1.91	5.6

Experiment 3 | Stand with Balance: Constant force

X - AXIS



Y - AXIS

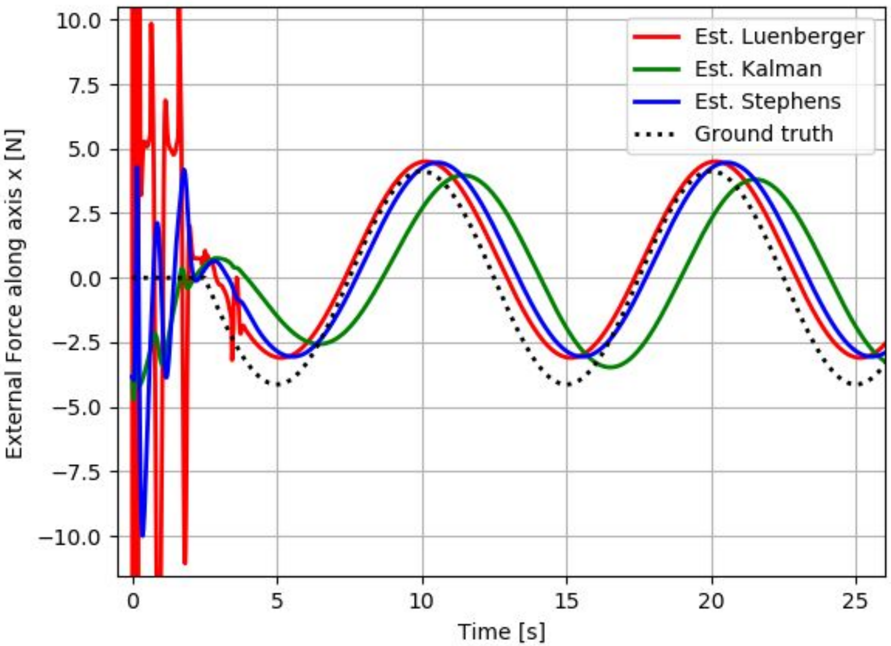


- ***x*-axis** suffered more of the peaking phenomenon
- Luenberger is the **quickest**, but has the **highest total RMSE**
- KF has **lower errors** then Luenberger but **highest TTC**
- Stephens' results are similar to KF ones but it was **faster**

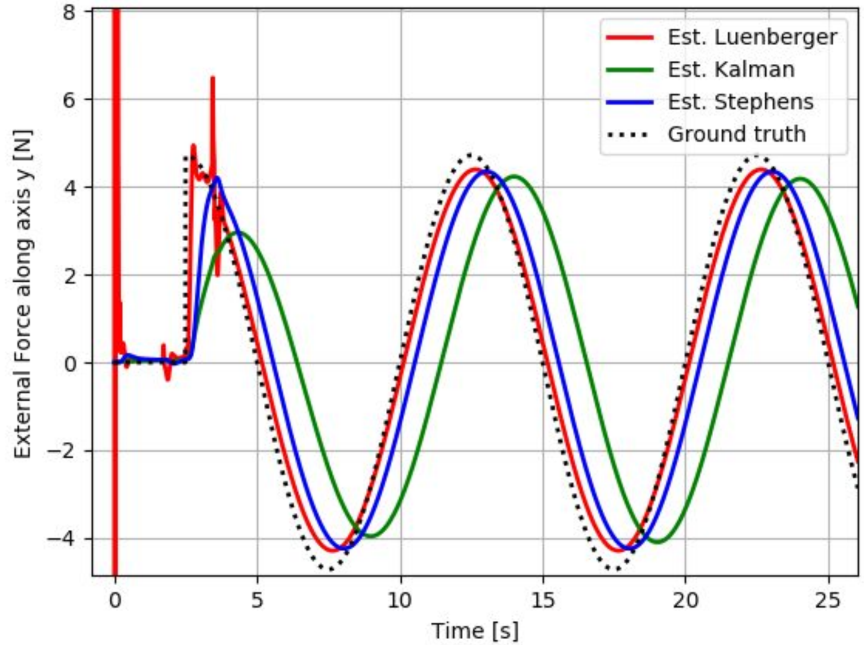
	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	50.82	2.85	**0.50**	**4.00**
KF based	1.33	1.10	0.99	24.0
Stephens	**1.24**	**0.72**	**0.50**	4.6

Experiment 4 | Stand with Balance: Periodic force

X - AXIS



Y - AXIS

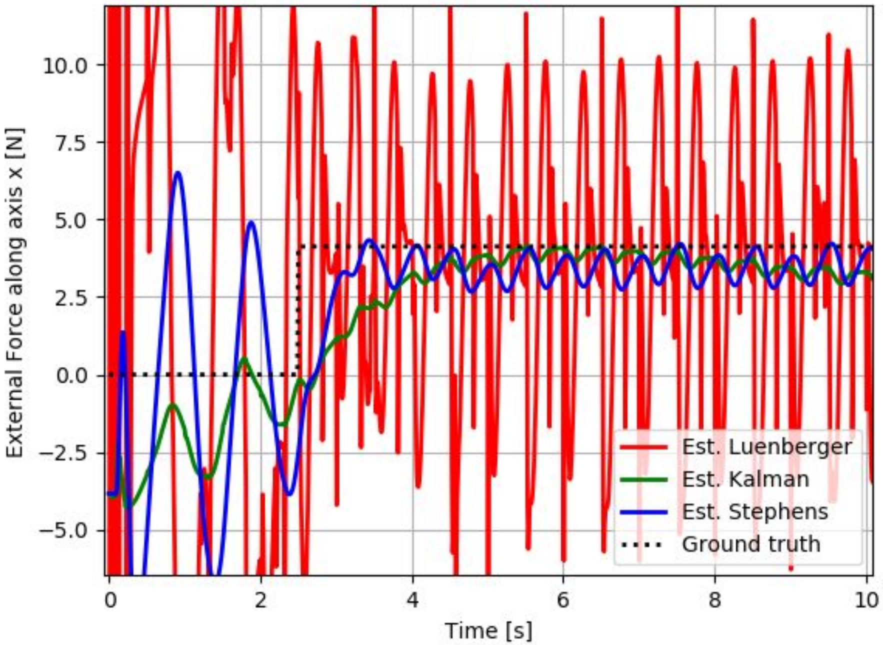


- Peaks and oscillations on ***x*-axis** before application force
- Luenberger and Stephens provided **better estimates** than KF
- **Major delay** in KF estimates
- Metrics consistent to previous experiments

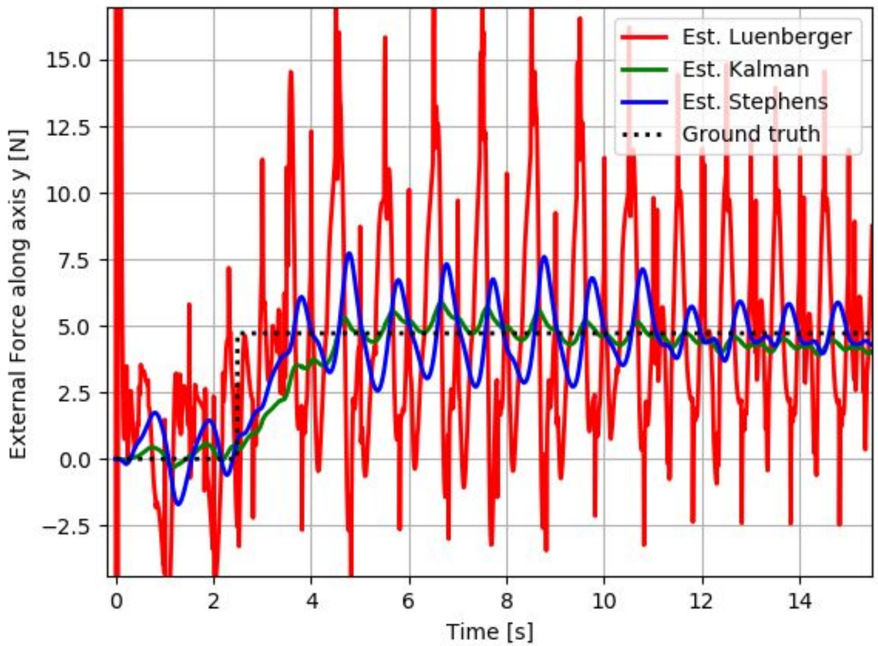
	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	51.15	3.20	**0.90**	**4.0**
KF based	3.69	3.62	3.78	5.0
Stephens	**1.94**	**1.72**	1.69	**4.0**

Experiment 5 | Walking: Constant force

X - AXIS



Y - AXIS

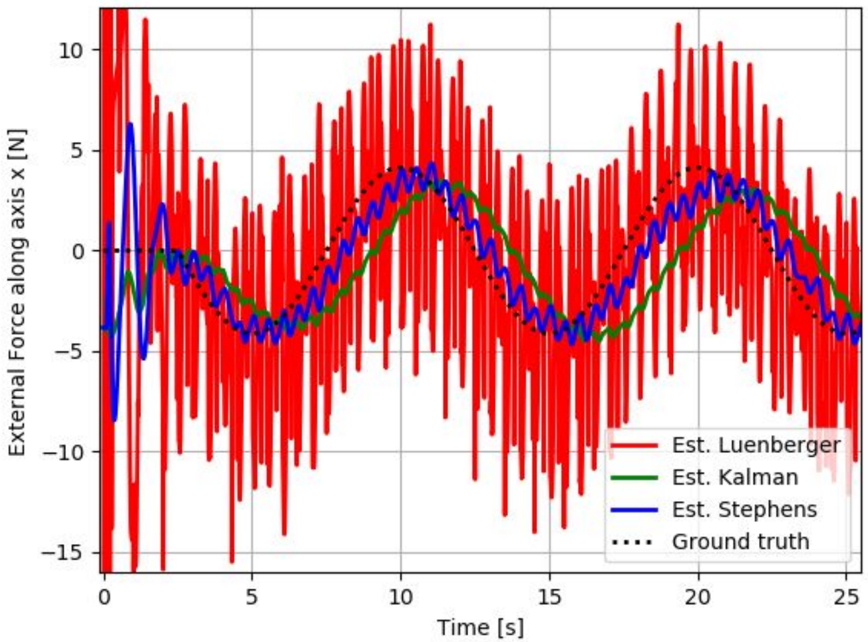


- High peaks and oscillations on both **axes**.
- Degraded results in Luenberger probably due to it's deterministics nature
- Stephens slightly better than KF and with **lower TTC**

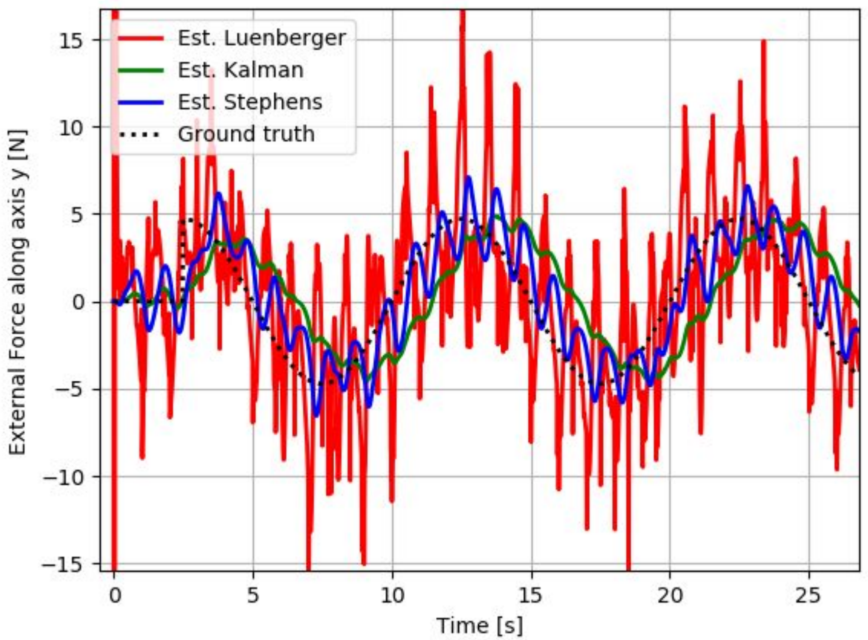
	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	52.10	7.33	5.85	**4.5**
KF based	**1.68**	1.49	1.54	20.0
Stephens	1.76	**1.35**	**1.09**	6.0

Experiment 6 | Walking: Periodic force

X - AXIS



Y - AXIS



- All the observers suffer from oscillating behaviour but clearly have detected shape of the force
- Luenberger presented the **highest spikes**
- KF showed the highest **delay** in following the force trend
- Stephens was the **best**, i.e. with lowest errors, but highest TTC

	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	50.60	7.44	6.09	**2.5**
KF based	3.84	3.74	3.99	4.5
Stephens	**1.85**	**1.50**	**1.31**	6.0

Observers Comparison

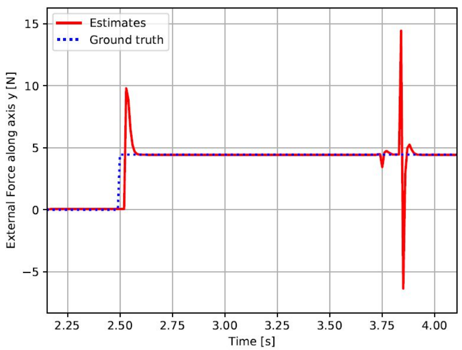
- **Stephens'** observer turned out to be the **best performing**
- Luenberger observer was the fastest but with **highest errors**
- Kalman Filter based observer provided good results but introduced the **highest delay** in replicating the shape of the periodic force
- Stephens' observer had the **lowest errors** and is **very fast**

	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Luenberger	51.07	4.51	2.63	**4.17**
KF based	2.57	2.42	2.48	13.67
Stephens	**1.7**	**1.39**	**1.27**	5.2

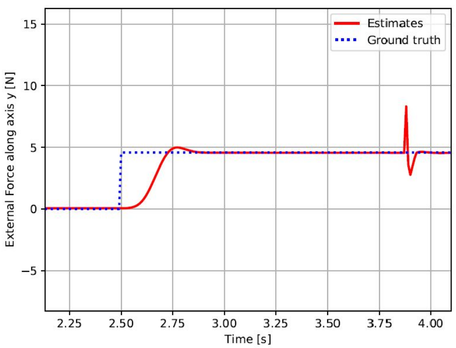
Average of the values for all the experiments

Experimental Considerations

- Filtering of the ZMP measurements
  - Found normalized cutoff frequency 0.04 through analysis of Fourier Transform
  - Low pass FIR filter of order 31 obtained with Hamming window



Before

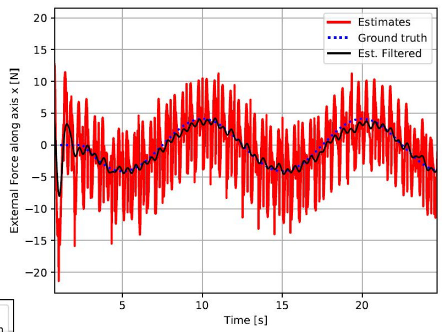


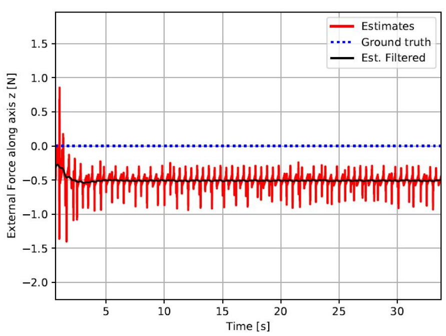
After

Results of the application of the ZMP filter in the experiment of constant force when standing no balance (Luenberger observer)

- Filtering of the external force estimates for walking
  - Better view of the estimated trends
  - Low pass FIR filter of order 100 obtained with Hamming window
  - Not suitable for real-time applications

Walking behaviour, periodic force, Luenberger observer





Walking behaviour, constant force, Kalman Filter based observer

Experimental Considerations (cont’d)

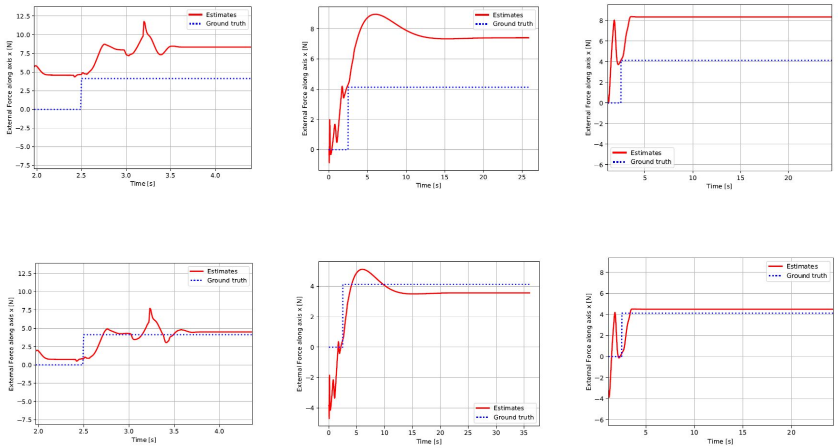
- Removal of systematic offset along *x*-axis in force estimates
  - Offset distribution with mean 3.84 N and st.dev. 0.86 N
  - Probably due to non-zero displacement between *x* CoM and point of application of the force

- Peaking phenomenon for the Luenberger observer
  - Affected several quantities
  - Unreliable extreme values
  - Shows up only at first instants with quick depletion

Luenberger *KF based* *Stephens*

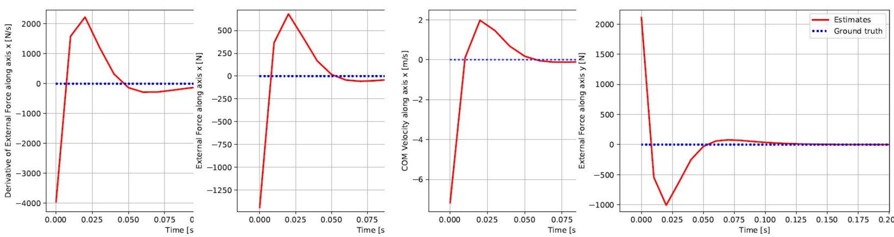
Before

After



Results of the offset correction in the experiment of constant force when standing with balance

Peak phenomenon in application constant force when standing no balance



Derivative of external force for x-axis *External force for x-axis* *CoM velocity x-axis* *External force for y-axis*

Conclusions and Future Work

- The problem of estimating external forces acting on a robot was addressed through the implementation of three state-disturbance observers
- The construction of a shared structure provided a testbed for future researchers
- The framework was validated through experiments, in which it was found that the Stephens' observer was the best solution in terms of high performances and speed of convergence
- Future improvements may include:
    - Finer hyperparameters tuning
    - Online filtering of the estimates
    - Testbed augmentation with new observers

References

[1] N. Scianca, M. Cognetti, D. De Simone, L. Lanari, G. Oriolo. *Intrinsically stable MPC for humanoid gait generation.* 2016 IEEE-RAS 16th International Conference on Humanoid Robots (Humanoids), Cancun, 2016, pp. 601-606, doi: 10.1109/HUMANOIDS.2016.7803336.

[2] F. M. Smaldone, N. Scianca, V. Modugno, L. Lanari, G. Oriolo. *Gait Generation using Intrinsically Stable MPC in the Presence of Persistent Disturbances.* 2019 IEEE-RAS 19th International Conference on Humanoid Robots (Humanoids), Toronto, ON, Canada, 2019, pp. 651-656, doi: 10.1109/Humanoids43949.2019.9035068.

[3] L. Hawley, W. Suleiman. *External force observer for medium-sized humanoid robots.* 2016 IEEE-RAS 16th International Conference on Humanoid Robots (Humanoids), Cancun, 2016, pp. 366-371, doi: 10.1109/HUMANOIDS.2016.7803302.

[4] L. Hawley, R. Rahem, W. Suleiman. *Kalman Filter Based Observer for an External Force Applied to Medium-sized Humanoid Robots.* 2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Madrid, 2018, pp. 1204-1211, doi: 10.1109/IROS.2018.8593610.

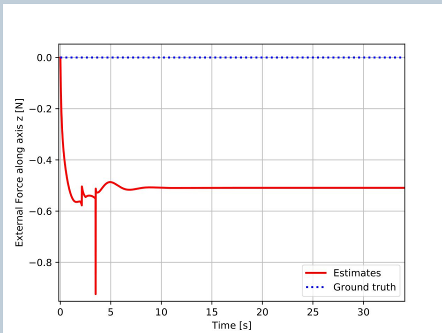
[5] B. J. Stephens. *State estimation for force-controlled humanoid balance using simple models in the presence of modeling error.* 2011 IEEE International Conference on Robotics and Automation, Shanghai, 2011, pp. 3994-3999, doi: 10.1109/ICRA.2011.5980358.

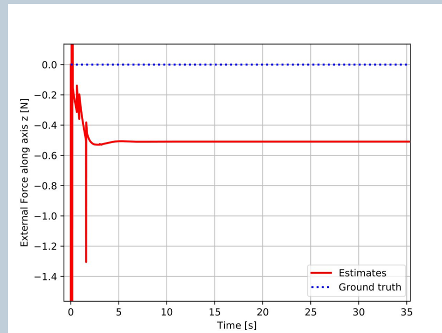
Appendix A: External Force along z-axis

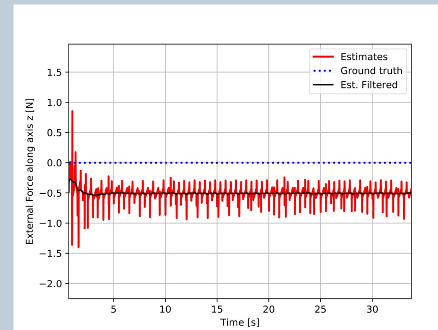
- KF based observer for *z*-axis estimation of the external force

	No Balance	*Balance*	*Walking*

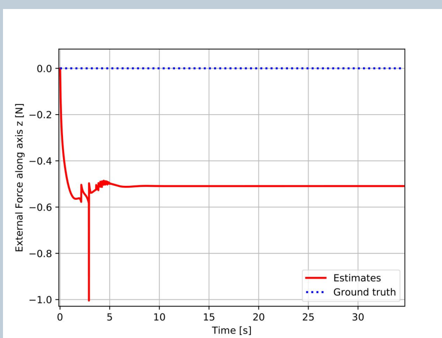
Constant Force

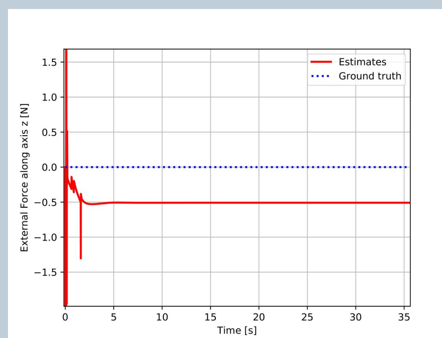


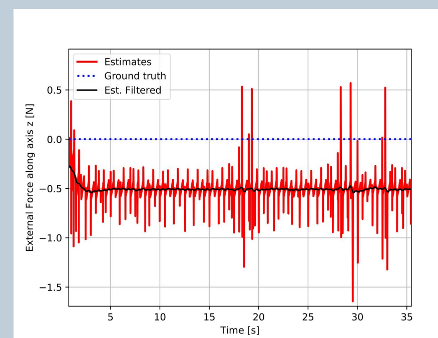




Periodic Force



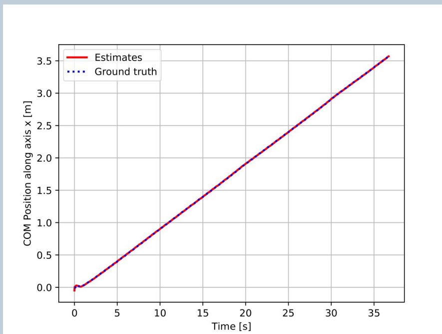




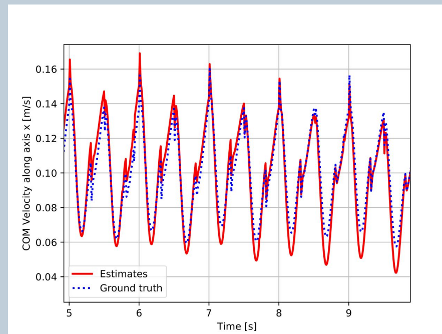
Appendix B: Full State Estimation

- Examples of the estimation of other quantities

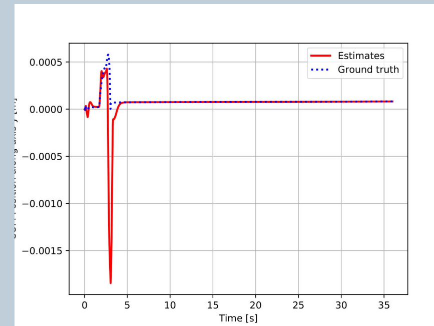
Luenberger | ***Kalman*** | ***Stephens***



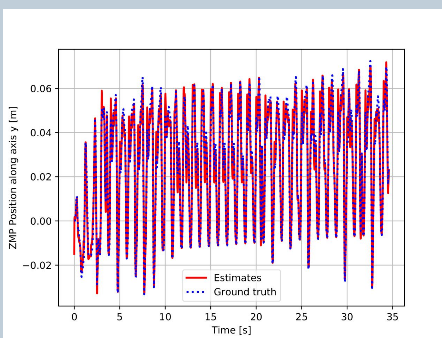
Walking - Periodic force



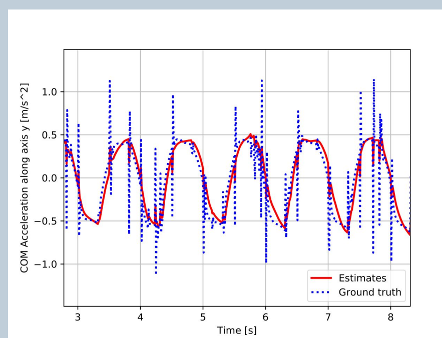
Walking - Periodic force



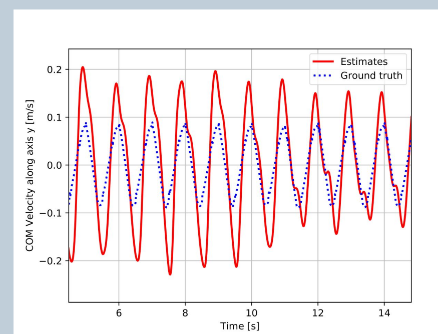
Standing - Constant force



Walking - Constant force



Walking - Periodic force



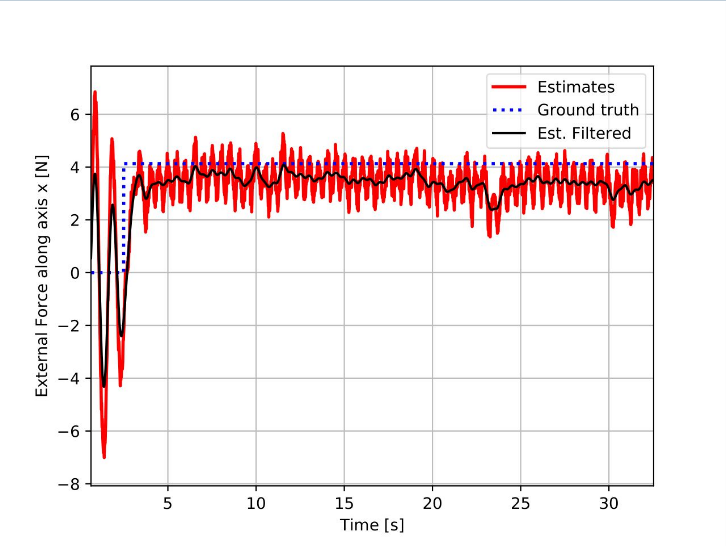
Walking - Constant force

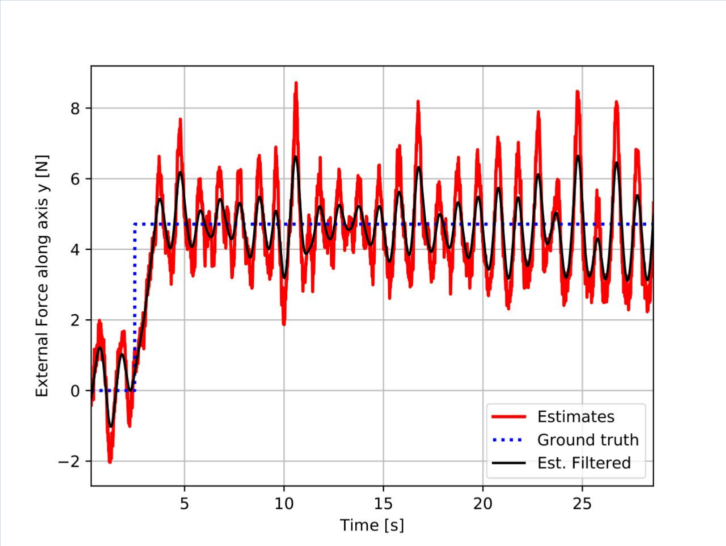
Appendix C: Additional Noise

- Added a **Gaussian white noise** with zero mean and st.dev. of $\sigma = 0.002$ to the measurements for Stephens' observer
- Tried **several st.dev. values** to evaluate impact of noise
- Obtained noisier estimates but the overall trend follows ground truth behaviour
- Sufficiently robust to additional disturbances

	Metric			
Observer	RMSE (tot.) [N]	MAE (tot.) [N]	RMSE [N]	TTC [s]
Stephens	**2.03**	**1.66**	**1.63**	4.5

Error metrics for this additional experiment



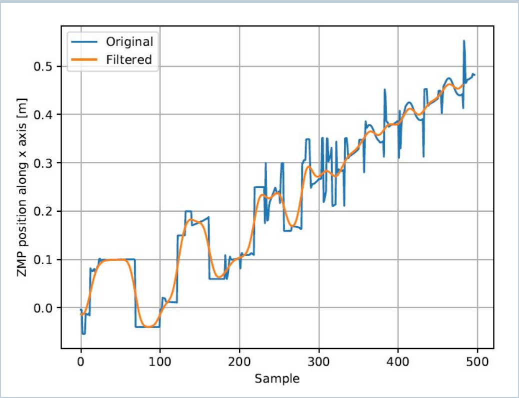


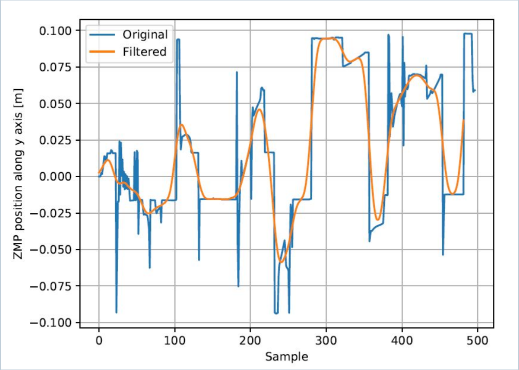
Walking behaviour, constant force with additionally perturbed measurements

Appendix D: Miscellaneous

- Results of the low pass filtering procedure on the ZMP position

ZMP position behaviour





- Covariance values of the Kalman Filter based observer used in the experiments

Measurement noise covariance matrix:

$$\boldsymbol{R}_z = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{R}_x = \boldsymbol{R}_y = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}$$

Inputs covariance values:

$$\sigma^2_{\ddot{x}_c} = \sigma^2_{\ddot{y}_c} = \sigma^2_{\ddot{z}_c} = 10^3$$
$$\sigma^2_{\ddot{F}_x} = \sigma^2_{\ddot{F}_y} = \sigma^2_{\ddot{F}_z} = 10^3$$

** the filtered behaviour is actually shifted by half of the order of the filter*