Pushdown automata (PAs)

Implementation of Computational Models (TC2037)

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Pushdown Automata?

Informal description of a PA

A Pushdown Automata (PA) is an Automata with a *stack*.

The automata has a stack, it is used as extra **memory** to make more complicated operations.

The Pushdown Automata are **equivalent** to the Context-free Grammars—they serve to represent context-free languages.

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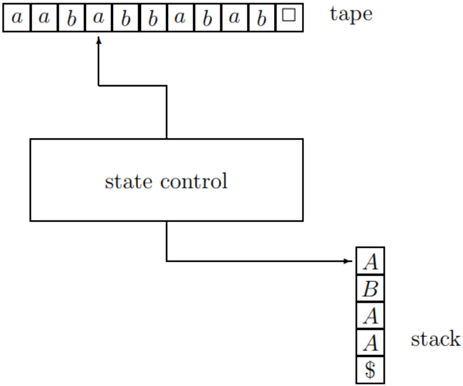

Figure 3.1: *A pushdown automaton.*

Image from Maheshwari and Smid's *Theory of Computation*, 2017

What is a Pushdown Automata?

Informal description of a PA

A PA has several elements:

1) A tape divided in **cells**

- Each cell contains a **symbol** of the input word that belongs to some **alphabet** Σ (also called **tape alphabet**).
- At the end of the tape, we have a **new symbol** to represent the end of the word: $\Box$. This new symbol does not belong to the alphabet of the word.

2) A tape head that can read the value of each cell according to its position, and it can perform two actions $\sigma = \{N, R\}$: N if it does not move and R if it moves to the right.

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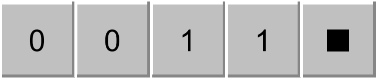

What is a Pushdown Automata?

Informal description of a PA

3) A stack that can store symbols.

- The symbols that can be stored in the stack belongs to **another alphabet**: Γ.
- One of these symbols is \$, which is **inside of the tape alphabet**.

4) A stack head that reads **the last symbol** of the same.

- The stack head can *stack* (`push`) more symbols, or it can *remove* the symbol from the top (`pop`).

5) A set of states, connected by transitions that depend of the symbols read by **both heads**.

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A stack as a data structure

Informal description of a PA

A stack (or in Spanish *pila*) works by the principle LIFO, that means *Last In, First Out*—the last element to enter is the first to be removed.

Example

Let A be a stack with the values $A = \langle 1, 2, 3 \rangle$ and $F = \{\mathrm{pop}, \mathrm{push}\}$ the set of functions *in-place* applicable to stacks.

If we apply the function pop on A, then we obtain the value 3, and the stack will change to $A = \langle 1, 2 \rangle$.

If after that we introduce one more value—let us say 4—to the stack (using the function push), then the stack will turn to $A = \langle 1, 2, 4 \rangle$.

What would happen if we apply pop again?

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What would happen if we apply pop again?

Formal definitions

Formal definition and design of PAs

There are **many** ways to express PAs and its elements:

- Brena (2003) uses transitions expressed in terms of the input and `pop` and `push` of the stack, belonging to a *relation of transition*, with final states and new symbols.
- Maheshwari and Smid (2017) use a **full** transition function in terms of the state, the input, tape movement ($\sigma = \{N, R\}$) and the `replace` function of the stack, without final states.
- Tinelli (2016) uses transitions expressed in terms of the input and the `replace` function of the stack, with final states.

In our case we will use **a combination of the three**: using the notations of Maheshwari and Smid for the design, we improve the notations using Brena with the symbols of Tinelli because we're cool like that.

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Definition of a Pushdown Automata

A pushdown automata M is a 6-tuple with the form $M = (Q, \Sigma, \Gamma, \delta, q, F)$ where:

- Q is a finite set of **states**,
- Σ is the **tape alphabet** (with no $\Box$),
- Γ is the **stack alphabet** (including $\$$),
- $q \in Q$ is the **initial state**,
- $F \subseteq Q$ is a finite set of **final states** and
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The transition function δ is a function with the form:

$$\delta : Q \times (\Sigma \cup \{\Box\}) \times \Gamma \to Q \times \{N, R\} \times \Gamma^*$$

Example

We can write $q_0 1 S \to q_1 RSS$ that means:

- Being in the state q_0,
- upon receiving a 1,
- and if the top of the stack has S

then the PA

- change to the state q_1,
- moves the tape to the right (Right) and
- replace the top of the stack with SS.

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Additional considerations

Formal definition and design of PAs

Initial configuration

- The PA starts in the state q.
- The tape head starts in the initial symbol of word w.
- The stack starts with only one symbol, $\$$.

Computation and termination

The PA makes a series of computation steps and *finishes* at the moment the stack is empty. If the stack is not empty, then the program does not end (infinite *loop*).

Acceptance

The PA accept the words w if the following conditions are met:

1. the automaton ends, and
2. at the time of finishing, (i.e. the stack is empty) the tape head is with the symbol $\square$.

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Example: *Matching Parentheses*

Formal definition and design of PAs

Examples of accepted words: $(), ((())), ()(), \dots$

Structure:

- Before finishing reading the whole word, the amount of "(" must be greater or equal than ")", and
- When you finished reading the whole word, the number of "(" must be equal to the number of ")".

Let us use a to represent "(" and b to represent ")" in the tape.

For each a read, we introduce as S in the stack. For each b read, we take out an S. How many states do we need? How many are finals?
Is a **complicated** process, so it is first suggested to define the PA formally (e.g. the complete transition function, and part by part), and then create the transition diagram and then pass it to a transition diagram with a smaller transition relation: $Q \times \Sigma \cup \{\Box\} \times \Gamma \to Q \times \Gamma^*$.
Why not consider whether or not the tape head moves?

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Why not consider whether or not the tape head moves?

Example: *Matching Parentheses*

Examples of accepted words: $(), ((())), ()(), \ldots$

Structure:

- Before finishing reading the whole word, the amount of "(" must be greater or equal than ")", and
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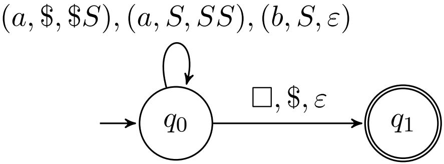
$M = (Q, \Sigma, \Gamma, \delta, q, F)$:

- $Q = \{q_0, q_1\}$
- $\Sigma = \{a, b\}$
- $\Gamma = \{\$, S\}$
- $\delta =$
  - $((q_0, a, \$)(q_0, \$S))$
  - $((q_0, a, S)(q_0, SS))$
  - $((q_0, b, S)(q_0, \varepsilon))$
  - $((q_0, \Box, \$), (q_1, \varepsilon))$
- $q = q_0$
- $F = \{q_1\}$



Example: $\{0^n 1^n\}$

Formal definition and design of PAs

$M = (Q, \Sigma, \Gamma, \delta, q, F)$:

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- $\Sigma = \{0, 1\}$
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  - $((q_0, 0, \$)(q_0, \$S))$
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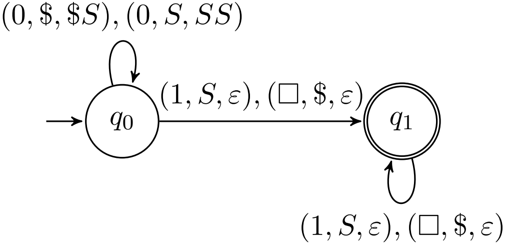

Combination and concatenation

Combination of PAs

In a similar way we combine two NFAs, the idea is to make a **previous** initial state that unites the initial states of the PAs using empty transitions $(\varepsilon, \varepsilon, \varepsilon)$.

Concatenation of PAs

The concatenation works in a similar way as the NFAs, however we must guarantee that the stack is found with *certain conditions* before moving to the next PA. The solution is to use a special symbol before starting with the first PA, and take it out before starting any operation with the sencond PA.

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Exercises

Formal definition and design of PAs

Construct (deterministic or nondeterministic) PA that accept the following languages

1. $\{w \in \{0,1\}^* : w \text{ is a palindrome}\}$.
2. $\{w \in \{0,1\}^* : w \text{ contains: }\}$
    - more 1s than 0s.
    - less 1s than 0s.
    - equal number of 1s and 0s.
3. Combine all PAs from exercise 2.
4. $\{0^{2n}1^n : n \geq 1\}$.
5. $\{0^n 1^m 0^n : n \geq 1, m \geq 1\}$.

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