Instituto Politécnico Nacional
Escuela Superior de Física y Matemáticas
Simulación I | 7MM1
Tarea Catálogo
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1 Distribuciones de Probabilidad Discretas

1.1 Bernoulli

Notación: $X \sim \text{Ber}(p)$, $0 < p < 1$

Función de masa de probabilidad (PMF):

$$P(X = k) = p^k(1-p)^{1-k}, \quad k = 0, 1$$

Función de distribución acumulativa (CDF):

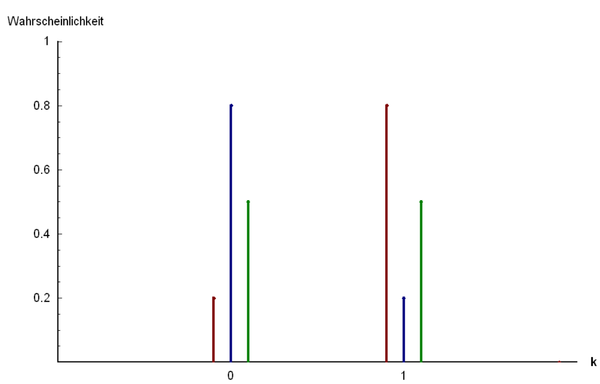
$$F(x) = \begin{cases} 0 & \text{si } x < 0 \\ 1-p & \text{si } 0 \leq x < 1 \\ 1 & \text{si } x \geq 1 \end{cases}$$

Momentos:

$$E[X] = p, \qquad \text{Var}(X) = p(1-p)$$

Función generadora de momentos (MGF):

$$M(t) = 1 - p + pe^t$$



1.2 Binomial

Notación: $X \sim \text{Bin}(n, p)$, $n \in \mathbb{N}$, $0 < p < 1$

PMF:

$$P(X = k) = \binom{n}{k} p^k(1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

CDF:

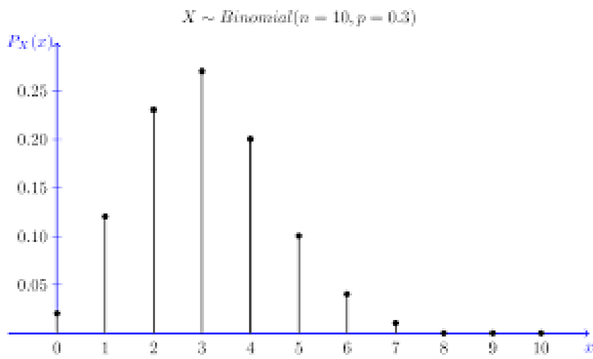
$$F(x) = \sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1-p)^{n-k}$$

Momentos:

$$E[X] = np, \qquad \mathrm{Var}(X) = np(1-p)$$

MGF:

$$M(t) = (1 - p + pe^t)^n$$



1.3 Poisson

Notación: $X \sim \mathrm{Poi}(\lambda)$, $\lambda > 0$

PMF:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

CDF:

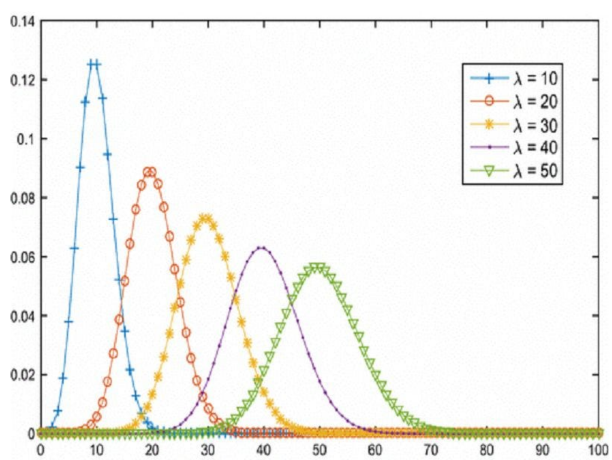
$$F(x) = \sum_{k=0}^{\lfloor x \rfloor} \frac{\lambda^k e^{-\lambda}}{k!}$$

Momentos:

$$E[X] = \lambda, \qquad \mathrm{Var}(X) = \lambda$$

MGF:

$$M(t) = e^{\lambda(e^t - 1)}$$



1.4 Geométrica

Notación: $X \sim \text{Geo}(p)$, $0 < p < 1$ (ensayos hasta el primer éxito)

PMF:

$$P(X = k) = (1-p)^{k-1}p, \quad k = 1, 2, \ldots$$

CDF:

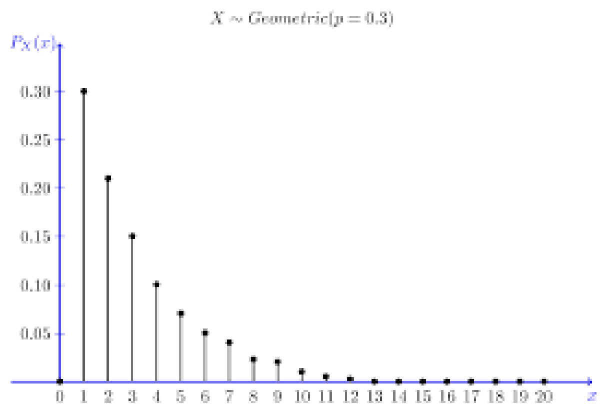
$$F(x) = 1 - (1-p)^{\lfloor x \rfloor}$$

Momentos:

$$E[X] = \frac{1}{p}, \qquad \text{Var}(X) = \frac{1-p}{p^2}$$

MGF:

$$M(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -\ln(1-p)$$



1.5 Hipergeométrica

Notación: $X \sim \text{HG}(N, K, n)$

PMF:

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

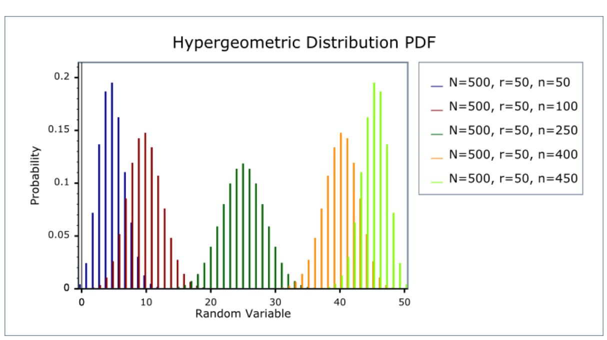
CDF:

$$F(x) = \sum_{k=0}^{\lfloor x \rfloor} P(X = k)$$

Momentos:

$$E[X] = n\frac{K}{N}, \qquad \text{Var}(X) = n\frac{K}{N}\left(1 - \frac{K}{N}\right)\frac{N-n}{N-1}$$

MGF: (expresión compleja, usualmente no se usa forma cerrada simple)



1.6 Uniforme Discreta

Notación: $X \sim U\{a, b\}$, $a, b \in \mathbb{Z}$, $a \le b$

PMF:

$$P(X = k) = \frac{1}{b - a + 1}, \quad k = a, a + 1, \dots, b$$

CDF:

$$F(x) = \frac{\lfloor x \rfloor - a + 1}{b - a + 1} \quad (a \le x < b + 1)$$

Momentos:

$$E[X] = \frac{a + b}{2}, \qquad \mathrm{Var}(X) = \frac{(b - a + 1)^2 - 1}{12}$$

MGF:

$$M(t) = \frac{e^{ta}(1 - e^{t(b-a+1)})}{(b - a + 1)(1 - e^t)}$$

2 Distribuciones de Probabilidad Continuas

2.1 Uniforme Continua

Notación: $X \sim U(a, b)$, $a < b$

PDF:

$$f(x) = \frac{1}{b - a}, \quad a \le x \le b$$

CDF:

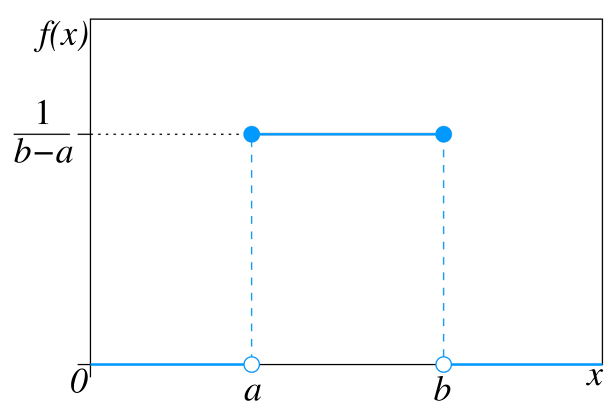
$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & x \ge b \end{cases}$$

Momentos:

$$E[X] = \frac{a + b}{2}, \qquad \mathrm{Var}(X) = \frac{(b - a)^2}{12}$$

MGF:

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b - a)}, \quad t \neq 0$$



2.2 Normal (Gaussiana)

Notación: $X \sim N(\mu, \sigma^2)$

PDF:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

CDF:

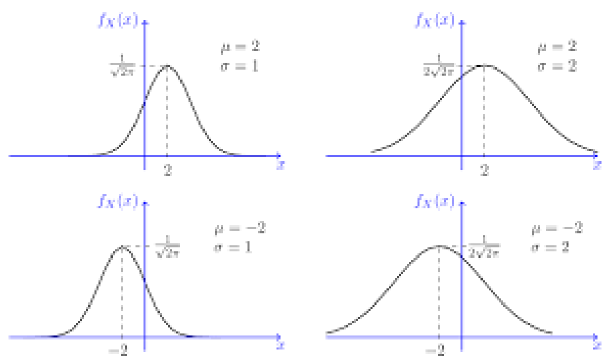
$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Momentos:

$$E[X] = \mu, \qquad \mathrm{Var}(X) = \sigma^2$$

MGF:

$$M(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$



2.3 Exponencial

Notación: $X \sim \mathrm{Exp}(\lambda)$, $\lambda > 0$

PDF:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

CDF:

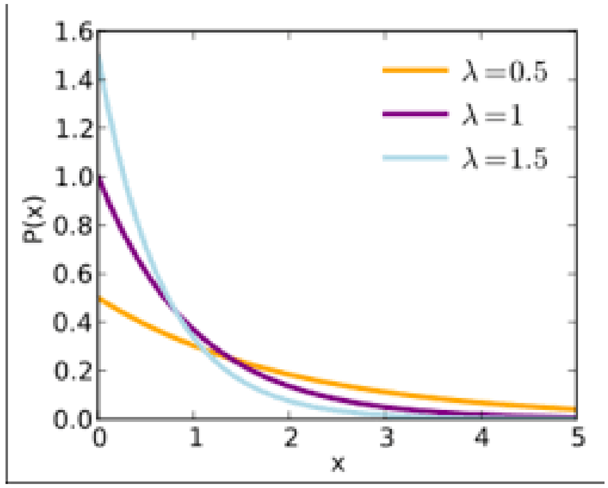
$$F(x) = 1 - e^{-\lambda x}, \quad x \geq 0$$

Momentos:

$$E[X] = \frac{1}{\lambda}, \qquad \mathrm{Var}(X) = \frac{1}{\lambda^2}$$

MGF:

$$M(t) = \frac{\lambda}{\lambda - t}, \quad t < \lambda$$



2.4 Gamma

Notación: $X \sim \text{Gamma}(\alpha, \beta)$ (forma α, tasa β)

PDF:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0$$

CDF:

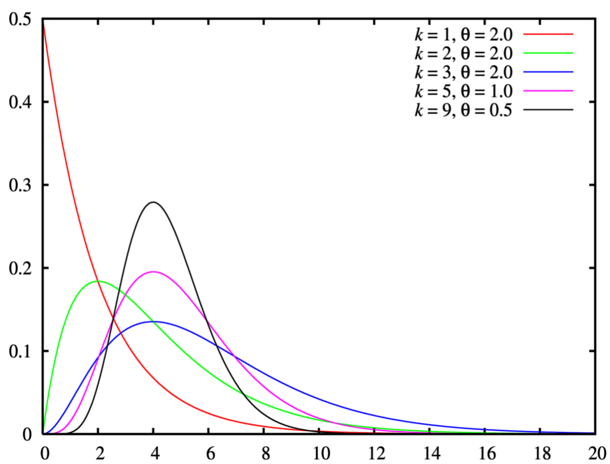
$$F(x) = \frac{\gamma(\alpha, \beta x)}{\Gamma(\alpha)}$$

Momentos:

$$E[X] = \frac{\alpha}{\beta}, \qquad \text{Var}(X) = \frac{\alpha}{\beta^2}$$

MGF:

$$M(t) = \left(\frac{\beta}{\beta - t}\right)^\alpha, \quad t < \beta$$



2.5 Beta

Notación: $X \sim \text{Beta}(\alpha, \beta)$, $\alpha > 0$, $\beta > 0$

PDF:

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$$

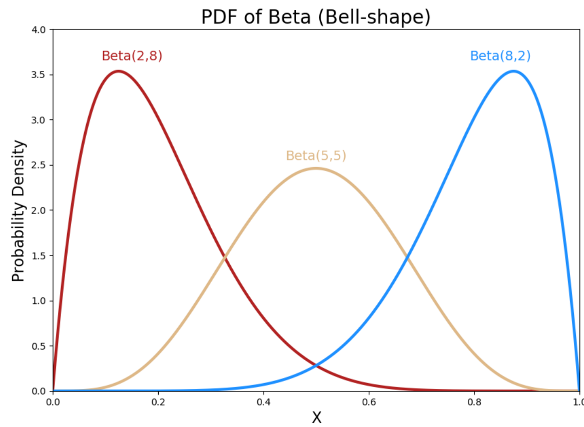
CDF:

$$F(x) = I_x(\alpha, \beta) \quad \text{(función beta incompleta regularizada)}$$

Momentos:

$$E[X] = \frac{\alpha}{\alpha+\beta}, \qquad \text{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

MGF: (no cerrada simple; usualmente serie hipergeométrica)



2.6 Chi-cuadrado

Notación: $X \sim \chi^2(k)$, $k \in \mathbb{N}^+$

PDF:

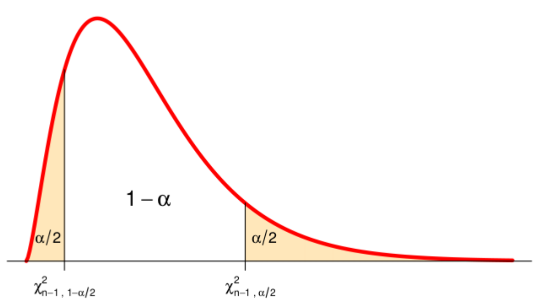
$$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}, \quad x > 0$$

Momentos:

$$E[X] = k, \qquad \text{Var}(X) = 2k$$

MGF:

$$M(t) = (1-2t)^{-k/2}, \quad t < 1/2$$



2.7 t-Student

Notación: $X \sim t(\nu)$, $\nu > 0$

PDF:

$$f(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\,\Gamma(\nu/2)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$$

Momentos:

$$E[X] = 0 \ (\nu > 1), \qquad \text{Var}(X) = \frac{\nu}{\nu-2} \ (\nu > 2)$$

MGF: No existe en todo $\mathbb{R}$ (colas pesadas)

2.8 F (Fisher-Snedecor)

Notación: $X \sim F(d_1, d_2)$, $d_1, d_2 > 0$

PDF:

$$f(x) = \frac{\Gamma((d_1+d_2)/2)(d_1/d_2)^{d_1/2} x^{d_1/2-1}}{\Gamma(d_1/2)\Gamma(d_2/2)\,(1+(d_1/d_2)x)^{(d_1+d_2)/2}}, \quad x > 0$$

Momentos:

$$E[X] = \frac{d_2}{d_2-2} \ (d_2 > 2), \qquad \mathrm{Var}(X) = \frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)} \ (d_2 > 4)$$

MGF: No existe finita