

$$| \psi \rangle = | + \rangle = \frac{| 0 \rangle + | 1 \rangle}{2}$$

$$\begin{aligned} Z | 0 \rangle &= | 0 \rangle \\ Z | 1 \rangle &= - | 1 \rangle \end{aligned}$$

$$\begin{aligned} X | + \rangle &= | 1 \rangle \\ X | - \rangle &= - | - \rangle \end{aligned}$$

evaluate
↓
vector

$$| 0 \rangle \xrightarrow{H} | + \rangle$$

Obs 1: $X \rightarrow \langle \psi | X | \psi \rangle = \langle + | X | + \rangle = \langle + | + \rangle = 1$
 Obs 2: $Z \rightarrow \langle \psi | Z | \psi \rangle = \langle + | Z | + \rangle = 0$

$$| \psi \rangle = \sum_k a_k | k \rangle \quad \hat{H} \rightarrow \sum_i w_i P_i \quad \langle \hat{H} \rangle = \sum_i w_i \langle P_i \rangle$$

↑
eigenvector of H

$$\begin{aligned} \langle \psi | \hat{H} | \psi \rangle &= \sum_k a_k^* \langle k | \hat{H} \sum_{k'} a_{k'} | k' \rangle \\ &= \sum_k \sum_{k'} a_k^* a_{k'} \langle k | \hat{H} | k' \rangle \\ &= \sum_k \sum_{k'} a_k^* a_{k'} \underbrace{\langle k | k' \rangle}_{= \delta_{kk'}} \\ &= \sum_k \sum_{k'} a_k^* a_{k'} \delta_{kk'} \\ &= \sum_k a_k^* a_k = \sum_k |a_k|^2 \end{aligned}$$

$$\hat{H} \rightarrow \boxed{\hat{H} = V^\dagger \Lambda V}$$

unitary operator

$$\begin{pmatrix} h_1 & & 0 \\ & \ddots & \\ 0 & & h_n \end{pmatrix}$$

$$\begin{aligned} \langle \psi | \hat{H} | \psi \rangle &= \langle \psi | V^\dagger \Lambda V | \psi \rangle \\ &= \sum_j \underbrace{| \langle j | V | \psi \rangle |^2}_{p_j} h_j \end{aligned}$$

$$| 0 \rangle \xrightarrow{H} \begin{matrix} \nearrow 0 \\ \searrow 1 \end{matrix}$$

$\langle \hat{z} \rangle$

$$\begin{aligned} \mu | 0 \rangle &= p_0 | 0 \rangle + p_1 | 1 \rangle \\ &= \begin{pmatrix} p_0 : | 0 \rangle \\ p_1 : | 1 \rangle \end{pmatrix} \end{aligned}$$

$$\hat{Z} = V^\dagger \Lambda V = \underbrace{\begin{pmatrix} +1 \\ -1 \end{pmatrix}}_{V^\dagger} \underbrace{\begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}}_V \underbrace{\begin{pmatrix} \\ \end{pmatrix}}_Z = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

\hat{X}

$$= \mathbb{I} \Lambda \mathbb{I} = Z$$

$$\hat{X} = V^\dagger \Lambda V = \begin{pmatrix} \\ \end{pmatrix}_{V^\dagger} \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}_V = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_X$$

$$= H^\dagger \Lambda H = X$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} (H) = X$$

$$|0\rangle \xrightarrow{M} \text{meter} \xrightarrow{H} \text{meter} \quad \langle \psi | X | \psi \rangle$$

$|0\rangle$

$$|0\rangle \xrightarrow{M} \boxed{H} \xrightarrow{\text{meter}} \begin{cases} \text{"0"} : x_0 \\ \text{"1"} : x_1 \end{cases}$$

$$\langle \psi | X | \psi \rangle = +1 \cdot \frac{x_0}{\text{\#shots} \rightarrow N} + -1 \cdot \frac{x_1}{\text{\#shots} \rightarrow N}$$

$$\hat{Y} = V^\dagger \Lambda V = (HS^\dagger)^\dagger \Lambda V$$

$$\rightarrow |0\rangle \xrightarrow{M} \boxed{S^\dagger} \xrightarrow{H} \text{meter}$$

\uparrow