$|\psi\rangle = |+\rangle = \frac{|0\rangle + |1\rangle}{2}$

$$\boxed{\begin{aligned} Z|0\rangle &= |0\rangle \\ Z|1\rangle &= -1 . |1\rangle \end{aligned}}$$

$X|+\rangle = \underset{\text{evalue}}{1}\ \underset{\text{evector}}{|+\rangle}$

$X|-\rangle = -1|-\rangle$

$|0\rangle$ —[H]— $|+\rangle$

Obs 1: $X \longrightarrow \langle\psi|X|\psi\rangle = \langle +|X|+\rangle = \langle +|+\rangle = 1$

Obs 2: $Z \longrightarrow \langle\psi|Z|\psi\rangle = \langle +|Z|+\rangle = 0$

$\hat{H} \longrightarrow \sum w_i P_i \qquad \langle \hat{H} \rangle = \sum_i w_i \langle P_i \rangle$

$|\psi\rangle = \sum_h a_h |h\rangle$ ← eigenvector of H

$$\langle\psi|\hat{H}|\psi\rangle = \sum_h a_h^* \langle h| \hat{H} \sum_{h'} a_{h'} |h'\rangle$$

$$= \sum_h \sum_{h'} a_h^* a_{h'} \langle h| \underbrace{\hat{H}|h'\rangle}_{= h'|h'\rangle}$$

$$= \sum_h \sum_{h'} a_h^* a_{h'} h' \underbrace{\langle h|h'\rangle}_{\substack{1 \text{ if } h = h' \\ 0 \text{ if } h \neq h'}}$$

$$= \sum_h a_h^* a_h h = \sum_h |a_h|^2 h$$

$\hat{H} \rightarrow \boxed{\hat{H} = V^\dagger \Lambda V}$

unitary operator

$$\begin{pmatrix} h_1 & & 0 \\ & \ddots & \\ 0 & & h_n \end{pmatrix}$$

$$\langle\psi|\hat{H}|\psi\rangle = \langle\psi| V^\dagger \Lambda V |\psi\rangle$$

$$= \sum_{\hat{j}} \underbrace{|\langle j|V|\psi\rangle|^2}_{p_j} h_j$$

$|0\rangle$ —[M]— → 0, → 1

$\langle \hat{Z} \rangle$

$M|0\rangle = p_0|0\rangle + p_1|1\rangle$

$p_0 : |0\rangle$

$p_1 : |1\rangle$

$$\hat{Z} = V^{\dagger} \Lambda V = \underbrace{\begin{pmatrix} & \\ & \end{pmatrix}}_{V^{\dagger}} \underbrace{\begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \underbrace{\begin{pmatrix} & \\ & \end{pmatrix}}_{V} = \underbrace{\begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}}_{Z}}_{=}$$

$\begin{matrix} +1 \\ -1 \end{matrix}$

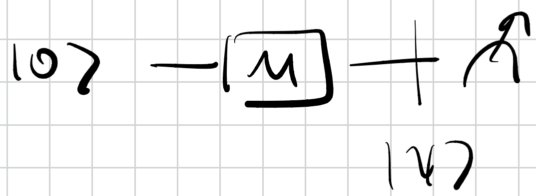
$\hat{X}$

$$\boxed{= I \Lambda I = Z}$$

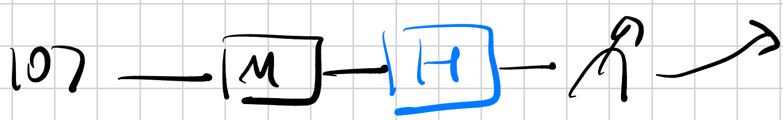
$$\hat{X} = V^{\dagger} \Lambda V = \underbrace{\begin{pmatrix} & \\ & \end{pmatrix}}_{V^{\dagger}} \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \underbrace{\begin{pmatrix} & \\ & \end{pmatrix}}_{V} = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{X}$$

$$\boxed{= H^{\dagger} \Lambda H = X}$$

$$= \underline{\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} H \end{pmatrix}} = X$$



$$\boxed{\langle \psi | X | \psi \rangle}$$



$\{ \text{"0"} : x_0$

$\text{"1"} : x_1 \}$

$$\langle \psi | X | \psi \rangle = +1 \cdot \frac{x_0}{N} + -1 \cdot \frac{x_1}{N}$$

#shots → N #shots

$$\boxed{\hat{Y} = V^{\dagger} \Lambda V} = (H S^{\dagger})^{\dagger} \Lambda V$$