MPC control of quadruped robot

Master’s Degree in Artificial Intelligence and Robotics

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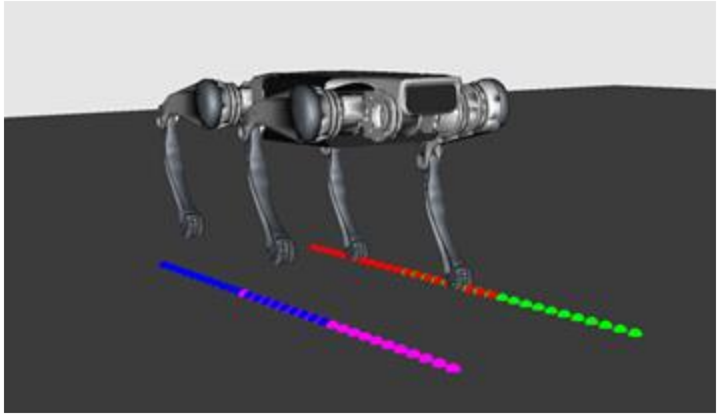
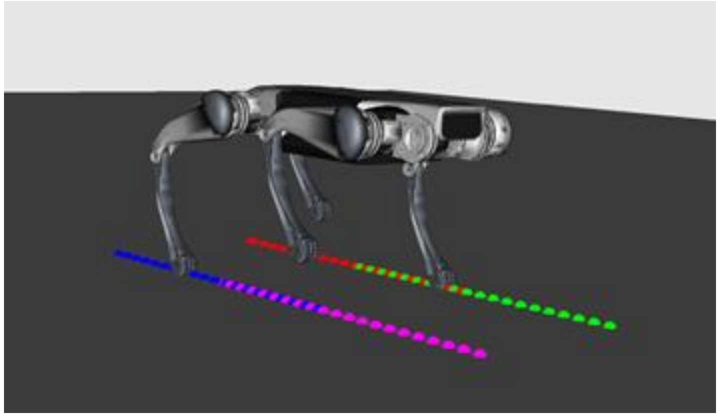
Project overview
Introduction

- the project focuses on **simulating quadruped locomotion** using **Model Predictive Control**
- the control problem is formulated as a **convex quadratic program** to compute the optimal contact forces
- feasible contact sequences for stable locomotion is generated by a custom **footstep planner**
- **ground** and **swing leg controllers** apply forces computed by the MPC and execute gait phases
- multiple gait scenarios are used to assess the framework's effectiveness





Problem definition

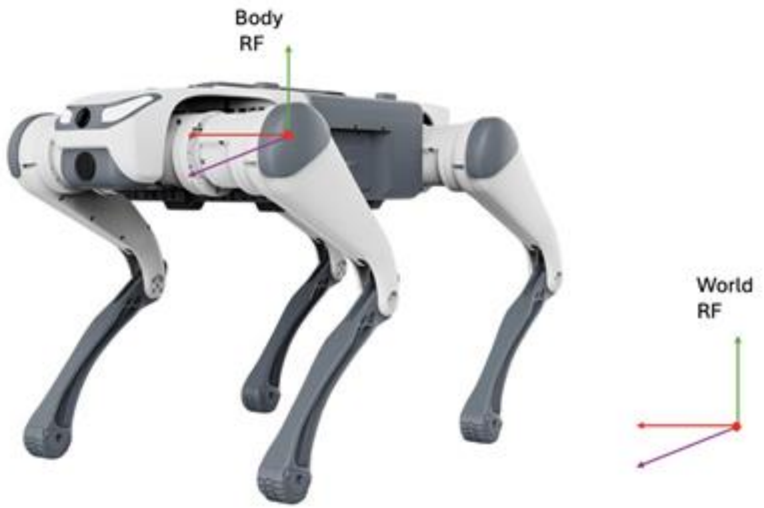
Introduction

- the analysis is carried out using the **Lite 3** quadruped robot from DeepRobotics

- the **state of the robot** is defined as $\boldsymbol{x} \in \mathrm{SE}(3) \times \mathbb{R}^6$:

$$\boldsymbol{x} = (\boldsymbol{\Theta}\ \boldsymbol{p}\ \boldsymbol{\omega}\ \dot{\boldsymbol{p}})^T$$

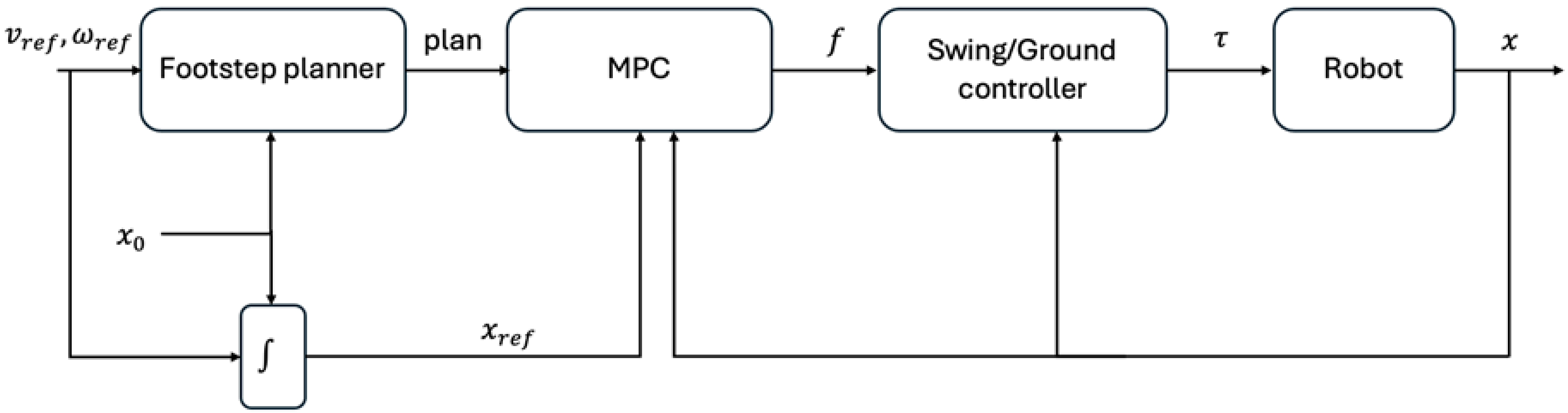
  - $\boldsymbol{\Theta}$: orientation expressed in ZYX Euler angles
  - $\boldsymbol{p}$: CoM position
  - $\boldsymbol{\omega}$: rigid body angular velocity
  - $\dot{\boldsymbol{p}}$: CoM linear velocity



- to enable effective walking, a reference trajectory $\boldsymbol{x_{des}}$ is properly designed by integrating predefined linear and angular velocities (v_{ref}, ω_{ref}), resulting in motion with **constant velocity**

Block scheme

Introduction



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Footstep planner

Overview

Footstep planner

- **input:** initial configuration, linear velocity v_{ref} ,angular velocity ω_{ref} ,number of steps, double and single support duration

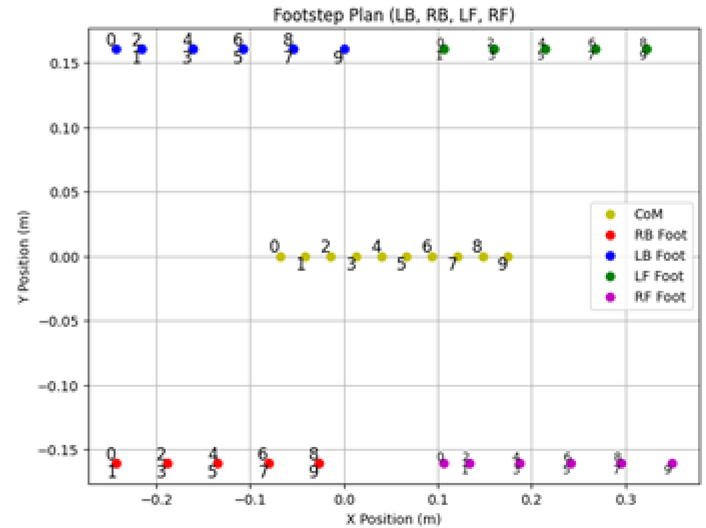
- **output**: a plan composed by a sequence of pose and the swing/support state for each leg

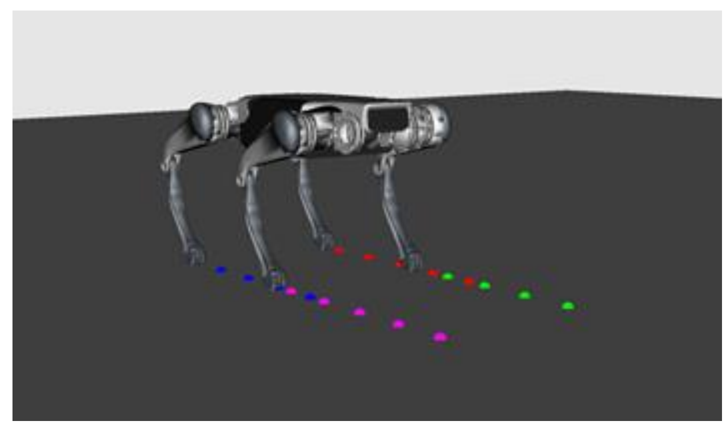
- **virtual unicycle modelling**: the robot is modelled as a virtual unicycle (x, y, θ) placed on the projection on the ground of the robot's center of mass

- **gait generation:** euler integration with $T = SS + DS$

$$\boldsymbol{p}_{\boldsymbol{com}}(t + \Delta t) = \boldsymbol{p}_{\boldsymbol{com}}(t) + \mathbf{R}(\theta) v_{ref} \cdot \Delta t$$

$$\theta(t + \Delta t) = \theta(t) + \omega_{ref} \cdot \Delta t$$

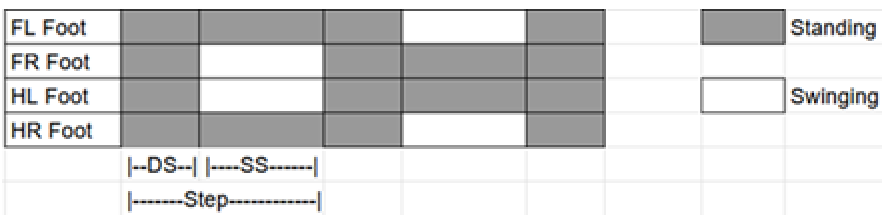




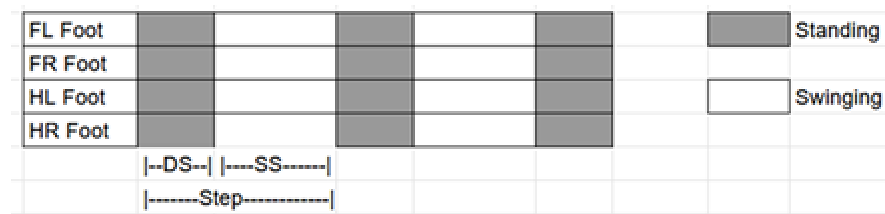
Types of gait

Footstep planner

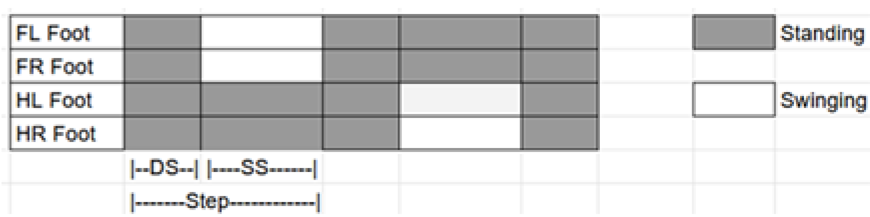
Trotting



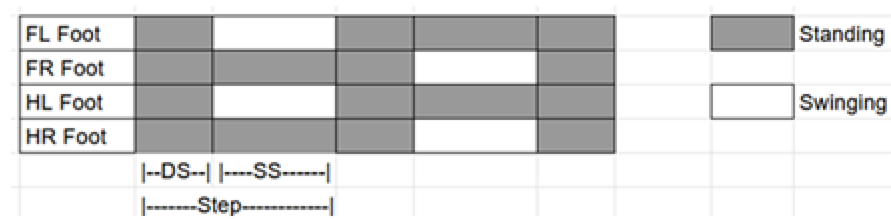
Pronking



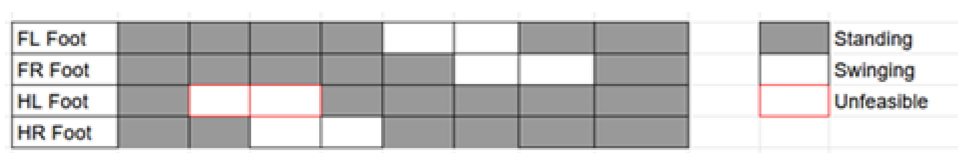
Pseudo-gallopping



Ambling



multi-phase gait such as **galloping** can't be generated



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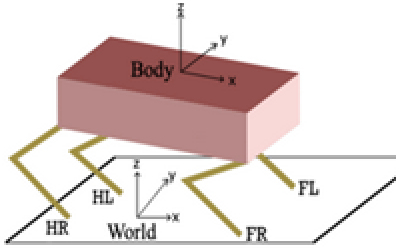
MPC

Robot's dynamics: simplification and model

MPC

The **linear system** $\boldsymbol{x}_{i+1} = \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{B}_i \boldsymbol{u}_i$ has been derived from the following assumptions:

- robot considered as a **rigid body** subject to external forces applied to the contact points - dynamics of the legs are neglected (low mass)
- Considering **small roll and pitch angles** and avoid near-vertical posture
- the state of the robot is then extended to include the gravitational term
- the problem is formulated as a **discrete-time linear system**.
- $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ should be computed using the future predicted position of the feet obtained through the dynamics at instant $t + i$; **desired position** of the feet are used instead



$$\begin{bmatrix} \dot{\boldsymbol{\phi}} \\ \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\psi}} \end{bmatrix} \approx \boldsymbol{R}_z(\psi)\boldsymbol{\omega}$$

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{\Theta} \\ \boldsymbol{p} \\ \boldsymbol{\omega} \\ \dot{\boldsymbol{p}} \\ g \end{bmatrix} = \begin{bmatrix} \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{R}_z(\psi) & \mathbf{0}_3 & 0 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{1}_3 & 0 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & 0 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & 0 \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Theta} \\ \boldsymbol{p} \\ \boldsymbol{\omega} \\ \dot{\boldsymbol{p}} \\ g \end{bmatrix} + \begin{bmatrix} \mathbf{0}_3 & \dots & \mathbf{0}_3 \\ \mathbf{0}_3 & \dots & \mathbf{0}_3 \\ \mathbf{I}^{-1}[\mathbf{r}_1]_\times & \dots & \mathbf{I}^{-1}[\mathbf{r}_1]_\times \\ \mathbf{1}_3/m & \dots & \mathbf{1}_3/m \\ \mathbf{0}_{1\times3} & \dots & \mathbf{0}_{1\times3} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1 \\ \vdots \\ \boldsymbol{f}_n \end{bmatrix}$$

Problem definition, cost function and constraints

MPC

- simplifications and linearization of the robot dynamic's leads to the following problems, formulated as **multiple-shooting**, **convex optimization problem**:

$$\min_{\boldsymbol{x},\boldsymbol{u}} \sum_{i=0}^{N-1} \left\| \boldsymbol{x}_{i+1} - \boldsymbol{x}_{i+1,ref} \right\|_{\boldsymbol{Q}_i}^2 + \|\boldsymbol{u}_i\|_{\boldsymbol{R}_i}^2$$

$$\begin{aligned} \textit{subject to} \quad & \boldsymbol{x}_{i+1} = \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{B}_i \boldsymbol{u}_i, & i = 0, \dots, N-1 \\ & \underline{\boldsymbol{c}_i} \le \boldsymbol{C}_i \boldsymbol{u}_i \le \overline{\boldsymbol{c}_i}, & i = 0, \dots, N-1 \\ & \boldsymbol{D}_i \boldsymbol{u}_i = 0 & i = 0, \dots, N-1 \end{aligned}$$

- $\boldsymbol{x}_i$ represents the system's state at the i-th step of the horizon
- $\boldsymbol{u}_i$ is the control input at step i
- $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ represent the discrete time system dynamics
- $\boldsymbol{C}_i$ and $\boldsymbol{c}_i$ represent inequality constraints (bound on the fz component, contact forces within friction cone)
- $\boldsymbol{D}_i$ selects forces corresponding to swinging foot (whose ground forces should be zero)
- $\boldsymbol{Q}_i$ and $\boldsymbol{R}_i$ are diagonal positive semidefinite matrices of weights

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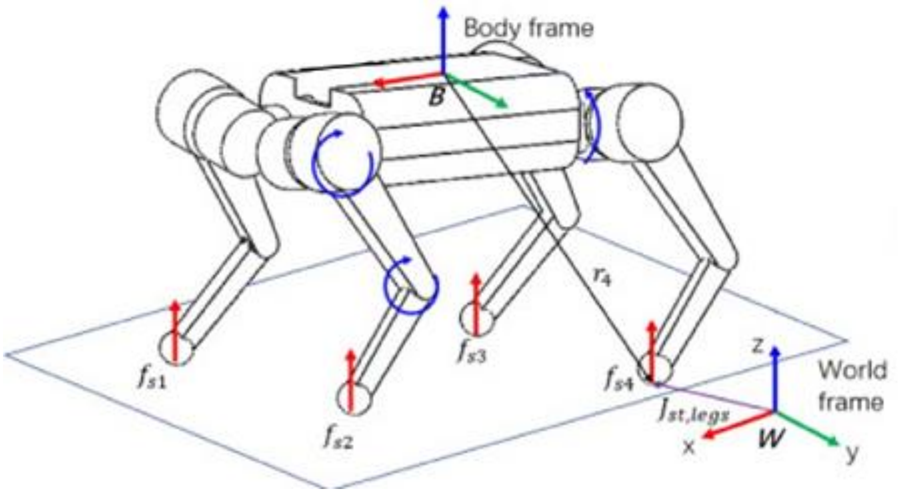
Controllers

Ground controller

- given a leg **planned to be in contact** at a specific *timestep* (*i*), is possible to map the computed **reaction force** (by the MPC) to the torque the joints of the leg should apply by simply computing:

$$\boldsymbol{\tau}_i = \boldsymbol{J}_i^T \boldsymbol{R}_i^T \boldsymbol{f}_i$$

- $\boldsymbol{J}_i^T$: i-th leg Jacobian
- $\boldsymbol{R}_i^T$: rotation from body frame to world frame
- $\boldsymbol{f}_i$: output of the MPC relative to the i-th leg



Swing trajectory definition

Controllers

- each foot of the robot should track a **predefined trajectory** for the movement during the swing phase

- using a **cubic polynomial** defined in the plane $\boldsymbol{x}-\boldsymbol{y}$ allows continuous velocity and acceleration profile and zero derivatives at the boundary:

$$\boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{p}_i + (\boldsymbol{p}_f - \boldsymbol{p}_i)\left(-\mathbf{2}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}}\right)^{\mathbf{3}} + \mathbf{3}\left(\frac{\boldsymbol{t}}{\boldsymbol{T}}\right)^{\mathbf{2}}\right), \quad t \in [0, T]$$

where $\boldsymbol{T}$ is the duration of the single support time

- using a **quartic polynomial** with a bell-shaped profile for the $\boldsymbol{z}$ component, allowing the liftoff and landing on 0 vertical velocity and acceleration:

$$\boldsymbol{z}(\boldsymbol{t}) = \mathbf{16}\frac{\boldsymbol{h}}{\boldsymbol{T}^4}\boldsymbol{t}^4 - \mathbf{32}\frac{\boldsymbol{h}}{\boldsymbol{T}^3}\boldsymbol{t}^3 + \mathbf{16}\frac{\boldsymbol{h}}{\boldsymbol{T}^2}\boldsymbol{t}^2, \quad t \in [0, T]$$

where the value of the ***h*** is an *hyperparameter* describing the maximum height of the step

Swing leg controller
Controllers

- simple control strategy using **feedback + feedforward** to track the reference trajectory
- commanding the torque for the swinging leg as:

$$\boldsymbol{\tau}_i = \boldsymbol{J}_i^T \left[\boldsymbol{K}_p \left(\boldsymbol{p}_{i,ref} - \boldsymbol{p}_i \right) + \boldsymbol{K}_d \left(\boldsymbol{v}_{i,ref} - \boldsymbol{v}_i \right) \right] + \boldsymbol{\tau}_{i\_ff}$$

where the $\boldsymbol{\tau}_{i\_ff}$ is the feedforward term computed taking account of the Dynamical term of the leg:

$$\boldsymbol{\tau}_{i\_ff} = \boldsymbol{J}_i^T \boldsymbol{M}_i \left(\boldsymbol{a}_{i,ref} - \dot{\boldsymbol{J}}_i \dot{\boldsymbol{q}}_i \right) + \boldsymbol{C}_i \dot{\boldsymbol{q}}_i + \boldsymbol{G}_i$$

- $\boldsymbol{J}_i^T$: Jacobian of the i-th leg
- $\dot{\boldsymbol{q}}_i$: joint velocity vector
- $\boldsymbol{M}_i$: operational space inertia matrix (apparent mass along the i-th direction)
- $\boldsymbol{C}_i$: Coriolis term for the i-th leg
- $\boldsymbol{G}_i$: gravitational term for the i-th leg

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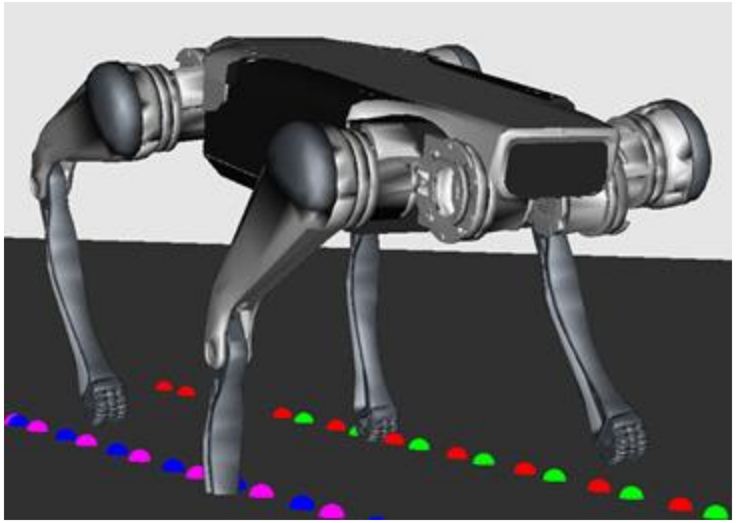
Results

Simulation environment and MPC frequencies

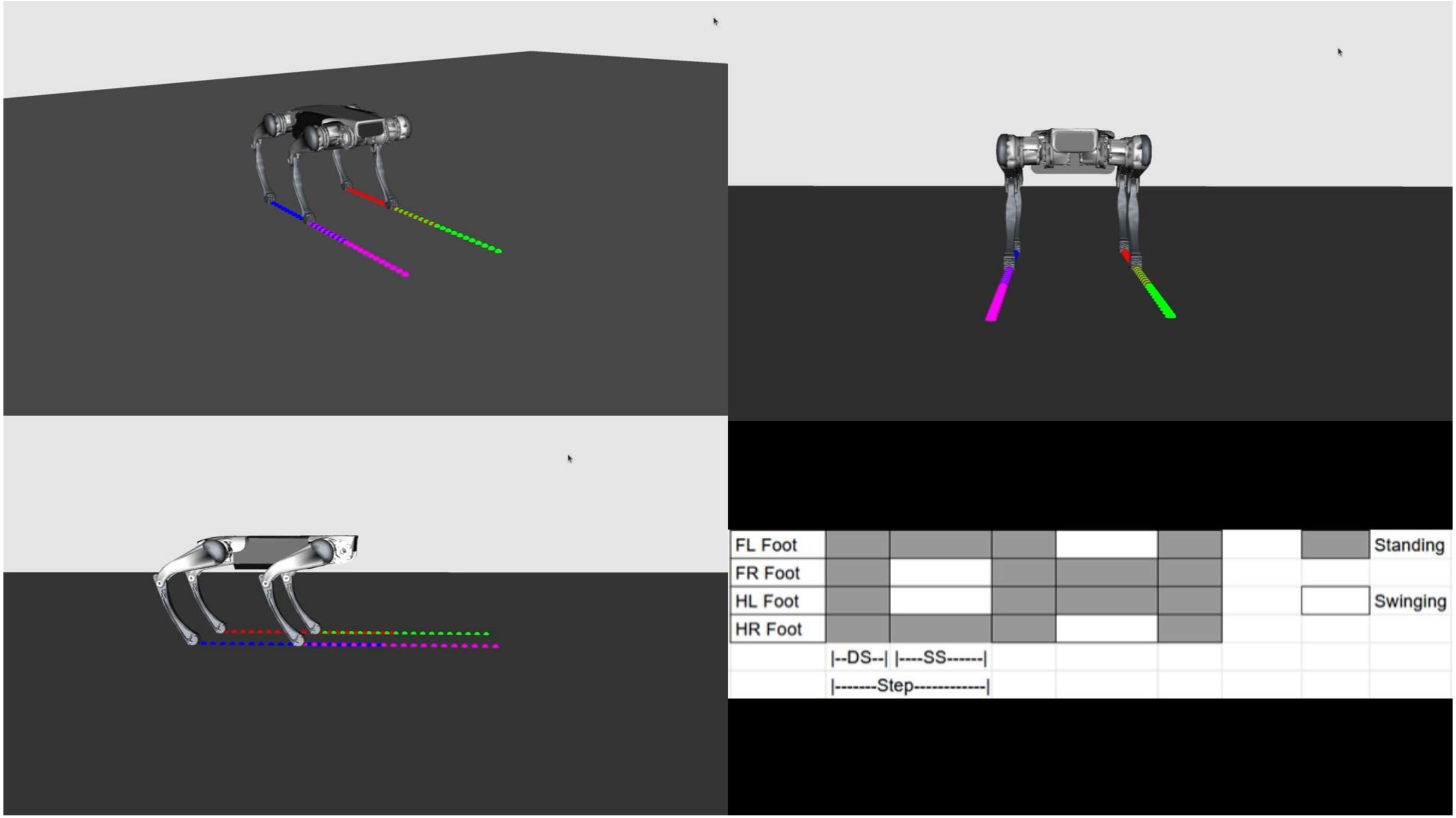
Results

- The proposed results are obtained using the python based **DARTPy simulation environment**
- MPC solver achieved solve frequencies in the range of **40-60 Hz** with horizon length of 60
- By decreasing the horizon length to 30, tracking performance were slightly worse but solve frequencies exceed **100 Hz**, making **real-time**, **on-board** implementation feasible (simulation time step of 0.01 seconds)
- One major problem was **compenetration between feet and ground**; it was solved by enlarging collision spheres of the feet and by adding small mass and inertia to them



Trotting gait - Video

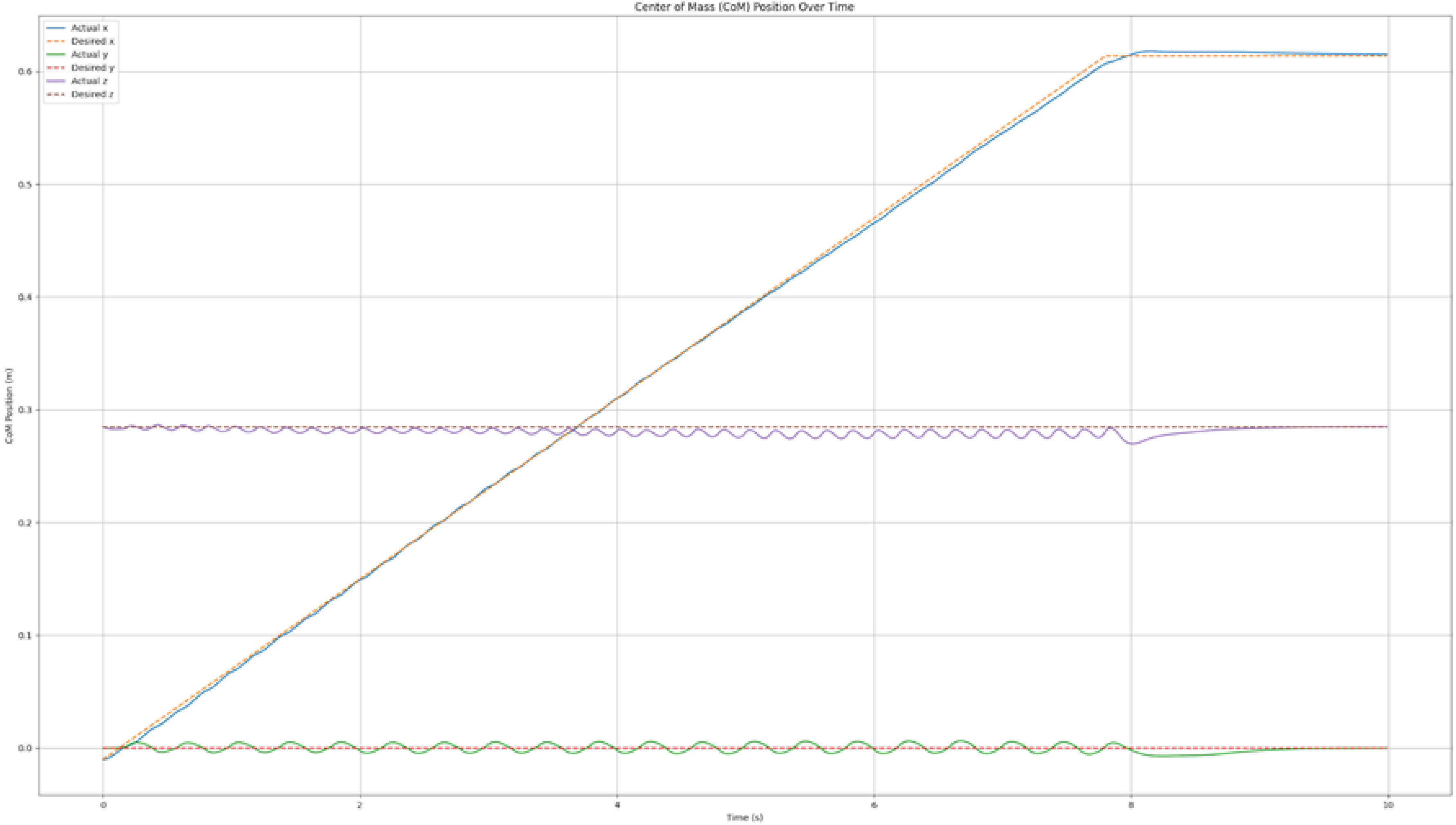
Results



$$ss\_duration = 0.1s, \quad ds\_duration = 0.1s, \quad v\_com\_ref\_x = 0.08m/s$$

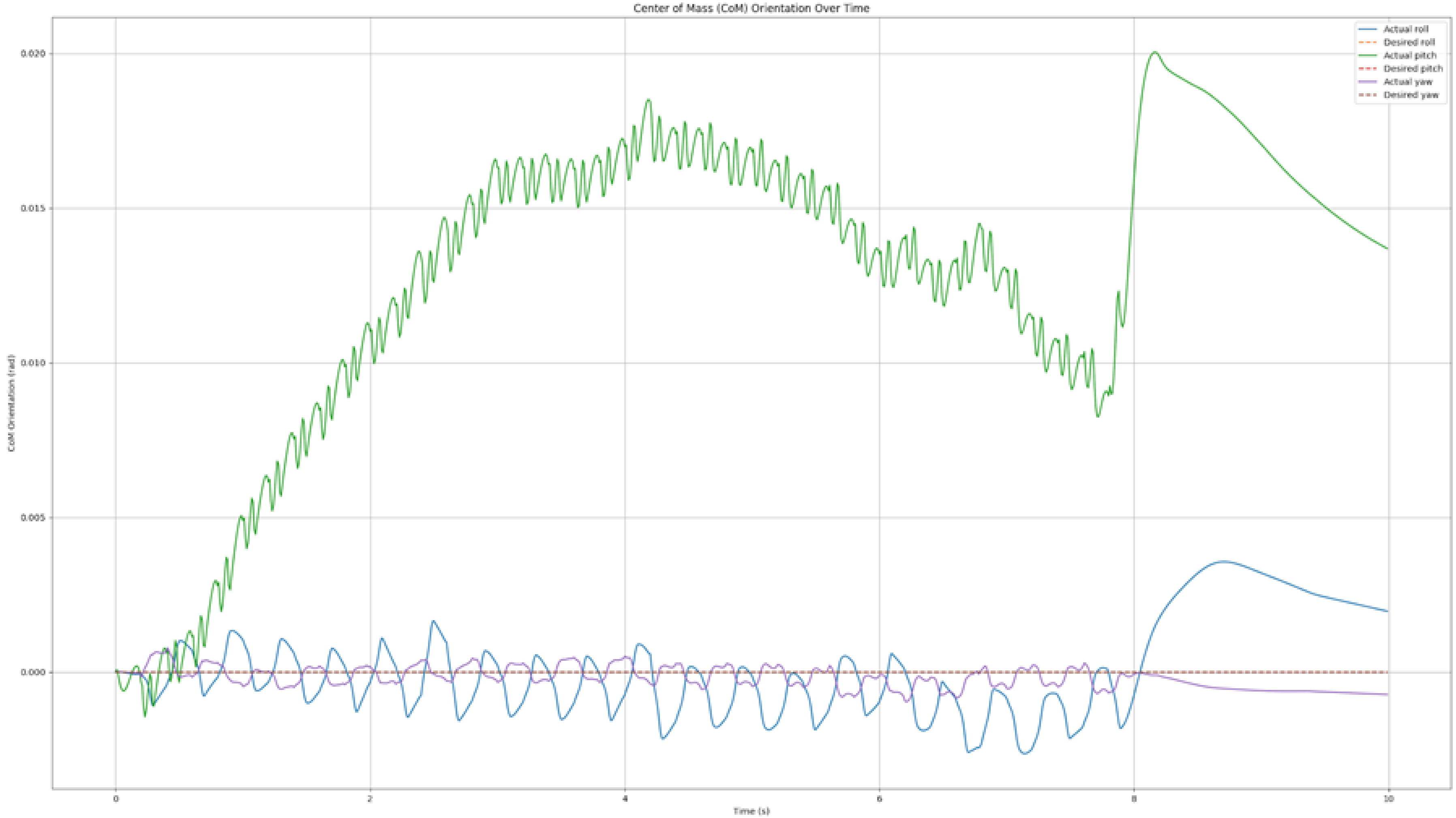
Trotting gait - Position of CoM

Results



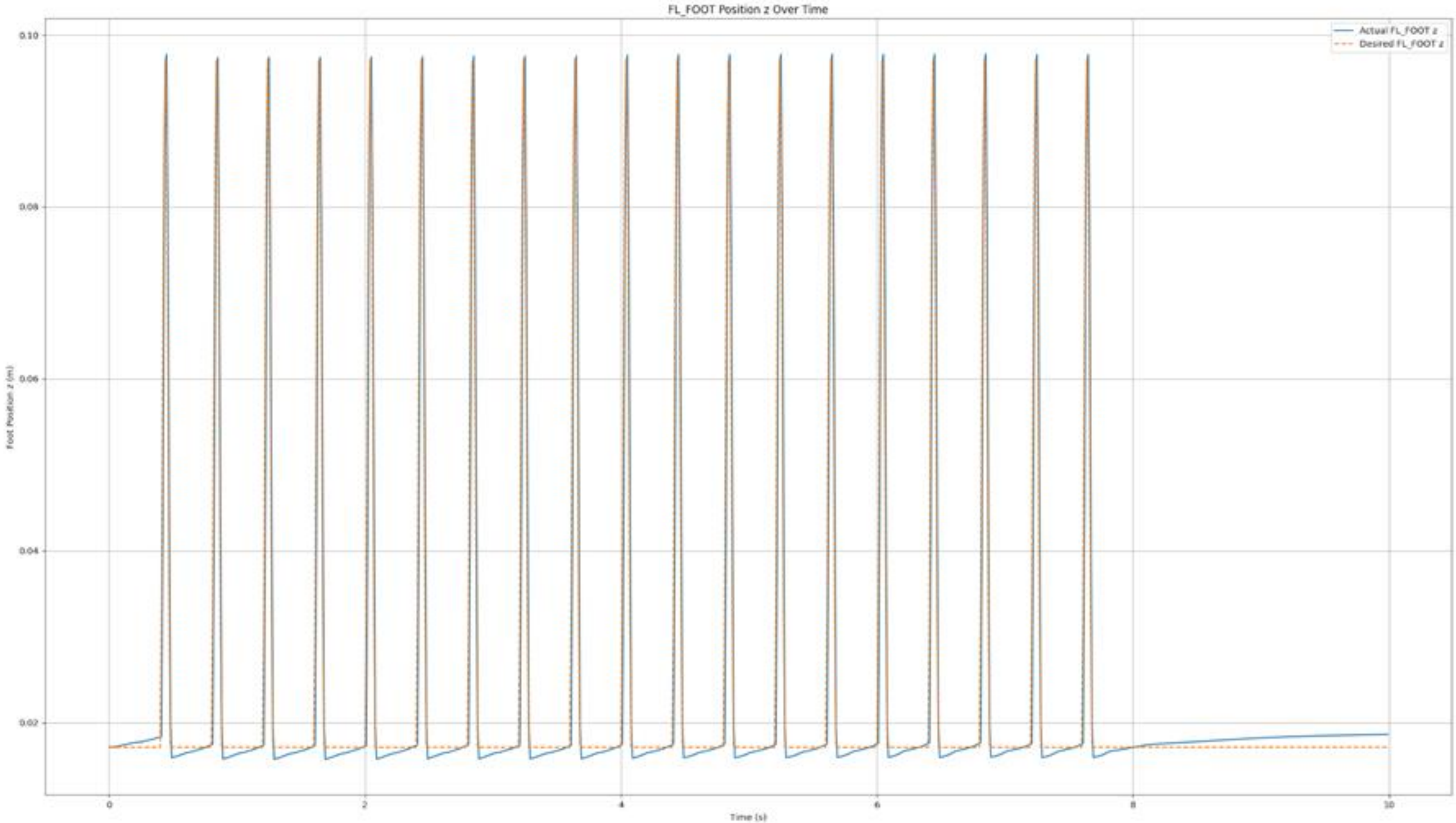
Trotting gait - Orientation of CoM

Results



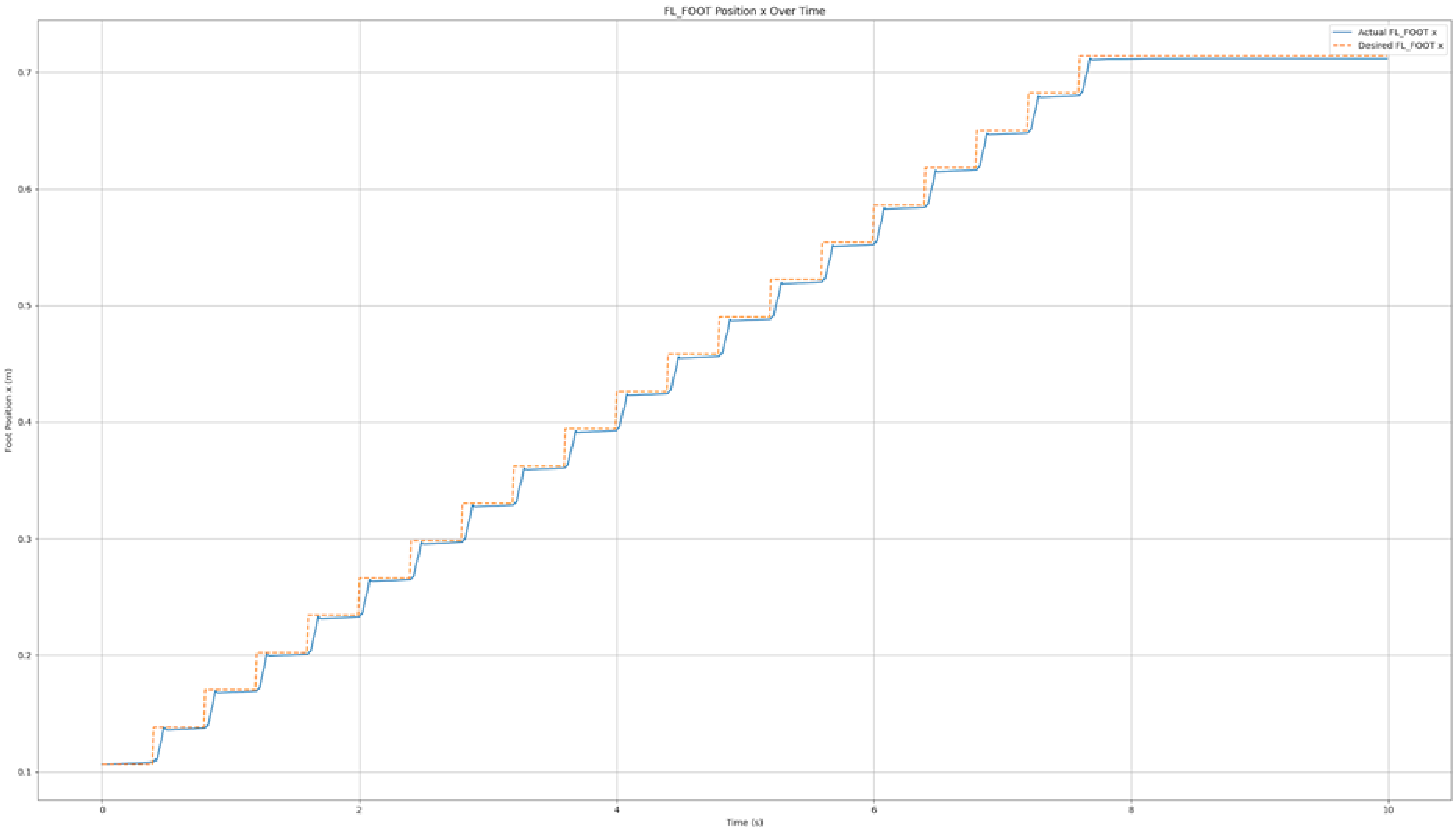
Trotting gait - FL foot z component

Results



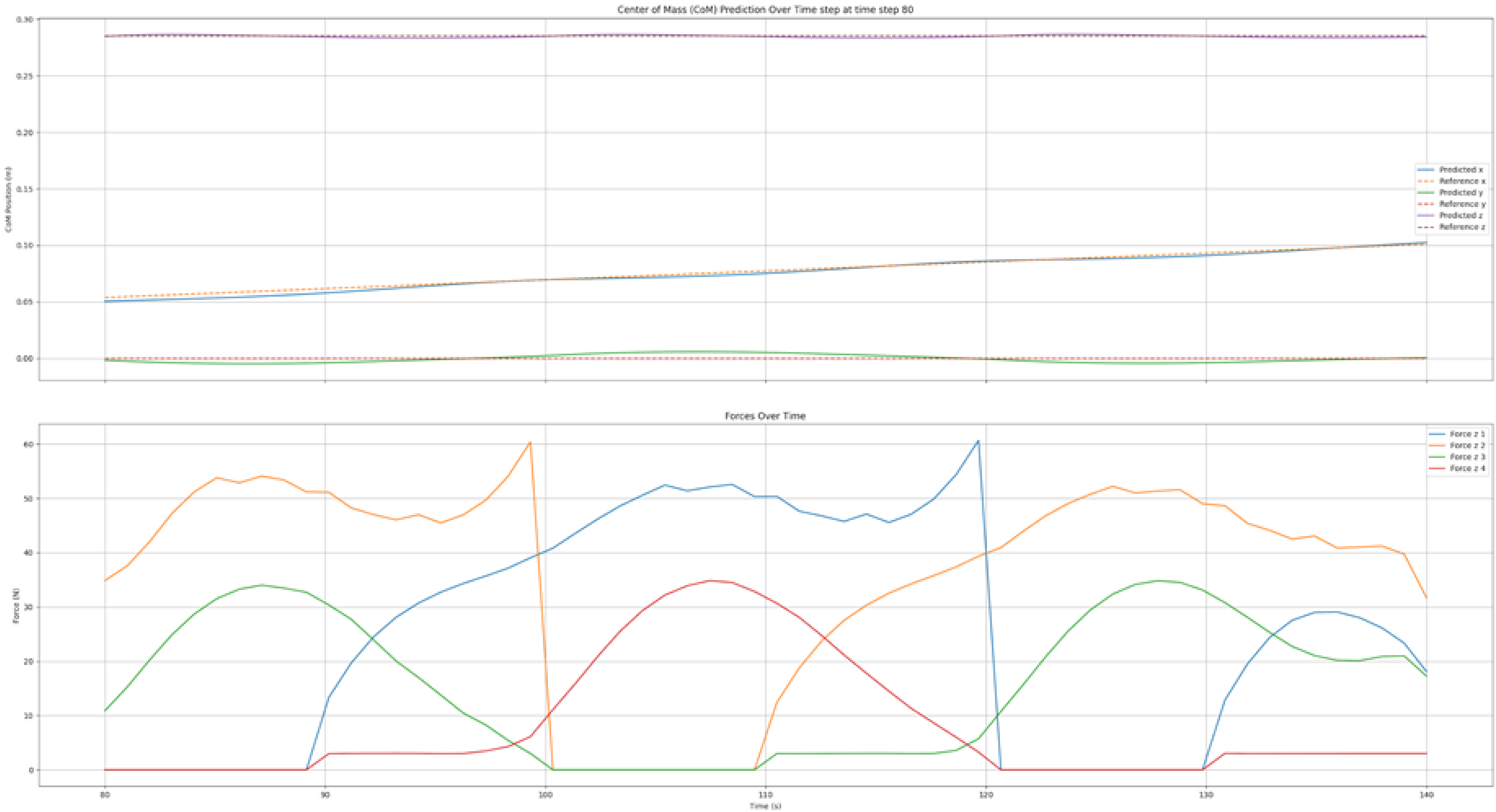
Trotting gait - FL foot x component

Results



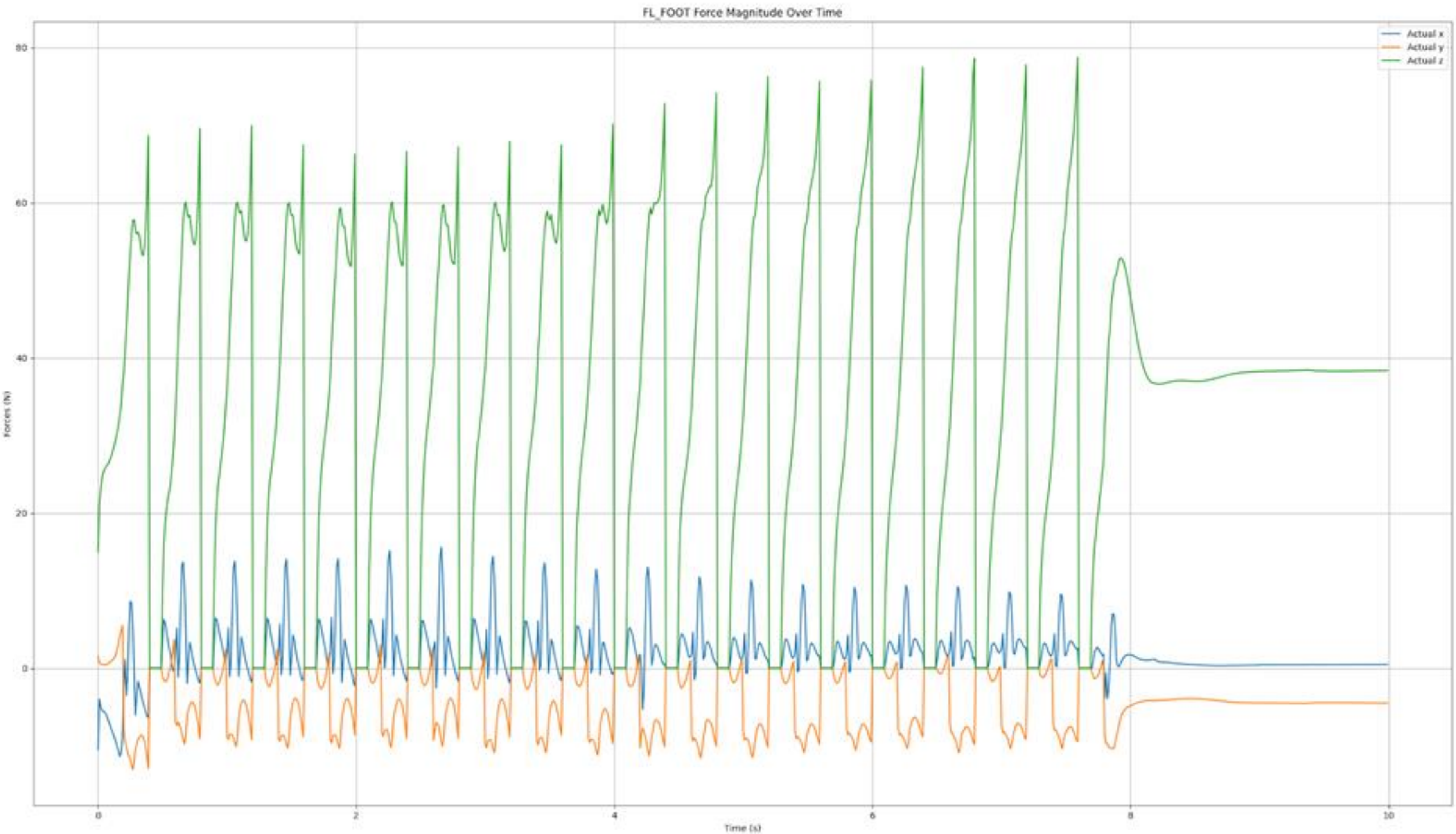
Trotting gait - MPC prediction at timestep 80

Results



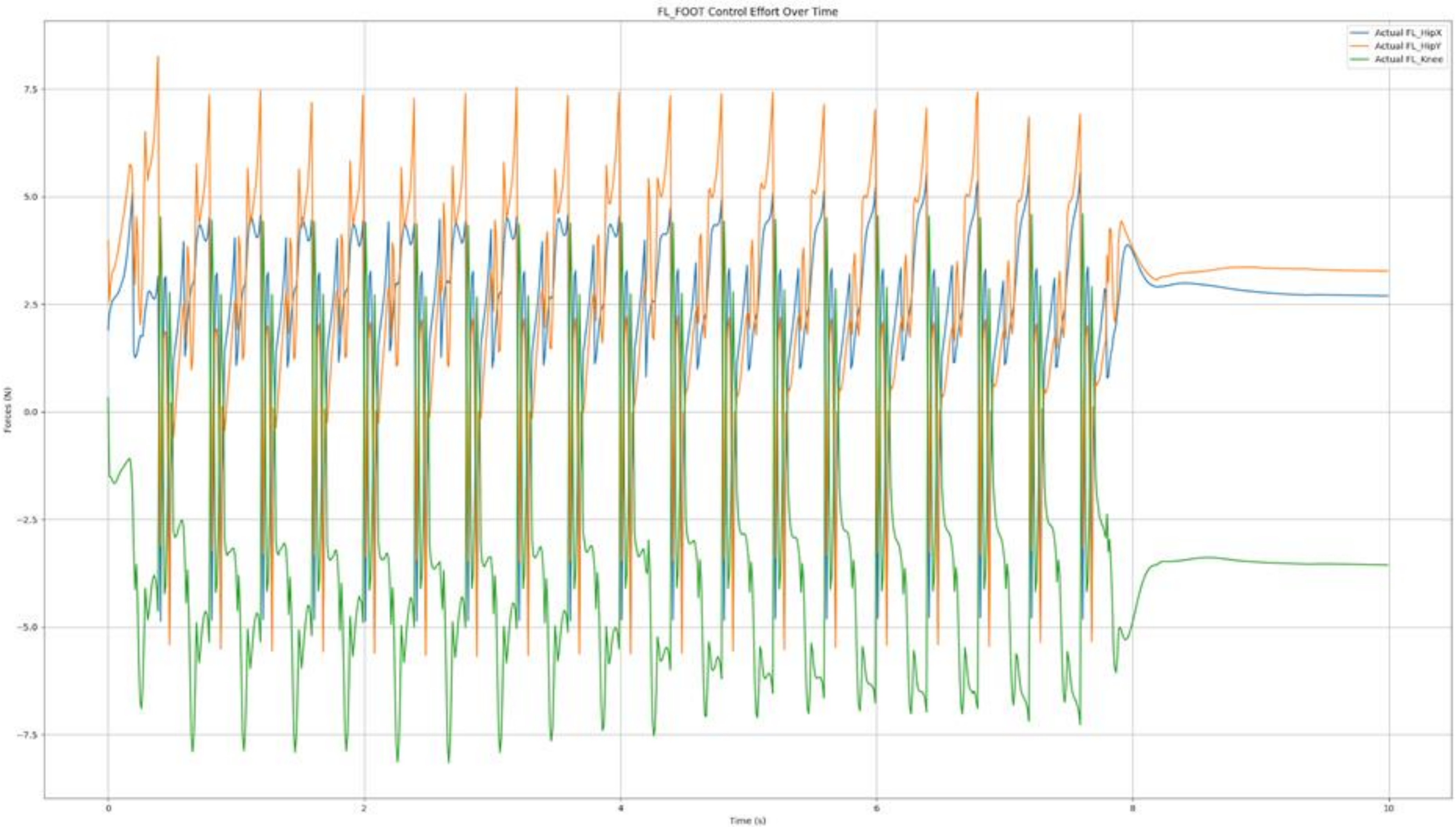
Trotting gait - FL foot forces

Results



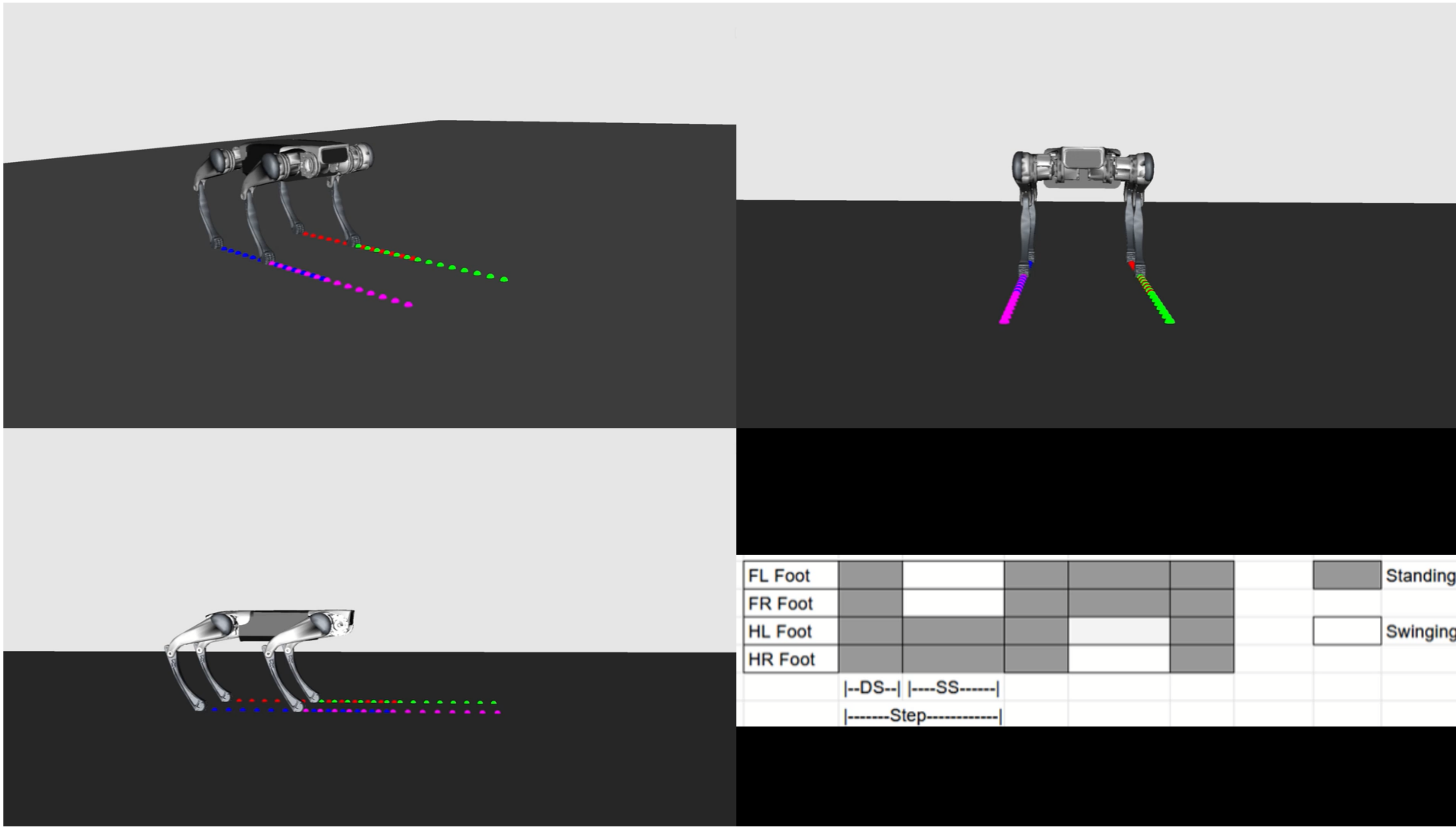
Trotting gait - FL leg control effort

Results



Pesudo-gallopping gait - Video

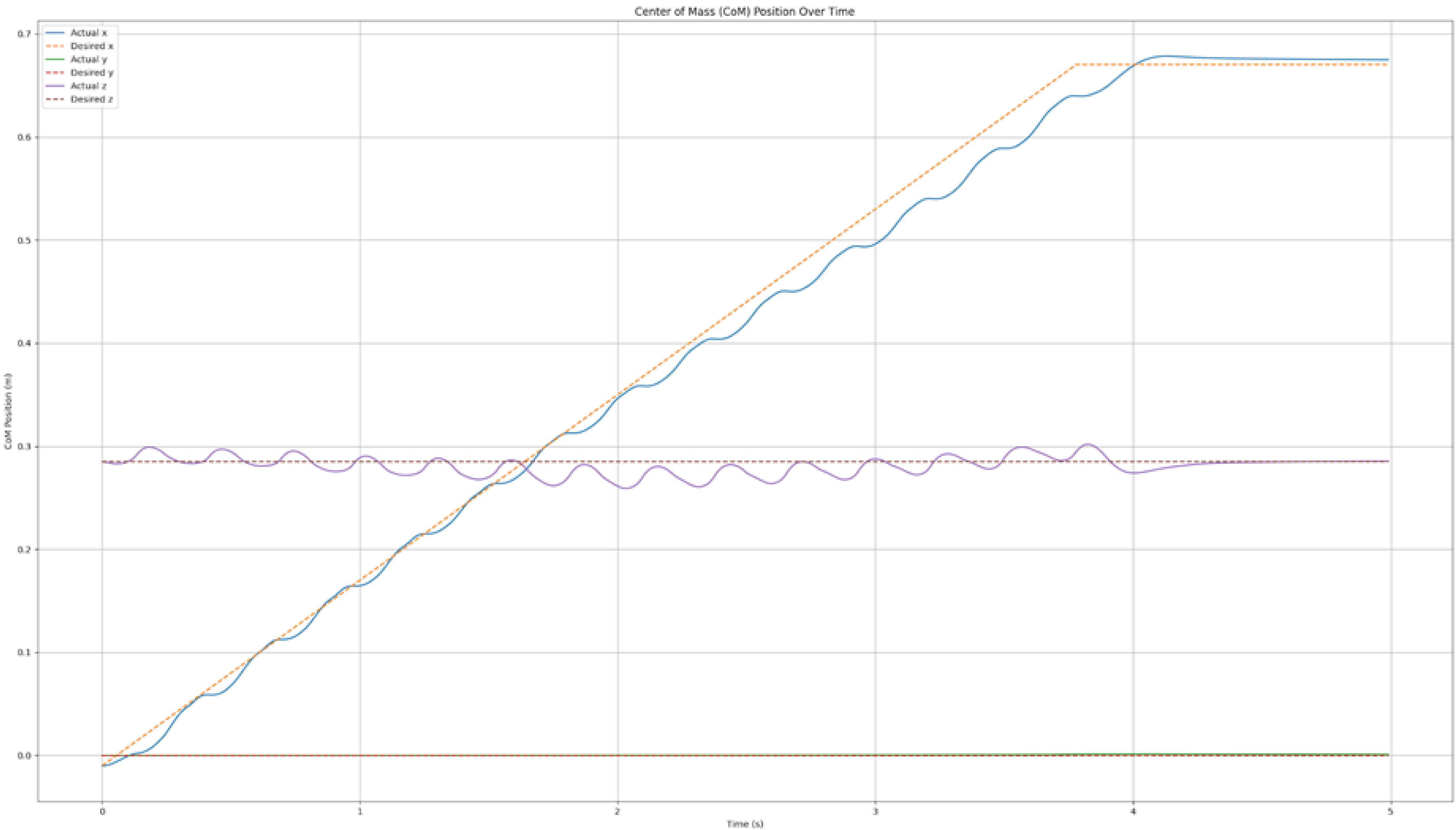
Results



$$ss\_duration = 0.1s, \quad ds\_duration = 0.04s, \quad v\_com\_ref\_x = 0.18m/s$$

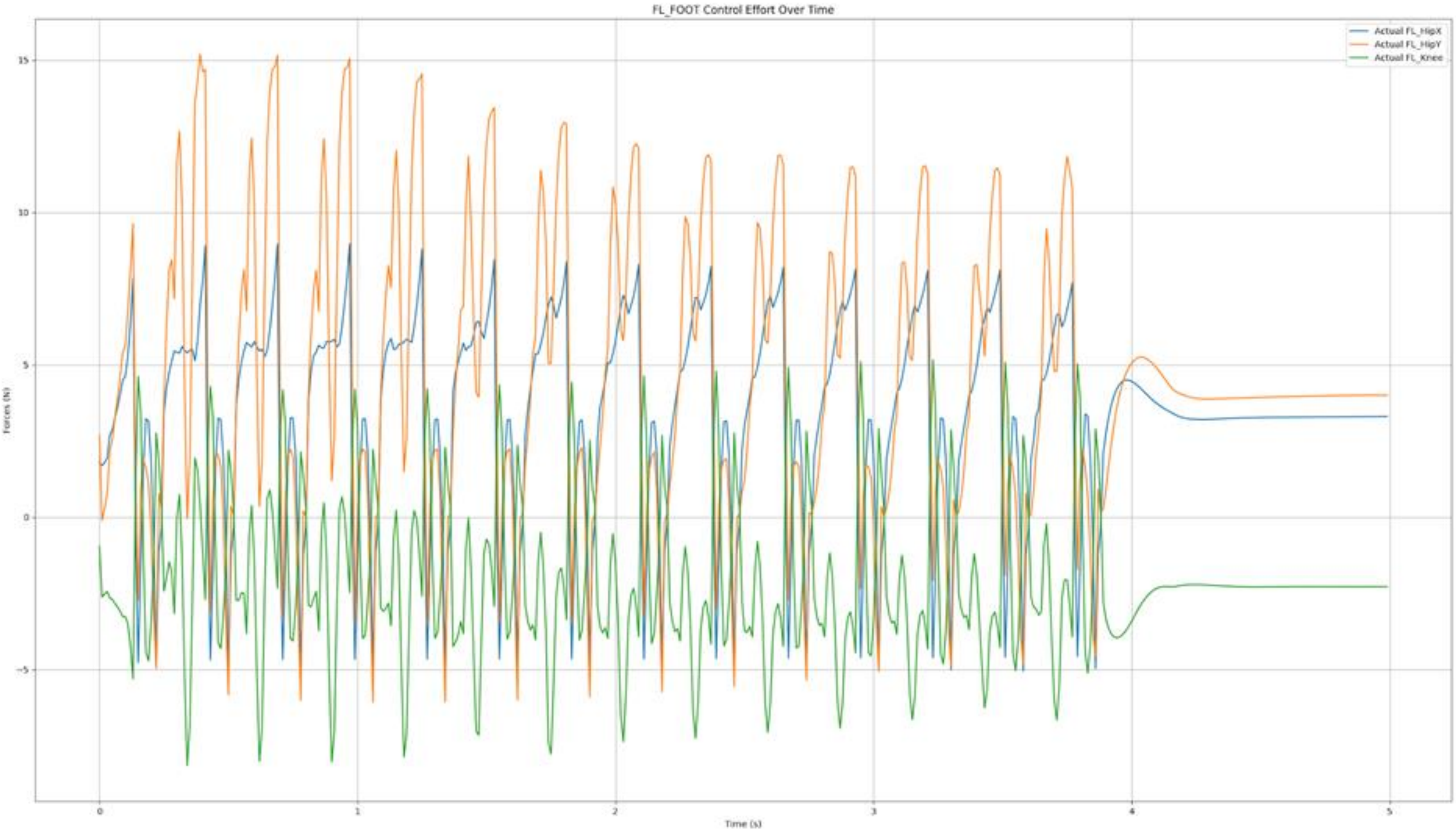
Pesudo-gallopping gait - Position of CoM

Results



Pesudo-gallopping gait - FL leg control effort

Results



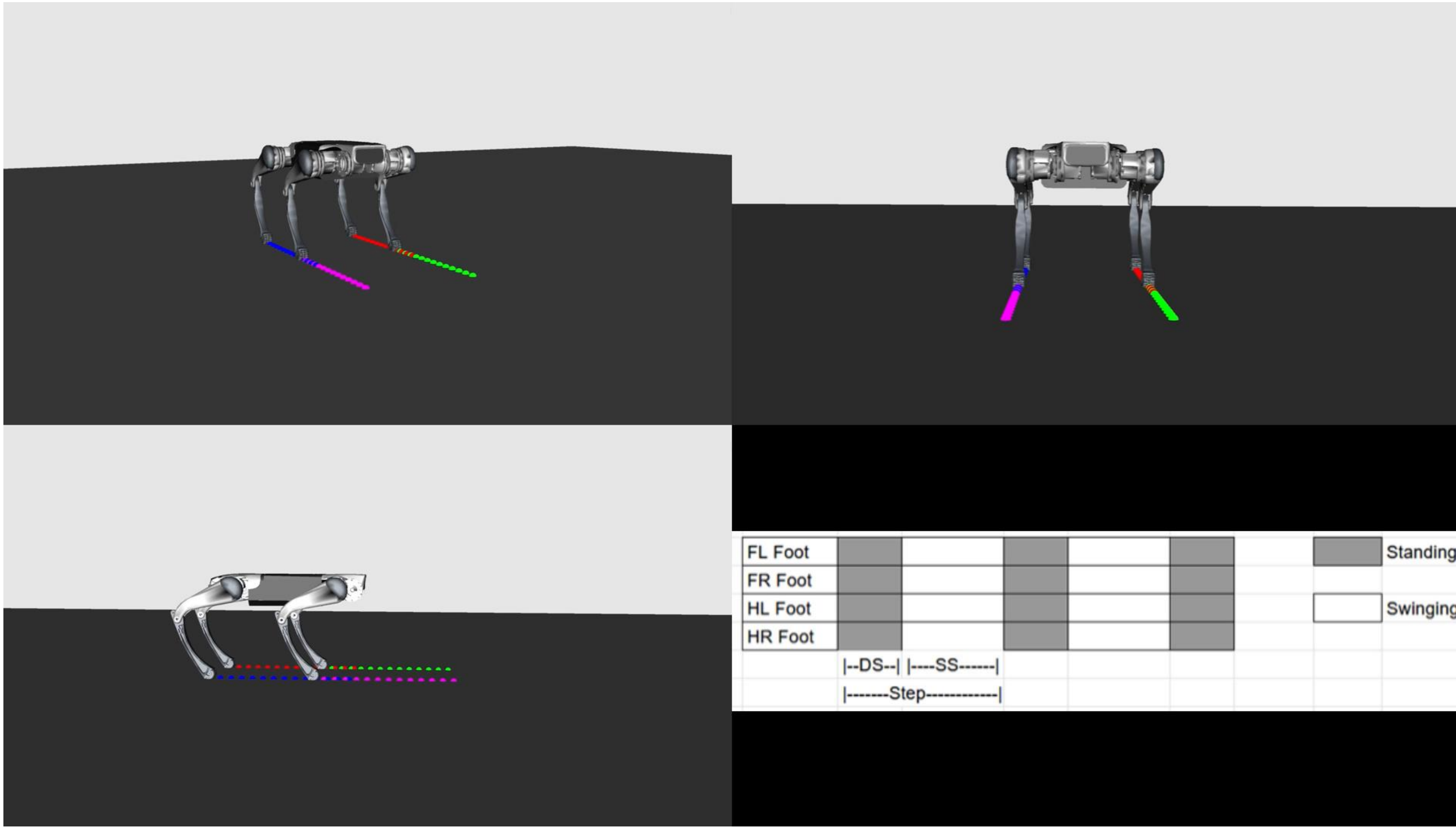
Ambling gait - Video

Results

$ss\_duration = 0.1s$, $\quad$ ds $\_duration = 0.1s$, $\quad v\_com\_ref\_x = 0.08m/s$

Pronking gait - Video

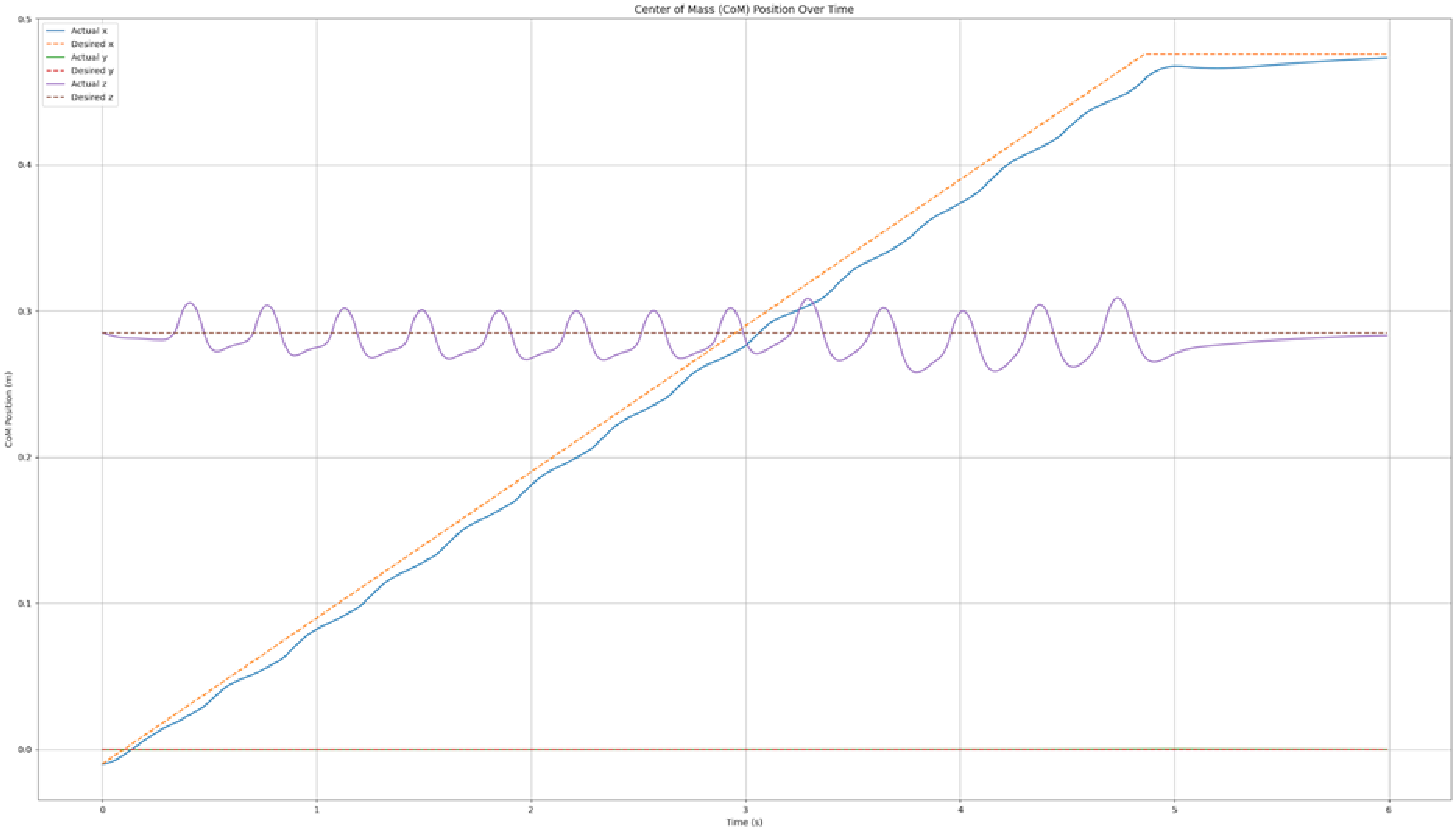
Results



$ss\_duration = 0.1s$, $\quad$ $ds\_duration = 0.08s$, $\quad$ $v\_com\_ref\_x = 0.1m/s$

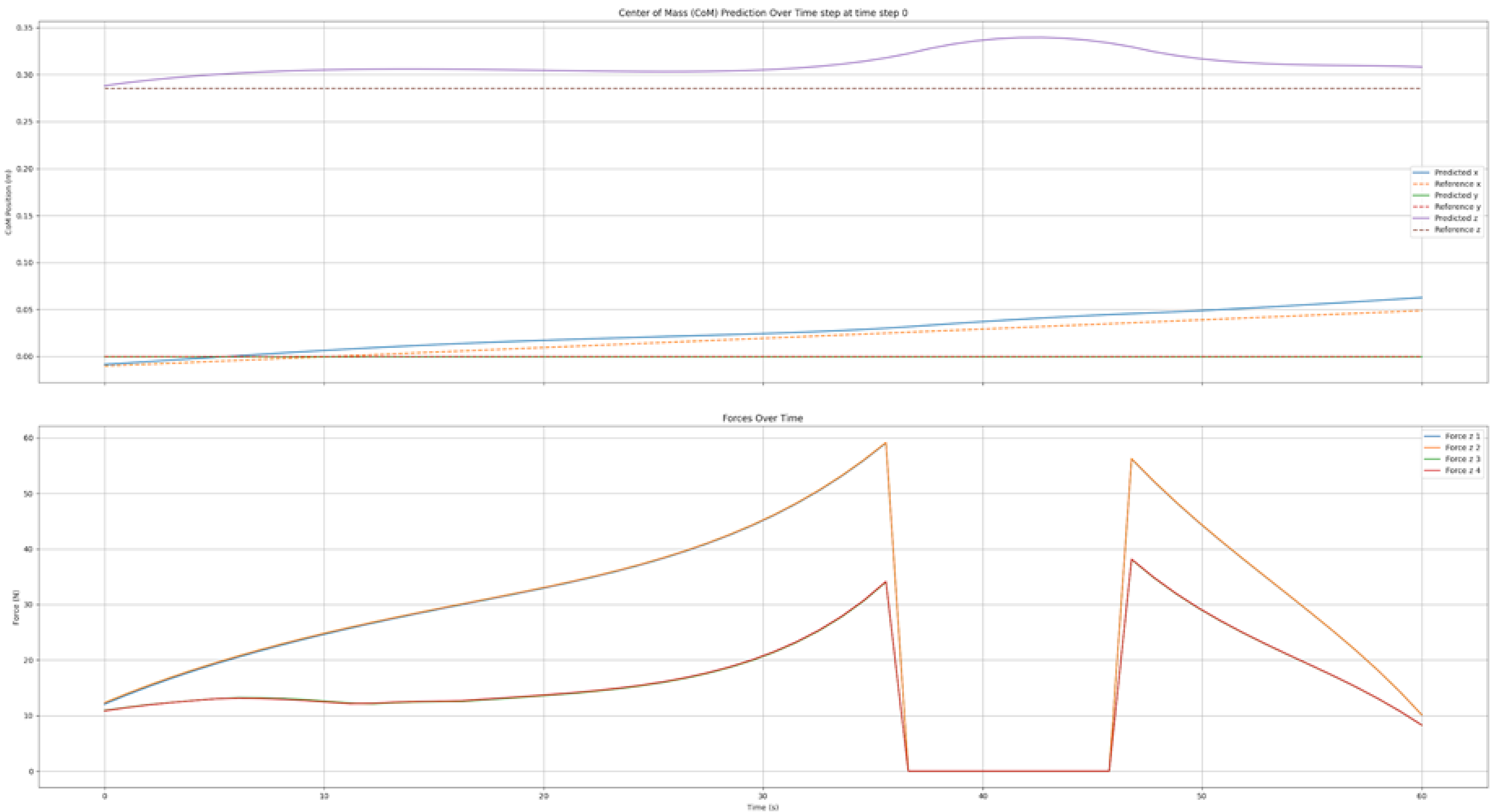
Pronking gait - Position of CoM

Results



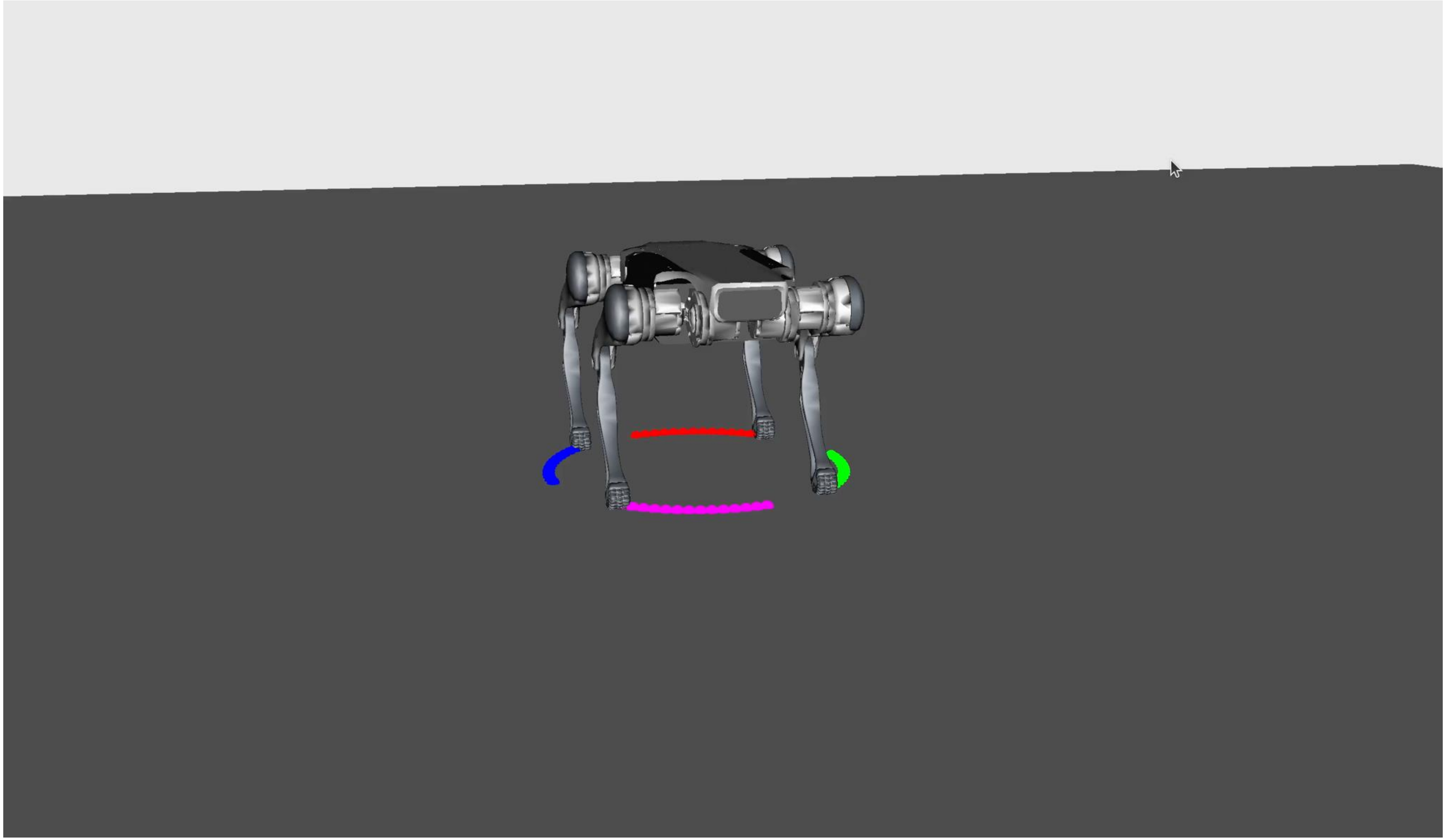
Pronking gait - MPC prediction at timestep 0

Results



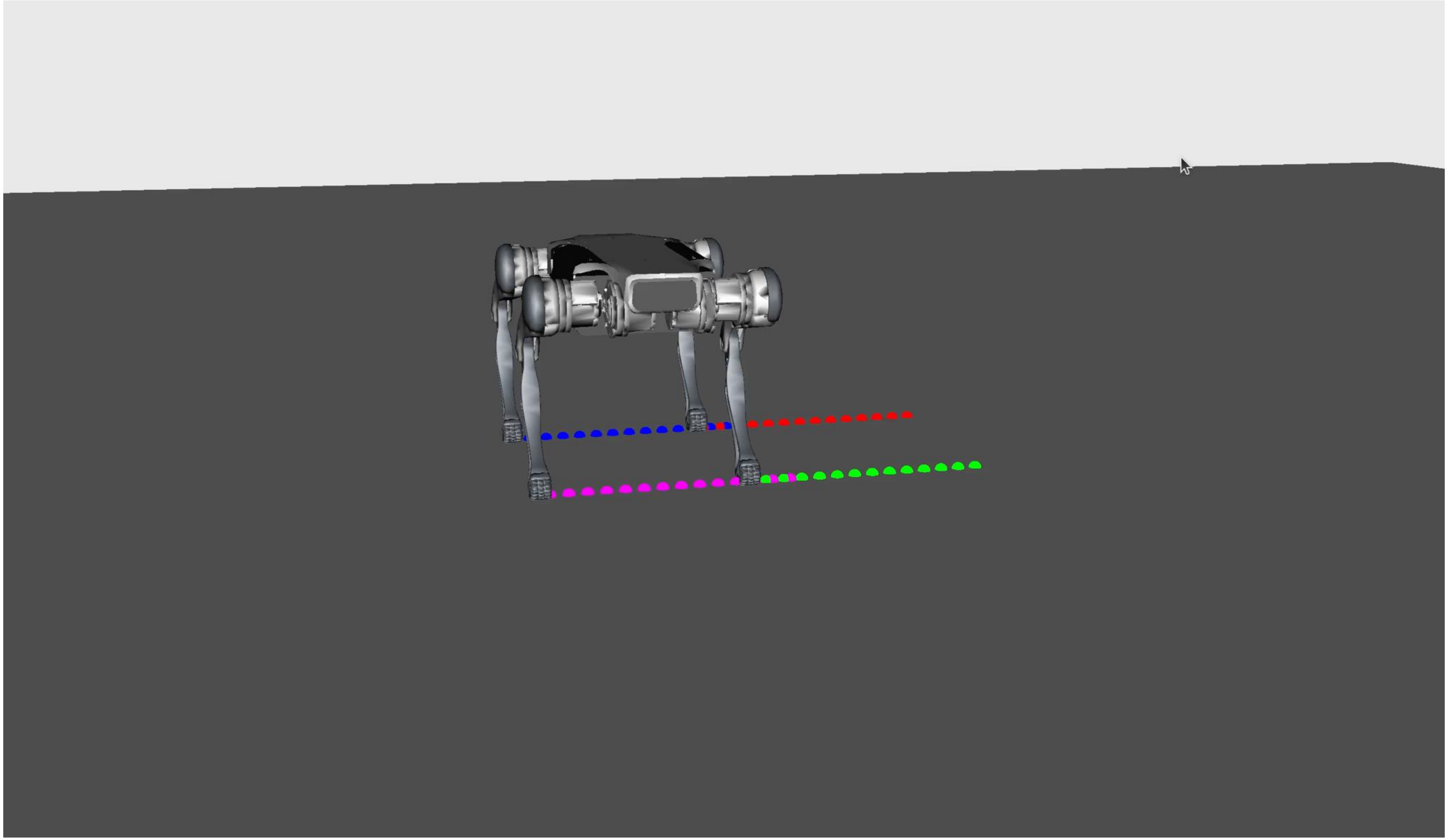
Spinning in place

Results



Side-trotting

Results



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Conclusion

Conclusion

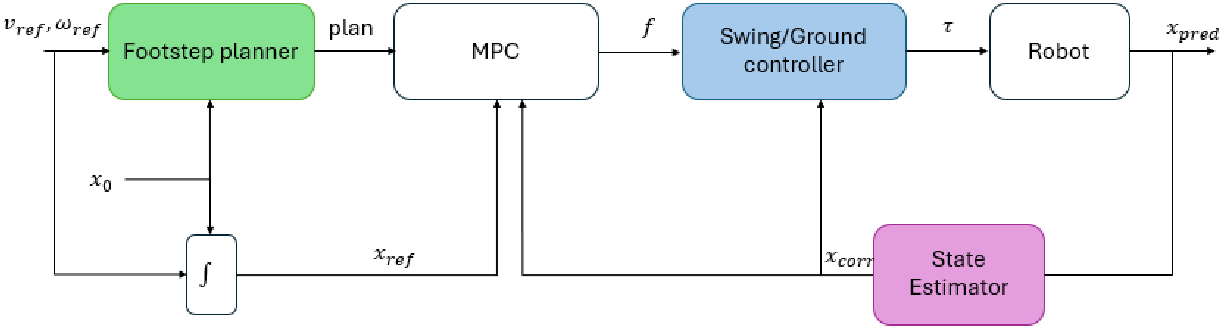
- **complete locomotion control pipeline**: implemented and validated Model Predictive Control for the Lite 3 quadruped robot
- **simulation handling of the problem**: effectively addressed the issue of ground penetration
- **integrated key components**: footstep planner, trajectory generator, ground and swing leg controllers
- **MPC convergence**: demonstrated successful convergence in simulation, with valid ground reaction forces enabling trajectory tracking.
- **smooth and stable gaits**: achieved reasonably smooth and stable locomotion with different gaits

Future works

Conclusion

Solid foundation for future work: framework sets the stage for real-world deployment and extensions to complex gaits or terrains. Some examples are:

- **multi-phase gait planner** for able the robot to handle different and more complex gaits
- **on-line planning** for real-time control
- **contact detection algorithm** for early or late contacts
- **state estimator** for real-time application



Thanks for your attention!

References

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