Robotics 2 project

Robust tracking control based on bounds on dynamic coefficients

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Abstract

In this work, we aim to design a robust control law for tracking a periodic joint space trajectory for a 3R spatial manipulator, based on bounds on its dynamic coefficients. To achieve this, we will derive the dynamic model and extract a linear parameterization in terms of a minimal set of dynamic coefficients. We will then apply robust control theory to the case in exam. Additionally, we will conduct simulations to evaluate the performance of our designed control law, highlighting the main benefits of robust control in contrast to a classic control law such as feedback linearization under both ideal and uncertain conditions.

Contents

1 Introduction

Manipulators frequently operate in dynamic environments where conditions are subject to change, leading to potential imprecision or incompleteness in their dynamic models. In such scenarios, robust control becomes crucial to ensure that the manipulator maintains desired performance despite uncertainties and disturbances. The dynamic coefficients of a robot can vary due to changes in the system, such as component wear or additional payloads. Robust control methods that utilize dynamic coefficients can adapt to these variations without necessitating constant reparametrization, thereby enhancing system reliability and performance. For manipulators subject to relatively predictable ranges of uncertainties that impact dynamic coefficients, implementing a robust control law can be highly effective. This approach is designed to deliver consistent performance even in the presence of external disturbances. Historically, robust control approaches have required uncertainty bounds that depend on both inertia parameters and the reference trajectory or state vector of the manipulator. This dependence often complicates the precise calculation of uncertainty bounds. Additionally, many previous methods have relied on assumptions about the "closeness in norm" of the computed inertia matrix to the actual inertia matrix, which can be restrictive and impractical. Our approach builds on the foundational work of M. Spong [1], who advanced robust control theory for manipulators by leveraging the skew-symmetry property and linear parameterizability of robot dynamics. By focusing solely on the robot's dynamic coefficients, our control law eliminates the restrictive assumptions of earlier methods and offers a simplified design. This report presents the implementation of a robust tracking control law based on dynamic coefficients, aimed at ensuring accurate trajectory tracking despite variations in the dynamic coefficients of the robot.

2 Robust Control Theory

We show here the basis of the theory used in this work to design a robust control law with bounds over the uncertainties of the dynamic coefficients exploiting the properties of linear parametrizability of the Robot dynamic model:

$$u = M(q,\dot{q})\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = Y_\pi(q,\dot{q},\ddot{q})\pi \quad \text{with } q \in \mathbb{R}^N; u \in \mathbb{R}^N \tag{1}$$

Where π is the vector of dimension $10N \times 1$ of dynamic parameters but in the following we will refer to a formulation of the problem in terms of a minimal set of dynamic coefficients θ of dimension $p \times 1$ with $p << 10N$ with $Y(q,\dot{q},\ddot{q})$ the $N \times p$ regressor matrix, and the property of the special factorization for $S(q,\dot{q})$ such that $\dot{M} - 2S(q,\dot{q})$ is skew symmetric.

In order to design a robust control law, firstly, we need to assume that the uncertainties are only on the dynamic coefficients and they are bounded in norm, or equivalently there exist $\theta_0 \in \mathbb{R}^p$ and $\rho \in \mathbb{R}^+$ such that:

$$\|\Delta\theta\| = \|\theta - \theta_0\| \leq \rho \tag{2}$$

Where θ is the actual parameter vector of the system, these are the true parameters that define the system's behavior but are typically not known precisely, θ_0 is the nominal parameter vector, these are the estimated or assumed values of the parameters that are used in the design of the control system. They represent the best guess or the baseline values around which the actual parameters vary.

We consider now a "nominal" control vector based on the adaptive algorithm of Slotine and Li [2] as:

$$u_0 = M_0(q)a + S_0(q,\dot{q})v + g_0(q) - Kr = Y(q,\dot{q},v,a)\theta_0 - Kr \tag{3}$$

With $\bar{q} = q - q^d$; $r = \dot{\bar{q}} + \Lambda\bar{q}$; $v = \dot{q}^d - \Lambda\bar{q}$; $a = \dot{v}$, q_d is a given twice continuously differentiable reference trajectory and the gain matrices K and Λ are positive definite (diagonal) matrices. We note here that the nominal control vector u_0 is defined in terms of fixed parameters given by θ_0. That makes the difference with an indirect adaptive control method where parameters are updated over time, potentially leading to issues such as parameter drift. However, by using fixed parameters, we avoid these problems but must have an a priori bound on the parametric uncertainty.

Next, we define the control input u in terms of the nominal control vector u_0 and an added robust term δ as:

$$u = u_0 + Y(q,\dot{q},v,a)\delta = Y(q,\dot{q},v,a)(\theta_0 + \delta) - Kr \tag{4}$$

Where δ is an additional control input designed to achieve robustness to the parametric uncertainty of $\Delta\theta$.

Finally we get the closed loop equation:

$$M(q)r + S(q,\dot{q})r + Kr = Y(q,\dot{q},v,a)(\Delta\theta + \delta) \tag{5}$$

With the continuous robust term defined as:

$$\delta = \begin{cases} -\rho\frac{Y^T r}{|||Y^T r|||} & \text{if } |||Y^T r||| > \epsilon \\ \\ -\rho\frac{Y^T r}{\epsilon} & \text{if } |||Y^T r||| \leq \epsilon \end{cases} \tag{6}$$

with $\epsilon > 0$. The closed loop system is uniformly ultimately bounded. The proof is omitted, but can be found in [1].

It is important to note that an extension of the control law is possible, instead of using a single value ρ as the norm's bound we can have a measure of uncertainty for each parameter θ_i, separately as:

$$|\Delta\theta_i| \leq \rho_i \quad i = 1, ...p$$

With this approach we can assign different weights to the components of δ that would result, calling ξ_i the i-th component of $Y^T r$, as:

$$\delta_i = \begin{cases} -\rho_i\frac{\xi_i}{|\xi_i|} & \text{if } |\xi_i| > \epsilon_i \\ \\ -\rho_i\frac{\xi_i}{\epsilon_i} & \text{if } |\xi_i| \leq \epsilon_i \end{cases} \tag{7}$$

But in the following we won't adopt this strategy and we will refer only to 6.

3 Robust tracking control for 3R spatial Robot

For this study, we will analyze a 3R Spatial Robot, as shown in Figure 1, with the DH parameters provided in Figure 2. We will derive the dynamic model of the manipulator with no special assumptions on the distribution of link masses (CoM location, link inertia matrices), excluding friction and elastic forces to simplify the calculations. However, it is important to note that a more accurate model can be obtained by incorporating viscous friction and stiction forces and joint elasticity.

The resulting dynamic model will be derived using a linear parametrization in terms of a minimal set of dynamic coefficients.

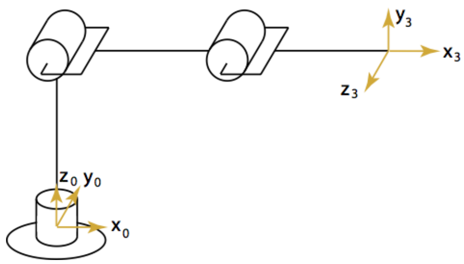

Figure 1: 3R Spatial Robot

Joint i	α_i	a_i	d_i	q_i
1	$\frac{\pi}{2}$	0	l_1	q_1
2	0	l_2	0	q_2
3	0	l_3	0	q_3

Figure 2: Denavit-Hartenberg Parameters

Assuming the gravity vector is $g = \begin{pmatrix} 0 \\ 0 \\ -g_0 \end{pmatrix}$ and the position of center of mass and inertia matrix for each link, with respect to the frame attached to link i, are given as :

$$ {}^i r_{ci} = \begin{pmatrix} {}^i r_{cxi} \\ {}^i r_{cyi} \\ {}^i r_{czi} \end{pmatrix}, \quad {}^i I_i = \begin{pmatrix} {}^i I_{xxi} & {}^i I_{xyi} & {}^i I_{xzi} \\ {}^i I_{xyi} & {}^i I_{yyi} & {}^i I_{yzi} \\ {}^i I_{xzi} & {}^i I_{yzi} & {}^i I_{zzi} \end{pmatrix} \quad \text{for } i = 1, ..., 3 $$

3.1 Derivation of the dynamic model

We derive now the kinetic energies for each link using the moving frames algorithm for dynamics:

$$ \omega_1 = \begin{pmatrix} 0 \\ \dot{q}_1 \\ 0 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad {}^1 v_{c1} = \begin{pmatrix} \dot{q}_1 r_{cz1} \\ 0 \\ -\dot{q}_1 r_{cx1} \end{pmatrix} $$

$$ T_1 = \frac{1}{2} m_1 \dot{q}_1^2 (r_{cx1}^2 + r_{cz1}^2) + \frac{1}{2} I_{yy1} \dot{q}_1^2 $$

$$ \omega_2 = \begin{pmatrix} \dot{q}_1 s_2 \\ \dot{q}_1 c_2 \\ \dot{q}_2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ l_2 \dot{q}_2 \\ -l_2 \dot{q}_1 c_2 \end{pmatrix}, \quad {}^2 v_{c2} = \begin{pmatrix} \dot{q}_1 r_{cz2} c_2 - \dot{q}_2 r_{cy2} \\ (l_2 + r_{cx2}) \dot{q}_2 s_2 - \dot{q}_1 r_{cz2} s_2 \\ -\dot{q}_1 ((l_2 + r_{cx2}) c_2 - r_{cy2} s_2) \end{pmatrix} $$

$$ \begin{aligned} T_2 = \frac{1}{2} m_2 [\dot{q}_1^2 r_{cz2}^2 + \dot{q}_2^2 ((l_2 + r_{cx2})^2 + r_{cy2}^2) - 2 \dot{q}_1 \dot{q}_2 r_{cz2} (r_{cy2} c_2 + (l_2 + r_{cx2}) s_2) + \dot{q}_1^2 [(l_2 + r_{cx2})^2 c_2^2 + r_{cy2}^2 s_2^2 \\ -2(l_2 + r_{cx2}) r_{cy2} c_2 s_2)]] + \frac{1}{2} [\dot{q}_1^2 (s_2^2 I_{xx2} + c_2^2 I_{yy2} + 2 c_2 s_2 I_{xy2}) + 2 \dot{q}_1 \dot{q}_2 (I_{xz2} s_2 + I_{yz2} c_2) + \dot{q}_2^2 I_{zz2}] \end{aligned} $$

$$ \omega_3 = \begin{pmatrix} \dot{q}_1 s_{23} \\ \dot{q}_1 c_{23} \\ \dot{q}_2 + \dot{q}_3 \end{pmatrix}, \quad {}^2 v_{c3} = \begin{pmatrix} \dot{q}_2 l_2 s_3 - r_{cy3} (\dot{q}_2 + \dot{q}_3) + \dot{q}_1 r_{cz3} c_{23} \\ r_{cx3} (\dot{q}_2 + \dot{q}_3) + l_3 (\dot{q}_2 + \dot{q}_3) + l_2 \dot{q}_2 c_3 - \dot{q}_1 r_{cz3} s_{23} \\ -\dot{q}_1 ((l_3 + r_{cx3}) c_{23} - r_{cy3} s_{23} + l_2 c_2) \end{pmatrix} $$

$$T_3 = \frac{1}{2} m_3[\dot{q}_1^2 r_{cz3}^2 + l_2 \dot{q}_2^2 + (\dot{q}_2 + \dot{q}_3)^2((l_3 + r_{cx3})^2 + r_{cy3}^2) + 2\dot{q}_1 \dot{q}_2 l_2 r_{cz3}(s_3 c_{23} - c_3 s_{23})$$

$$- 2\dot{q}_1(\dot{q}_2 + \dot{q}_3) r_{cz3}(r_{cy3} c_{23} + (l_3 + r_{cx3}) s_{23}) + 2\dot{q}_2(\dot{q}_2 + \dot{q}_3) l_2((l_3 + r_{cy3}) c_3 - r_{cy3} s_3)$$

$$+ \dot{q}_1^2((l_3 + r_{cx3}) c_{23} - r_{cy3} s_{23} + l_2 c_2)^2] + \frac{1}{2}[\dot{q}_1^2(I_{xx3} s_{23}^2 + I_{yy3} c_{23}^2 + 2I_{xy3} s_{23} c_{23}) + 2\dot{q}_1(\dot{q}_2 + \dot{q}_3)(I_{xz3} s_{23} + I_{yz3} c_{23})$$

$$+ (\dot{q}_2 + \dot{q}_3)^2 I_{zz3}]$$

From this we can extract the 3×3 Inertia matrix whose components are:

$$m_{11} = m_1(r_{cx1}^2 + r_{cz1}^2) + I_{yy1} + m_2 r_{cz2}^2 + m_2[(l_2 + r_{cx2})^2 c_2^2 + r_{cy2}^2 s_2^2 - 2(l_2 + r_{cx2}) r_{cy2} c_2 s_2] + I_{xx2} s_2^2 + I_{yy2} c_2^2 + 2I_{xy2} c_2 s_2 +$$

$$+ m_3 r_{cz3}^2 + m_3[(l_3 + r_{cx3})^2 c_{23}^2 + r_{cy3}^2 s_{23}^2 - 2(l_3 + r_{cx3}) r_{cy3} c_{23} s_{23} + l_2^2 c_2^2 + 2(l_3 + r_{cx3}) l_2 c_2 c_{23} - 2r_{cy3} l_2 c_2 s_{23}] +$$

$$+ I_{xx3} s_{23}^2 + I_{yy3} c_{23}^2 + 2I_{xy3} s_{23} c_{23} = \theta_1 + \theta_2 s_2^2 + \theta_3 s_2 c_2 + \theta_4 s_{23}^2 + \theta_5 s_{23} c_{23} + 2\theta_6 c_2 s_{23} + 2\theta_7 c_2 c_{23}$$

$$m_{12} = -m_2 r_{cz2}(r_{cy2} c_2 + (l_2 + r_{cx2}) s_2) + I_{xz2} s_2 + I_{yz2} c_2 - m_3 r_{cz3} l_2 s_2 - m_3 r_{cz3}(r_{cy3} c_{23} + (l_3 + r_{cx3}) s_{23}) +$$

$$+ I_{xz3} s_{23} + I_{yz3} c_{23} = \theta_8 c_2 + \theta_9 s_2 + \theta_{10} c_{23} + \theta_{11} s_{23}$$

$$m_{13} = -m_3 r_{cz3}(r_{cy3} c_{23} + (l_3 + r_{cx3}) s_{23}) + I_{xz3} s_{23} + I_{yz3} c_{23} = \theta_{10} c_{23} + \theta_{11} s_{23}$$

$$m_{22} = m_2((l_2 + r_{cx2})^2 + r_{cy2}^2) + I_{zz2} + m_3(l_2^2 + (l_3 + r_{cx3})^2 + r_{cy3}^2) + 2m_3 l_2((l_3 + r_{cx3}) c_3 - r_{cy3} s_3) + I_{zz3} = \theta_{12} + 2\theta_6 s_3 + 2\theta_7 c_3$$

$$m_{23} = m_3((l_3 + r_{cx3})^2 + r_{cy3}^2) + m_3 l_2((l_3 + r_{cx3}) c_3 - r_{cy3} s_3) + I_{zz3} = \theta_{13} + \theta_6 s_3 + \theta_7 c_3$$

$$m_{33} = m_3((l_3 + r_{cx3})^2 + r_{cy3}^2) + I_{zz3} = \theta_{13}$$

$$M = \begin{pmatrix} \theta_1 + \theta_2 s_2^2 + \theta_3 s_2 c_2 + \theta_4 s_{23}^2 + \theta_5 s_{23} c_{23} + 2\theta_6 c_2 s_{23} + 2\theta_7 c_2 c_{23} & \theta_8 c_2 + \theta_9 s_2 + \theta_{10} c_{23} + \theta_{11} s_{23} & \theta_{10} c_{23} + \theta_{11} s_{23} \\ \theta_8 c_2 + \theta_9 s_2 + \theta_{10} c_{23} + \theta_{11} s_{23} & \theta_{12} + 2\theta_6 s_3 + 2\theta_7 c_3 & \theta_{13} + \theta_6 s_3 + \theta_7 c_3 \\ \theta_{10} c_{23} + \theta_{11} s_{23} & \theta_{13} + \theta_6 s_3 + \theta_7 c_3 & \theta_{13} \end{pmatrix}$$

Coriolis and centrifugal terms are computed as:

$$c_i(q, \dot{q}) = \dot{q}^T C_i(q) \dot{q}, \qquad C_i(q) = \frac{1}{2}\left(\frac{\partial M_i}{\partial q} + \left(\frac{\partial M_i}{\partial q}\right)^T - \frac{\partial M}{\partial q_I}\right)$$

$$C_1(q) = \frac{1}{2}\left[\begin{pmatrix} 0 & \begin{matrix} 2\theta_2 c_2 s_2 + \theta_3(c_2^2 - s_2^2) + 2\theta_4 c_{23} s_{23} + \theta_5(c_{23}^2 - s_{23}^2) \\ +2\theta_6(c_2 c_{23} - s_2 s_{23}) - 2\theta_7(s_2 c_{23} + c_2 s_{23}) \end{matrix} & 2\theta_4 c_{23} s_{23} + \theta_5(c_{23}^2 - s_{23}^2) + 2\theta_6 c_2 c_{23} - 2\theta_7 c_2 s_{23} \\ 0 & -\theta_8 s_2 + \theta_9 c_2 - \theta_{10} s_{23} + \theta_{11} c_{23} & -\theta_{10} s_{23} + \theta_{11} c_{23} \\ 0 & -\theta_{10} s_{23} + \theta_{11} c_{23} & -\theta_{10} s_{23} + \theta_{11} c_{23} \end{pmatrix} + (T)\right.$$

$$= \begin{pmatrix} 0 & \begin{matrix} \theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) + \theta_4 c_{23} s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) \\ +\theta_6(c_2 c_{23} - s_2 s_{23}) - \theta_7(s_2 c_{23} + c_2 s_{23}) \end{matrix} & \theta_4 c_{23} s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23} \\ & -\theta_8 s_2 + \theta_9 c_2 - \theta_{10} s_{23} + \theta_{11} c_{23} & -\theta_{10} s_{23} + \theta_{11} c_{23} \\ \text{sym} & & -\theta_{10} s_{23} + \theta_{11} c_{23} \end{pmatrix}$$

$$\Rightarrow c_1(q,\dot q) = \dot q_3(\dot q_3 + 2\dot q_2)(-\theta_{10}s_{23} + \theta_{11}c_{23}) + 2\dot q_1\dot q_3(\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23})$$

$$+\dot q_2^2(-\theta_8 s_2 + \theta_9 c_2 - \theta_{10}s_{23} + \theta_{11}c_{23}) + 2\dot q_1\dot q_2[\theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2)$$

$$+\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6(c_2c_{23} - s_2s_{23}) - \theta_7(s_2c_{23} + c_2s_{23})]$$

$$C_2(q) = \frac{1}{2}[\begin{pmatrix} 0 & -\theta_8 s_2 + \theta_9 c_2 - \theta_{10}s_{23} + \theta_{11}c_{23} & -\theta_{10}s_{23} + \theta_{11}c_{23} \\ 0 & 0 & 2\theta_6 c_3 - 2\theta_7 s_3 \\ 0 & 0 & \theta_6 c_3 - \theta_7 s_3 \end{pmatrix} + (T)$$

$$-\begin{pmatrix} \begin{matrix} 2\theta_2c_2s_2 + \theta_3(c_2^2 - s_2^2) + 2\theta_4c_{23}s_{23} + \theta_5(c_{23}^2 - s_{23}^2) \\ +2\theta_6(c_2c_{23} - s_2s_{23}) - 2\theta_7(s_2c_{23} + c_2s_{23}) \end{matrix} & -\theta_8 s_2 + \theta_9 c_2 - \theta_{10}s_{23} + \theta_{11}c_{23} & -\theta_{10}s_{23} + \theta_{11}c_{23} \\ -\theta_8 s_2 + \theta_9 c_2 - \theta_{10}s_{23} + \theta_{11}c_{23} & 0 & 0 \\ -\theta_{10}s_{23} + \theta_{11}c_{23} & 0 & 0 \end{pmatrix}]$$

$$= \begin{pmatrix} \begin{matrix} -\theta_2c_2s_2 - \frac{1}{2}\theta_3(c_2^2 - s_2^2) - \theta_4c_{23}s_{23} - \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) \\ -\theta_6(c_2c_{23} - s_2s_{23}) + \theta_7(s_2c_{23} + c_2s_{23}) \end{matrix} & 0 & 0 \\ 0 & 0 & \theta_6c_3 - \theta_7s_3 \\ 0 & \theta_6c_3 - \theta_7s_3 & \theta_6c_3 - \theta_7s_3 \end{pmatrix}$$

$$\Rightarrow c_2(q,\dot q) = \dot q_3(\dot q_3 + 2\dot q_2)(\theta_6c_3 - \theta_7s_3) - \dot q_1^2[\theta_2c_2s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) + \theta_4c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2)$$

$$+\theta_6(c_2c_{23} - s_2s_{23}) - \theta_7(s_2c_{23} + c_2s_{23})]$$

$$C_3(q) = \frac{1}{2}[\begin{pmatrix} 0 & -\theta_{10}s_{23} + \theta_{11}c_{23} & -\theta_{10}s_{23} + \theta_{11}c_{23} \\ 0 & 0 & \theta_6 c_3 - \theta_7 s_3 \\ 0 & 0 & 0 \end{pmatrix} + (T)$$

$$- \begin{pmatrix} 2\theta_4 c_{23}s_{23} + \theta_5(c_{23}^2 - s_{23}^2) + 2\theta_6 c_2 c_{23} - 2\theta_7 c_2 s_{23} & -\theta_{10}s_{23} + \theta_{11}c_{23} & -\theta_{10}s_{23} + \theta_{11}c_{23} \\ -\theta_{10}s_{23} + \theta_{11}c_{23} & 2\theta_6 c_3 - 2\theta_7 s_3 & \theta_6 c_3 - \theta_7 s_3 \\ -\theta_{10}s_{23} + \theta_{11}c_{23} & \theta_6 c_3 - \theta_7 s_3 & 0 \end{pmatrix}]$$

$$= \begin{pmatrix} -\theta_4 c_{23}s_{23} - \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) - \theta_6 c_2 c_{23} + \theta_7 c_2 s_{23} & 0 & 0 \\ 0 & -\theta_6 c_3 + \theta_7 s_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow c_3(q,\dot{q}) = -\dot{q}_1^2[\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23}] + \dot{q}_2^2(\theta_7 s_3 - \theta_6 c_3)$$

Finally we get the Coriolis and centrifugal terms vector:

$$c(q,\dot{q}) = \begin{pmatrix} \dot{q}_3(\dot{q}_3 + 2\dot{q}_2)(-\theta_{10}s_{23} + \theta_{11}c_{23}) + 2\dot{q}_1\dot{q}_3(\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23}) \\ +\dot{q}_2^2(-\theta_8 s_2 + \theta_9 c_2 - \theta_{10}s_{23} + \theta_{11}c_{23}) + 2\dot{q}_1\dot{q}_2[\theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) \\ +\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6(c_2 c_{23} - s_2 s_{23}) - \theta_7(s_2 c_{23} + c_2 s_{23})] \\ \dot{q}_3(\dot{q}_3 + 2\dot{q}_2)(\theta_6 c_3 - \theta_7 s_3) - \dot{q}_1^2[\theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) + \theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) \\ +\theta_6(c_2 c_{23} - s_2 s_{23}) - \theta_7(s_2 c_{23} + c_2 s_{23})] \\ -\dot{q}_1^2[\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23}] + \dot{q}_2^2(\theta_7 s_3 - \theta_6 c_3) \end{pmatrix}$$

From the above study we can extract a factorization $S(q,\dot{q})$ such that $c(q,\dot{q}) = S(q,\dot{q})\dot{q}$:

$$S(q,\dot{q}) =$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} \\ \begin{matrix} -\dot{q}_1[\theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) + \theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) \\ +\theta_6(c_2 c_{23} - s_2 s_{23}) - \theta_7(s_2 c_{23} + c_2 s_{23})] \end{matrix} & \dot{q}_3(\theta_6 c_3 - \theta_7 s_3) & (\dot{q}_2 + \dot{q}_3)(\theta_6 c_3 - \theta_7 s_3) \\ -\dot{q}_1[\theta_4 c_{23}s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23}] & -\dot{q}_2(\theta_6 c_3 - \theta_7 s_3) & 0 \end{pmatrix}$$

$$s_{11} = \dot{q}_2[\theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) + \theta_4 c_{23} s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6(c_2 c_{23} - s_2 s_{23}) - \theta_7(s_2 c_{23} + c_2 s_{23})]$$
$$+\dot{q}_3[\theta_4 c_{23} s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23}]$$

$$s_{12} = \dot{q}_1[\theta_2 c_2 s_2 + \frac{1}{2}\theta_3(c_2^2 - s_2^2) + \theta_4 c_{23} s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6(c_2 c_{23} - s_2 s_{23}) - \theta_7(s_2 c_{23} + c_2 s_{23})]$$
$$+\dot{q}_2(-\theta_8 s_2 + \theta_9 c_2) + (\dot{q}_2 + \dot{q}_3)(-\theta_{10} s_{23} + \theta_{11} c_{23})$$

$$s_{13} = \dot{q}_1[\theta_4 c_{23} s_{23} + \frac{1}{2}\theta_5(c_{23}^2 - s_{23}^2) + \theta_6 c_2 c_{23} - \theta_7 c_2 s_{23}] + (\dot{q}_2 + \dot{q}_3)(-\theta_{10} s_{23} + \theta_{11} c_{23})$$

With this choice for $S(q, \dot{q})$ we have the particular Skew symmetry property of:

$$\dot{M} - 2S = \begin{pmatrix} \begin{array}{r} 2\theta_2 c_2 s_2 \dot{q}_2 + \theta_3(c_2^2 - s_2^2)\dot{q}_2 + 2\theta_4 c_{23} s_{23}(\dot{q}_2 + \dot{q}_3) + \theta_5(c_{23}^2 - s_{23}^2)(\dot{q}_2 + \dot{q}_3) \\ +2\theta_6(c_2 c_{23}(\dot{q}_2 + \dot{q}_3) - s_2 s_{23}\dot{q}_2) - 2\theta_7(s_2 c_{23}\dot{q}_2 + c_2 s_{23}(\dot{q}_2 + \dot{q}_3)) \end{array} & (-\theta_8 s_2 + \theta_9 c_2)\dot{q}_2 + (-\theta_{10} s_{23} + \theta_{11} c_{23})(\dot{q}_2 + \dot{q}_3) & (-\theta_{10} s_{23} + \theta_{11} c_{23})(\dot{q}_2 + \dot{q}_3) \\ & (2\theta_6 c_3 - 2\theta_7 s_3)\dot{q}_3 & (\theta_6 c_3 - \theta_7 s_3)\dot{q}_3 \\ \text{sym} & & 0 \end{pmatrix} - 2S$$

$$= \begin{pmatrix} 0 & \begin{array}{r} -\dot{q}_1[2\theta_2 c_2 s_2 + \theta_3(c_2^2 - s_2^2) + 2\theta_4 c_{23} s_{23} \\ +\theta_5(c_{23}^2 - s_{23}^2) + 2\theta_6(c_2 c_{23} - s_2 s_{23}) - 2\theta_7(s_2 c_{23} + c_2 s_{23})] \\ -\dot{q}_2[-\theta_8 s_2 + \theta_9 c_2] - (\dot{q}_2 + \dot{q}_3)[-\theta_{10} s_{23} + \theta_{11} c_{23}] \end{array} & \begin{array}{r} -\dot{q}_1[2\theta_4 c_{23} s_{23} \\ +\theta_5(c_{23}^2 - s_{23}^2) + 2\theta_6 c_2 c_{23} \\ -2\theta_7 c_2 s_{23}] \\ -(\dot{q}_2 + \dot{q}_3)(-\theta_{10} s_{23} + \theta_{11} c_{23}) \end{array} \\ \begin{array}{r} +\dot{q}_1[2\theta_2 c_2 s_2 + \theta_3(c_2^2 - s_2^2) + 2\theta_4 c_{23} s_{23} + \theta_5(c_{23}^2 - s_{23}^2) \\ +2\theta_6(c_2 c_{23} - s_2 s_{23}) - 2\theta_7(s_2 c_{23} + c_2 s_{23})] \\ +\dot{q}_2[-\theta_8 s_2 + \theta_9 c_2] + (\dot{q}_2 + \dot{q}_3)[-\theta_{10} s_{23} + \theta_{11} c_{23}] \end{array} & 0 & -(2\dot{q}_2 + \dot{q}_3)(\theta_6 c_3 - \theta_7 s_3) \\ \begin{array}{r} \dot{q}_1[2\theta_4 c_{23} s_{23} + \theta_5(c_{23}^2 - s_{23}^2) + 2\theta_6 c_2 c_{23} - 2\theta_7 c_2 s_{23}] \\ +(\dot{q}_2 + \dot{q}_3)(-\theta_{10} s_{23} + \theta_{11} c_{23}) \end{array} & (2\dot{q}_2 + \dot{q}_3)(\theta_6 c_3 - \theta_7 s_3) & 0 \end{pmatrix}$$

For the gravity terms , assuming g_0, l_2 known, we have:

$$U_1 = m_1 g_0 (l_1 + r_{cy1})$$

$$U_2 = m_2 g_0 (l_1 + r_{cy2} c_2 + (l_2 + r_{cx2}) s_2)$$

$$U_3 = m_3 g_0 (l_1 + r_{cy3} c_{23} + (l_3 + r_{cx3}) s_{23} + l_2 s_2)$$

$$g(q) = \begin{pmatrix} 0 \\ g_0 \left[m_2((l_2 + r_{cx2})c_2 - r_{cy2} s_2) + m_3((l_3 + r_{cx3})c_{23} - r_{cy3} s_{23} + l_2 c_2) \right] \\ g_0 m_3((l_3 + r_{cx3})c_{23} - r_{cy3} s_{23}) \end{pmatrix} = \begin{pmatrix} 0 \\ g_0 \left(\theta_{14} c_2 + \theta_{15} s_2 + \frac{\theta_7}{l_2} c_{23} + \frac{\theta_6}{l_2} s_{23} \right) \\ g_0 \left(\frac{\theta_7}{l_2} c_{23} + \frac{\theta_6}{l_2} s_{23} \right) \end{pmatrix}$$

3.1.1 Linear parametrization

Finally we have the linear parametrization of the dynamic model in terms of a minimal set of dynamic coefficients:

$$Y(q, \dot{q}, \ddot{q})\theta = u$$

With the parametrization given by

$$\theta_1 = m_1(r_{cx1}^2 + r_{cz1}^2) + I_{yy1} + m_2 r_{cz2}^2 + m_2(r_{cx2} + l_2)^2 + I_{yy2} + m_3 r_{cz3}^2 + m_3(r_{cx3} + l_3)^2 + I_{yy3} + m_3 l_2^2$$

$$\theta_2 = -m_2(r_{cx2} + l_2)^2 + m_2 r_{cy2}^2 - I_{yy2} + I_{xx2} - m_3 l_2^2$$

$$\theta_3 = -2m_2 r_{cy2}(l_2 + r_{cx2}) + 2I_{xy2}$$

$$\theta_4 = m_3 r_{cy3}^2 - m_3(r_{cx3} + l_3)^2 + I_{xx3} - I_{yy3}$$

$$\theta_5 = -2m_3 r_{cy3}(l_3 + r_{cx3}) + 2I_{xy3}$$

$$\theta_6 = -m_3 r_{cy3} l_2$$

$$\theta_7 = m_3(l_3 + r_{cx3}) l_2$$

$$\theta_8 = -m_2 r_{cy2} r_{cz2} + I_{yz2}$$

$$\theta_9 = -m_2(l_2 + r_{cx2}) r_{cz2} + I_{xz2} - m_3 r_{cz3} l_2$$

$$\theta_{10} = -m_3 r_{cy3} r_{cz3} + I_{yz3}$$

$$\theta_{11} = -m_3(l_3 + r_{cx3}) r_{cz3} + I_{xz3}$$

$$\theta_{12} = m_2(r_{cy2}^2 + (l_2 + r_{cx2})^2) + I_{zz2} + m_3(l_2^2 + r_{cy3}^2 + (l_3 + r_{cx3})^2) + I_{zz3}$$

$$\theta_{13} = m_3((l_3 + r_{cx3})^2 + r_{cy3}^2) + I_{zz3}$$

$$\theta_{14} = m_2(l_2 + r_{cx2}) + m_3 l_2$$

$$\theta_{15} = -m_2 r_{cy2}$$

Where the components y_{ij} of $Y(q, \dot{q}, \ddot{q})$ are:

$y_{11} = \ddot{q}_1$

$y_{12} = \ddot{q}_1 s_2^2 + 2\dot{q}_1\dot{q}_2 s_2 c_2$

$y_{13} = \ddot{q}_1 c_2 s_2 + \dot{q}_1\dot{q}_2(c_2^2 - s_2^2)$

$y_{14} = \ddot{q}_1 s_{23}^2 + 2\dot{q}_1(\dot{q}_2 + \dot{q}_3)s_{23}c_{23}$

$y_{15} = \ddot{q}_1 c_{23}s_{23} + \dot{q}_1(\dot{q}_2 + \dot{q}_3)(c_{23}^2 - s_{23}^2)$

$y_{16} = 2\ddot{q}_1 c_2 s_{23} + 2\dot{q}_1\dot{q}_2(c_2 c_{23} - s_2 s_{23}) + 2\dot{q}_1\dot{q}_3 c_2 c_{23}$

$y_{17} = 2\ddot{q}_1 c_2 c_{23} - 2\dot{q}_1\dot{q}_2(s_2 c_{23} + c_2 s_{23}) - 2\dot{q}_1\dot{q}_3 c_2 s_{23}$

$y_{18} = \ddot{q}_2 c_2 - \dot{q}_2^2 s_2$

$y_{19} = \ddot{q}_2 s_2 + \dot{q}_2^2 c_2$

$y_{110} = (\ddot{q}_2 + \ddot{q}_3)c_{23} - \dot{q}_2^2 s_{23} - \dot{q}_3(2\dot{q}_2 + \dot{q}_3)s_{23}$

$y_{111} = (\ddot{q}_2 + \ddot{q}_3)s_{23} + \dot{q}_2^2 c_{23} + \dot{q}_3(2\dot{q}_2 + \dot{q}_3)c_{23}$

$y_{22} = -\dot{q}_1^2 s_2 c_2$

$y_{23} = -\frac{1}{2}\dot{q}_1^2(c_2^2 - s_2^2)$

$y_{24} = -\dot{q}_1^2 s_{23}c_{23}$

$y_{25} = -\frac{1}{2}\dot{q}_1^2(c_{23}^2 - s_{23}^2)$

$y_{26} = 2\ddot{q}_2 s_3 + \ddot{q}_3 s_3 - \dot{q}_1^2(c_2 c_{23} - s_2 s_{23}) + \dot{q}_3(\dot{q}_3 + 2\dot{q}_2)c_3 + \frac{g_0}{l_2}s_{23}$

$y_{27} = 2\ddot{q}_2 c_3 + \ddot{q}_3 c_3 + \dot{q}_1^2(s_2 c_{23} + c_2 s_{23}) - \dot{q}_3(\dot{q}_3 + 2\dot{q}_2)s_3 + \frac{g_0}{l_2}c_{23}$

$y_{28} = \ddot{q}_1 c_2$

$y_{29} = \ddot{q}_1 s_2$

$y_{210} = \ddot{q}_1 c_{23}$

$y_{211} = \ddot{q}_1 s_{23}$

$y_{212} = \ddot{q}_2$

$y_{213} = \ddot{q}_3$

$y_{214} = g_0 c_2$

$y_{215} = g_0 s_2$

$y_{34} = -\dot{q}_1^2 s_{23}c_{23}$

$y_{35} = -\frac{1}{2}\dot{q}_1^2(c_{23}^2 - s_{23}^2)$

$y_{36} = \ddot{q}_2 s_3 - \dot{q}_1^2 c_2 c_{23} - \dot{q}_2^2 c_3 + \frac{g_0}{l_2}s_{23}$

$y_{37} = \ddot{q}_2 c_3 + \dot{q}_1^2 c_2 s_{23} + \dot{q}_2^2 s_3 + \frac{g_0}{l_2}c_{23}$

$y_{310} = \ddot{q}_1 c_{23}$

$y_{311} = \ddot{q}_1 s_{23}$

$y_{313} = \ddot{q}_2 + \ddot{q}_3$

$$y_{21} = y_{31} = y_{32} = y_{33} = y_{38} = y_{39} = y_{112} = y_{113} =$$
$$y_{114} = y_{115} = y_{312} = y_{314} = y_{315} = 0$$

3.2 Design of robust control law

In order to design a Robust control law for tracking a trajectory we note that the dynamic coefficients are the result of the nominal parameters plus the uncertainties:

$$\theta_i = \theta_{i0} + \Delta\theta_i \quad \text{for } i = 1, ..., 15$$

Now assuming we know the dynamic coefficients to lie only within certain interval bounds, we can estimate the upper bound of the norm of the uncertainties as:

$$\|\Delta\theta\|^2 = \sum_{i=1}^{15}(\theta_{i0} - \theta_i)^2 \leq \rho^2$$

Practically, a suitable choice for the nominal parameter vector θ_0 is the mean value within the known interval of variation of possible θ_i. This approach reduces the maximum uncertainty the controller must handle, making ρ equidistant from both bounds of the variation range. Consequently, this minimizes the controller's efforts, which increase with higher uncertainty.

We consider now the Robust control law (4) where the components y_{ij} of $Y(q,\dot{q},v,a)$ are given as:

$y_{11} = a_1$

$y_{12} = a_1 s_2^2 + (v_1\dot{q}_2 + \dot{q}_1 v_2)s_2c_2$

$y_{13} = a_1 c_2 s_2 + \frac{1}{2}(v_1\dot{q}_2 + \dot{q}_1 v_2)(c_2^2 - s_2^2)$

$y_{14} = a_1 s_{23}^2 + (v_1\dot{q}_2 + \dot{q}_1 v_2)s_{23}c_{23} + (v_1\dot{q}_3 + \dot{q}_1 v_3)s_{23}c_{23}$

$y_{15} = a_1 c_{23}s_{23} + (v_1\dot{q}_2 + \dot{q}_1 v_2)(c_{23}^2 - s_{23}^2) + (v_1\dot{q}_3 + \dot{q}_1 v_3)(c_{23}^2 - s_{23}^2)$

$y_{16} = 2a_1 c_2 s_{23} + (v_1\dot{q}_2 + \dot{q}_1 v_2)(c_2c_{23} - s_2s_{23}) + (v_1\dot{q}_3 + \dot{q}_1 v_3)c_2c_{23}$

$y_{17} = 2a_1 c_2 c_{23} - (v_1\dot{q}_2 + \dot{q}_1 v_2)(s_2c_{23} + c_2s_{23}) - (v_1\dot{q}_3 + \dot{q}_1 v_3)c_2s_{23}$

$y_{18} = a_2c_2 - \dot{q}_2 v_2 s_2$

$y_{19} = a_2s_2 + \dot{q}_2 v_2 c_2$

$y_{110} = (a_2 + a_3)c_{23} - \dot{q}_2 v_2 s_{23} - (v_2\dot{q}_3 + \dot{q}_2 v_3)s_{23} - \dot{q}_3 v_3 s_{23}$

$y_{111} = (a_2 + a_3)s_{23} + \dot{q}_2 v_2 c_{23} + (v_2\dot{q}_3 + \dot{q}_2 v_3)c_{23} + \dot{q}_3 v_3 c_{23}$

$y_{22} = -\dot{q}_1 v_1 s_2 c_2$

$y_{23} = -\frac{1}{2}\dot{q}_1 v_1 (c_2^2 - s_2^2)$

$y_{24} = -\dot{q}_1 v_1 s_{23} c_{23}$

$y_{25} = -\frac{1}{2}\dot{q}_1 v_1 (c_{23}^2 - s_{23}^2)$

$y_{26} = (2a_2 + a_3)s_3 - \dot{q}_1 v_1 (c_2c_{23} - s_2s_{23}) + (v_2\dot{q}_3 + \dot{q}_2 v_3 + \dot{q}_3 v_3)c_3 + \frac{g_0}{l_2}s_{23}$

$y_{27} = (2a_2 + a_3)c_3 + \dot{q}_1 v_1 (s_2c_{23} + c_2s_{23}) - (v_2\dot{q}_3 + \dot{q}_2 v_3 + \dot{q}_3 v_3)s_3 + \frac{g_0}{l_2}c_{23}$

$y_{28} = a_1 c_2$

$y_{29} = a_1 s_2$

$y_{210} = a_1 c_{23}$

$y_{211} = a_1 s_{23}$

$y_{212} = a_2$

$y_{213} = a_3$

$y_{214} = g_0 c_2$

$y_{215} = g_0 s_2$

$y_{34} = -\dot{q}_1 v_1 s_{23} c_{23}$

$y_{35} = -\frac{1}{2}\dot{q}_1 v_1 (c_{23}^2 - s_{23}^2)$

$y_{36} = a_2 s_3 - \dot{q}_1 v_1 c_2 c_{23} - \dot{q}_2 v_2 c_3 + \frac{g_0}{l_2}s_{23}$

$y_{37} = a_2 c_3 + \dot{q}_1 v_1 c_2 s_{23} + \dot{q}_2 v_2 s_3 + \frac{g_0}{l_2}c_{23}$

$y_{310} = a_1 c_{23}$

$y_{311} = a_1 s_{23}$

$y_{313} = a_2 + a_3$

$$y_{21} = y_{31} = y_{32} = y_{33} = y_{38} = y_{39} = y_{112} = y_{113} = y_{114} = y_{115} = y_{312} = y_{314} = y_{315} = 0$$

These values are determined by utilizing the special factorization of $c(q,\dot{q},v) = S(q,\dot{q})v$, which ensures the skew-symmetry property, in the dynamic model. In this model, the control terms r, v, a have been substituted according to (3), while the diagonal gain matrices K and Λ need to be properly tuned to achieve the desired behavior. Additionally, the robust term defined in equation (6), which depends on the regressor matrix and the bound on the norm of the uncertainties (ρ), ensures that the control action can handle variations in the dynamic coefficients within the expected range. The parameter ϵ in the robust term is a design choice that balances the trade-off between the radius ω, which defines the ultimate boundedness of the tracking error, and the level of chattering in the control signal.

3.3 Trajectory

We can use the designed control law for tracking a twice continuously differentiable periodic joint space trajectory like:

$$q_d(t) = \begin{pmatrix} sin(t) \\ 5cos(t) \\ sin(t) \end{pmatrix} \quad \dot{q}_d(t) = \begin{pmatrix} cos(t) \\ -5sin(t) \\ cos(t) \end{pmatrix} \quad \ddot{q}_d(t) = \begin{pmatrix} -sin(t) \\ -5cos(t) \\ -sin(t) \end{pmatrix}$$

The simulation results are provided in the next chapter.

4 Simulation

The simulation is conducted using the Robot parameters reported in Table 1:

$Joint_i$	m_i	l_i	r_{ci}	I_i
1	10	1	$\begin{pmatrix} 0.5 \\ -0.5 \\ -0.1 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.1 & 0.1 \\ 0.1 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.5 \end{pmatrix}$
2	10	1	$\begin{pmatrix} -0.5 \\ -0.3 \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.2 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.5 \end{pmatrix}$
3	10	1	$\begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5 & 0.2 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.5 \end{pmatrix}$

Table 1: Robot parameters

The resulting associated dynamic coefficients are:

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	θ_{15}
33	-11.6	3.4	-14	-4.4	-2	12	1.4	-3.8	-0.2	-2.2	29.2	15.3	15	3

Table 2: dynamic coefficients

Assuming that uncertainties of the standard parameters can vary only in bounded intervals:

$$m_i + \Delta m_i \quad \text{with} \quad 0 \le \Delta m_i \le 10 \quad \text{for } i = 1, ..., 3$$

$$r_{cij} + \Delta r_{cij} \quad \text{with} \quad 0 \le \Delta r_{cij} \le 1 \quad \text{for } i, j = 1, ..., 3$$

$$I_{ij} + \Delta I_{ij} \quad \text{with} \quad 0 \le \Delta I_{ij} \le 1 \quad \text{for } i = 1, ..., 6, \text{ and } j = 1, ..., 3$$

Thus, we take the nominal parameter vector as composed by the mean value of possible θ_i varying in its range:

θ_{01}	θ_{02}	θ_{03}	θ_{04}	θ_{05}	θ_{06}	θ_{07}	θ_{08}	θ_{09}	θ_{010}	θ_{011}	θ_{012}	θ_{013}	θ_{014}	θ_{015}
164.25	-33.9	-19.1	-41.5	-54.8	-13	28	-9	-34.8	-14.4	-27.4	116.3	71.2	32.5	-5.5

Table 3: nominal parameters

With this Data we have the value $||\theta_0 - \theta|| \le \rho = 186.8129$.

4.1 Overview

We provide simulation results comparing our designed control law with a classic feedback linearization control law, both under ideal and perturbed conditions, for tracking the desired joint space trajectory starting from matching conditions for a total simulation time of 10 seconds. By ideal conditions, we mean scenarios where the actual dynamic behavior match the theoretical model. In contrast, perturbed conditions refer to scenarios where the actual dynamic behavior deviate from the theoretical predictions.

Firstly we show the simulation results using a feedback linearization control law with $K_P = diag(100, 100, 100), K_D = diag(100, 100, 100)$ in ideal conditions (when the dynamic model corresponds exactly to the reality):

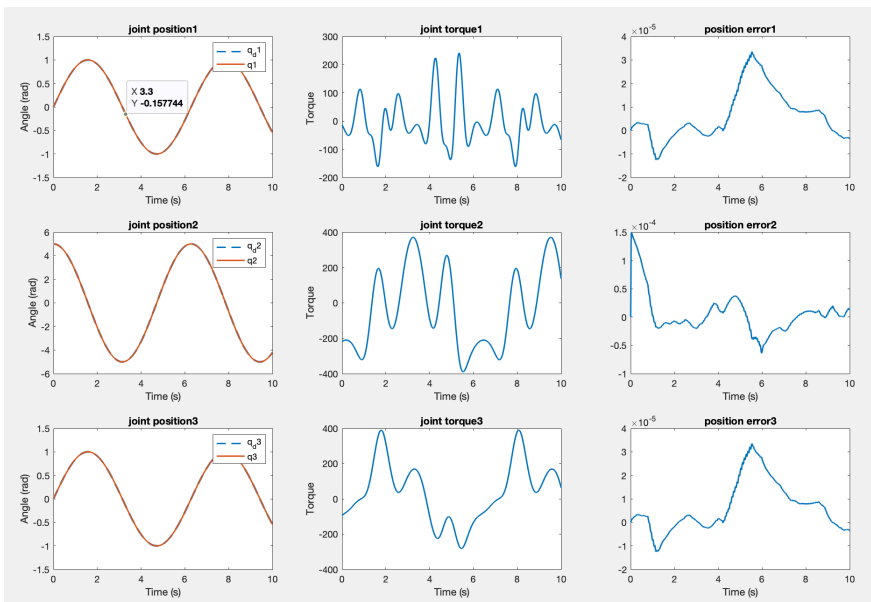

Figure 3: fbl-unperturbed

The results of our Robust control law using $\rho = 186.8129$, $K =$ diag(100,100,100), $\Lambda =$ diag(50,50,50), $\epsilon = 0.5$ in ideal conditions are as follows:

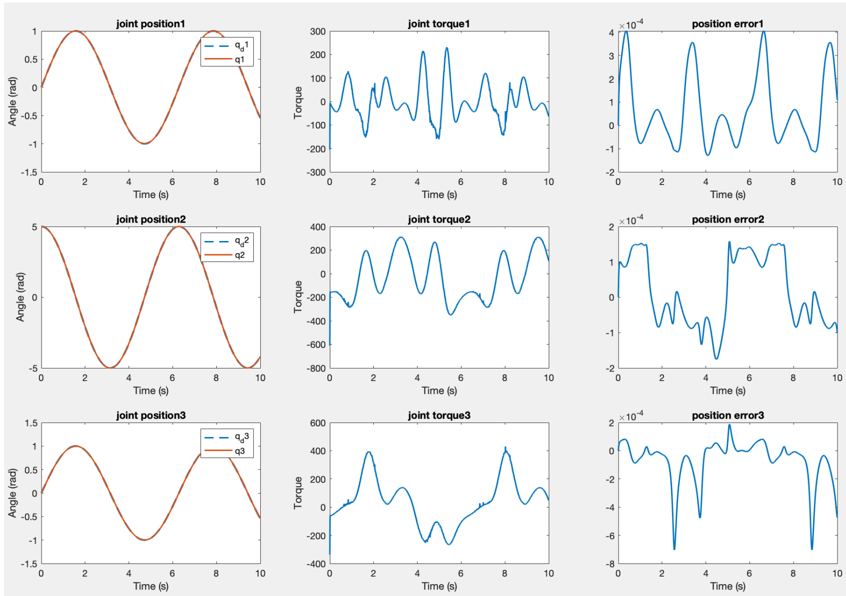

Figure 4: Robust-unperturbed

The control signal exhibits a slight amount of chattering, which is a consequence of the chosen design parameter ϵ. While the performance of the designed controller is generally satisfactory under ideal conditions, it is important to note that using a robust control law is only meaningful if we anticipate variations in dynamic coefficients due to uncertainties. In scenarios where dynamic coefficients are expected not to be alterate by uncertainties, the benefits of robust control may be less apparent.

Now we present the simulation results under perturbed conditions, where uncertainties in the dynamic coefficients are present, known to lie within bounded intervals, and cause the actual dynamic behavior to differ from the theoretical predictions.

The following results illustrate the performance of feedback linearization with the same gains as previously used, now evaluated under these perturbed conditions:

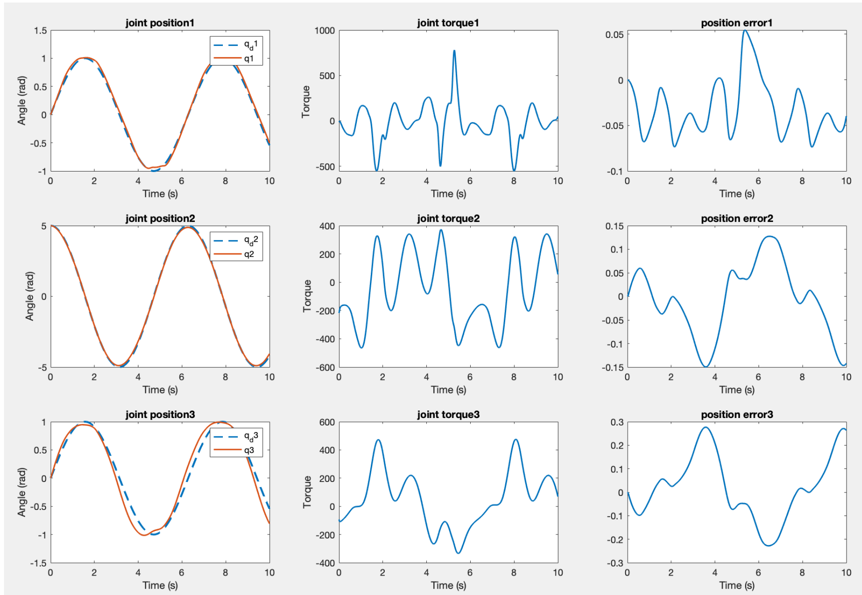

Figure 5: fbl-perturbed

It is clear that classic feedback linearization is inadequate for achieving good performance in presence of uncertainties in dynamic coefficients. To address this issue, a more advanced controller, such as our designed robust controller, is required.

The results of applying the robust control law, with the same gains as used previously, under perturbed conditions are as follows:

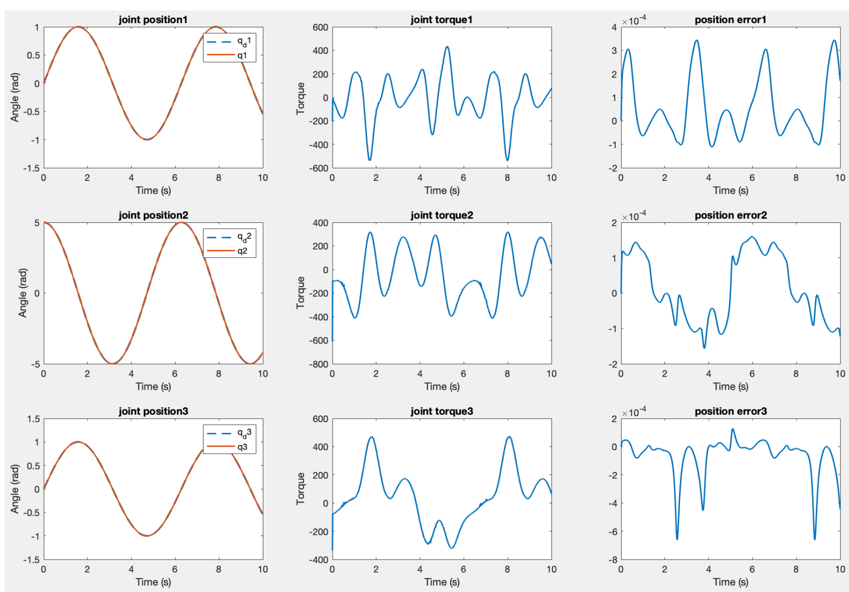

Figure 6: Robust-perturbed

Lastly, we examine the impact of tuning the parameter ϵ. As previously mentioned, ϵ represents a trade-off between the ultimate boundedness of the tracking error and the level of high-frequency oscillations (chattering) in the control signal. To illustrate its effects, we set $\epsilon = 0.05$ and observe the following:

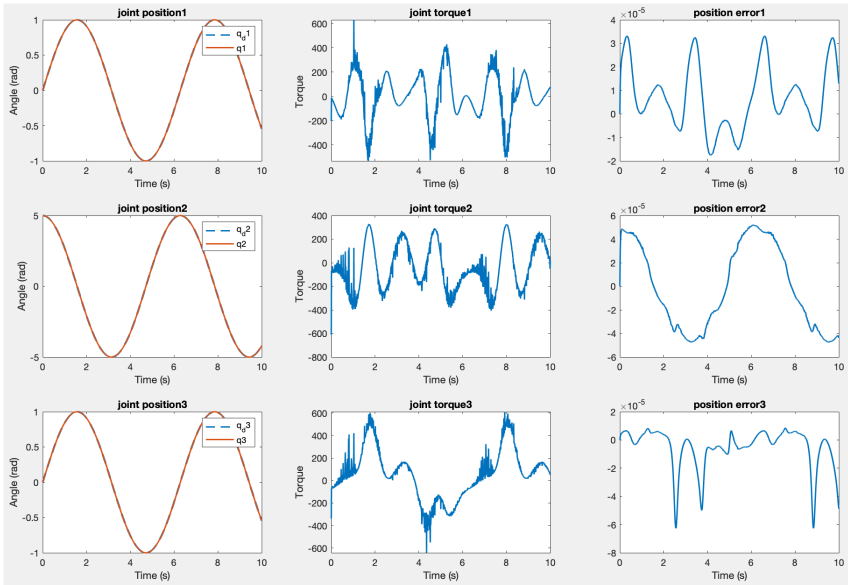

Figure 7: Robust-perturbed with $\epsilon = 0.05$

As expected, the tracking error decreases; however, this comes at the cost of increased chattering in the control signal. This is undesirable, as excessive chattering can lead to issues such as accelerated wear, degradation, potential failure over time and can make the system unstable.

4.2 Results

In summary, our comparison of the designed control law for trajectory tracking versus a classic feedback linearization control reveals the following:

- Ideal Conditions:
  - When there are no uncertainties and the dynamic behavior of the actual system perfectly matches the theoretical model, both control laws perform well. Specifically, the feedback linearization method achieves smaller position errors due to its precise alignment with the model.
- Perturbed Conditions:
  - Under perturbed conditions, where there are uncertainties and deviations between the actual system dynamics and the model, the performance of the classic feedback linearization method is no longer accurate. The results show that it is no longer suitable for maintaining accuracy when dynamic coefficients are uncertain.
  - In contrast, the robust control law maintains accurate performance even in the presence of uncertainties. It is particularly effective at correcting errors caused by variations in dynamic coefficients in known bounded intervals .

5 Conclusions

In this project, we have explored robust tracking control based on bounds on dynamic coefficients and assessed its effectiveness in various scenarios. Robust control methods prove to be a valuable strategy, providing optimal performance in situations where other approaches may be inefficient or complex. A key advantage of robust control is its ability to handle uncertainties within a known and bounded range, which must be precisely defined beforehand. However, it is important to note that robust control guarantees uniform ultimate bounded stability rather than asymptotic stability, and the stability bound is influenced by a trade-off parameter, ϵ, which affects the level of chattering in the control signal. When compared to adaptive control, which provides a more straightforward implementation and can perform well under significant uncertainties, adaptive control, while flexible in updating the parameters, often struggles with external disturbances and unmodeled dynamics such as structural flexibility, unless modifications are made. These modifications can complicate the adaptive control design to a level similar to robust control approaches. Furthermore the initial tracking error in adaptive control is often larger compared to robust control, this reflects the great initial uncertainty in estimating the parameters. In contrast, robust control does not rely on parameter updates; instead, the robust term alone compensates for deviations caused by uncertainties. Ultimately, the choice between robust and adaptive control strategies depends heavily on the operating conditions and the required level of robustness. Robust control, particularly when based on bounds on dynamic coefficients, is a suitable and effective choice when uncertainties are moderate and predictable, and when reliable performance and good disturbance rejection are a critical requirement. This approach offers a balance between performance and complexity, making it a valid option for systems where uncertainties are known to lie within a manageable range.

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