$x(n) = Rx(n-1) + a$

$x(1) = Rx(0) + a$

$x(2) = Rx(1) + a = R \cdot (Rx(0) + a) + a = R^2 x(0) + Ra + a$

$x(3) = Rx(2) + a = R(R^2 x(0) + Ra + a) + a = R^3 x(0) + R^2 a + Ra + a$

$x(n) = R^n x(0) + \left(\sum_{i=0}^{n-1} R^i a\right) = R^n \cdot x(0) + a\left(\frac{1-R^n}{1-R}\right)$

Proof: WTS $x(k+1) = R^{k+1} x(0) + a\left(\frac{1-R^{k+1}}{1-R}\right)$

Let $x(k+1) = Rx(k+1-1) + a$

$= Rx(k) + a$

Then, $x(k+1) = R\left(R^k x(0) + a\left(\frac{1-R^k}{1-R}\right)\right) + a$

$= R^{k+1} x(0) + Ra\left(\frac{1-R^k}{1-R}\right) + a$

$= \frac{Ra(1-R^k) + a(1-R)}{1-R}$

$\frac{Ra - aR^{k+1} + a - aR}{1-R}$

$= \frac{a - aR^{k+1}}{1-R} = \frac{a(1-R^{k+1})}{1-R}$

$x(k+1) = R^{k+1} x(0) + a\left(\frac{1-R^{k+1}}{1-R}\right)$

By induction, we proved $x(k+1) = Rx(k) + a = R^{k+1} x(0) + a\left(\frac{1-R^{k+1}}{1-R}\right)$

# Question 3

Suppose $k = x(k)$

a) $x(n) = x(n-1)e^{r\,x(n-1)}$

Let $x(n) = x(n-1) = x_*$

$$x_* = x_* e^{rx_*}$$

$$0 = x_* e^{rx_*} - x_*$$

$$0 = x_*(e^{rx_*} - 1)$$

$1 = e^{rx_*}$, $\ln(1) = rx_*$

$x_* = 0$

Therefore $x_* = 0$ or $r = 0$

Stability: $f(x) = xe^{rx}$ st. $f'(x) = e^{rx} + rxe^{rx}$

$$f'(x) = (1+rx)e^{rx}$$

↳ Let $x = 0$ (our fixed point). So $f'(0) = (1 + r(0))e^{r(0)}$
$= 1$

Analysis: Since $|f'(0)| = 1$ which is not less than 1, the fixed point is not stable

b) Given $x(n) = x(n-1)^2 + 0.7x(n-1) + 0.02$

Let $x_* = x(n) = x(n-1)$.

$$x_* = x_*^2 + 0.7x_* + 0.02$$

$$0 = x_*^2 - 0.3x_* + 0.02$$

$$x_* = 0.2, 0.1$$

Stability:

$f(x) = x^2 + 0.7x + 0.02$

$f'(x) = 2x + 0.7$

$f'(0.2) = 1.1$

$f'(0.1) = 0.9$

Analysis:

$f'(0.2) = 1.1 < 1$ is false so $x_* = 1.1$ is NOT stable

$f'(0.1) = 0.9 < 1$ is true so $x_* = 0.9$ is stable