Do Treasury Yields Affect the S&P 500?

Download and Clean Data

In [3]:
```
# Import libraries
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as stats
import seaborn as sns
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score
import statsmodels.api as sm
```

In [4]:
```
# Download historical S&P 500 historical data
hist_sp500 = pd.read_csv('HistoricalPrices.csv')

# Download historical daily treasury yields one month rate to 30 years rate
daily_ycr = pd.read_csv('yield-curve-rates-1990-2024ytd.csv')
```

In [67]:
```
# Transform 'Date" Columns into date format
hist_sp500['Date'] = pd.to_datetime(hist_sp500['Date'])
daily_ycr['Date'] = pd.to_datetime(daily_ycr['Date'])

# Merge data frames
sp500_yieldcr_df = pd.merge(hist_sp500, daily_ycr, on='Date', how='outer')

# Sort data frame by date
sp500_yieldcr_df = sp500_yieldcr_df.sort_values(by='Date')

# Forward and backfill empty cells
sp500_yieldcr_df.ffill(inplace=True)
sp500_yieldcr_df.bfill(inplace=True)

# Condense data frame. Subtract dependent value and 5 independent variables
columns_to_select = ['Date', ' Close', '1 Mo', '3 Mo', '6 Mo', '20 Yr', '30 Yr']
clean_df = sp500_yieldcr_df[columns_to_select]

# Show final data set
print(clean_df.head())
```

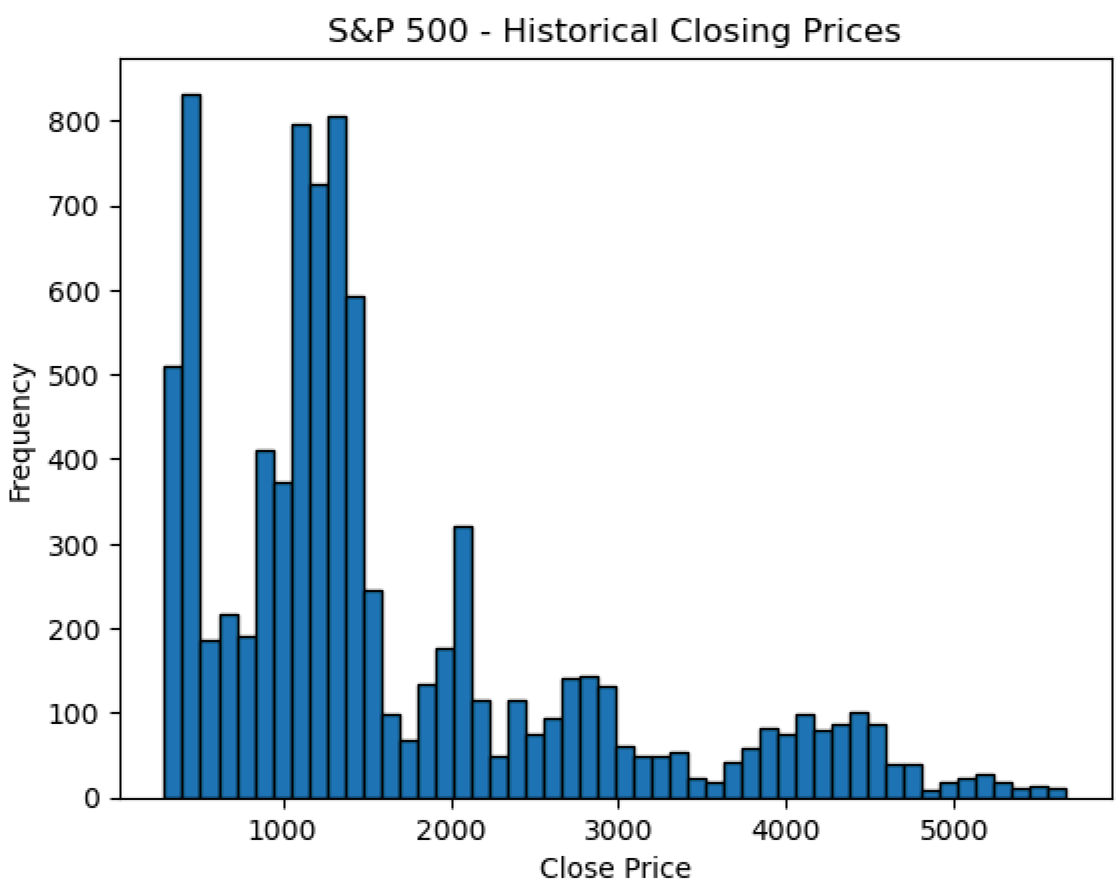
```
           Date   Close  1 Mo  3 Mo  6 Mo  20 Yr  30 Yr
8706 1990-01-02  359.69  3.67  7.83  7.89   6.12   8.00
8705 1990-01-03  358.76  3.67  7.89  7.94   6.12   8.04
8704 1990-01-04  355.67  3.67  7.84  7.90   6.12   8.04
8703 1990-01-05  352.20  3.67  7.79  7.85   6.12   8.06
8702 1990-01-08  353.79  3.67  7.79  7.88   6.12   8.09
```

Describe All Variables

- Date: Dates between 01/01/1990 and 07/24/2024 in a date-time format.
- Close: The Standard and Poor's 500, or simply the S&P 500,
  is a stock market index tracking the stock performance of 500 of the largest companies listed on stock exchanges in the United States.
  The "Close" variable is the closing price of the S&P 500 index on a specific date.
- 1 Mo: The return on investment in U.S. government debt obligations expected to mature in 1 month.
- 3 Mo: The return on investment in U.S. government debt obligations expected to mature in 3 months.
- 6 Mo: The return on investment in U.S. government debt obligations expected to mature in 6 months.
- 20 Yr: The return on investment in U.S. government debt obligations expected to mature in 20 years.
- 30 Yr: The return on investment in U.S. government debt obligations expected to mature in 30 years.

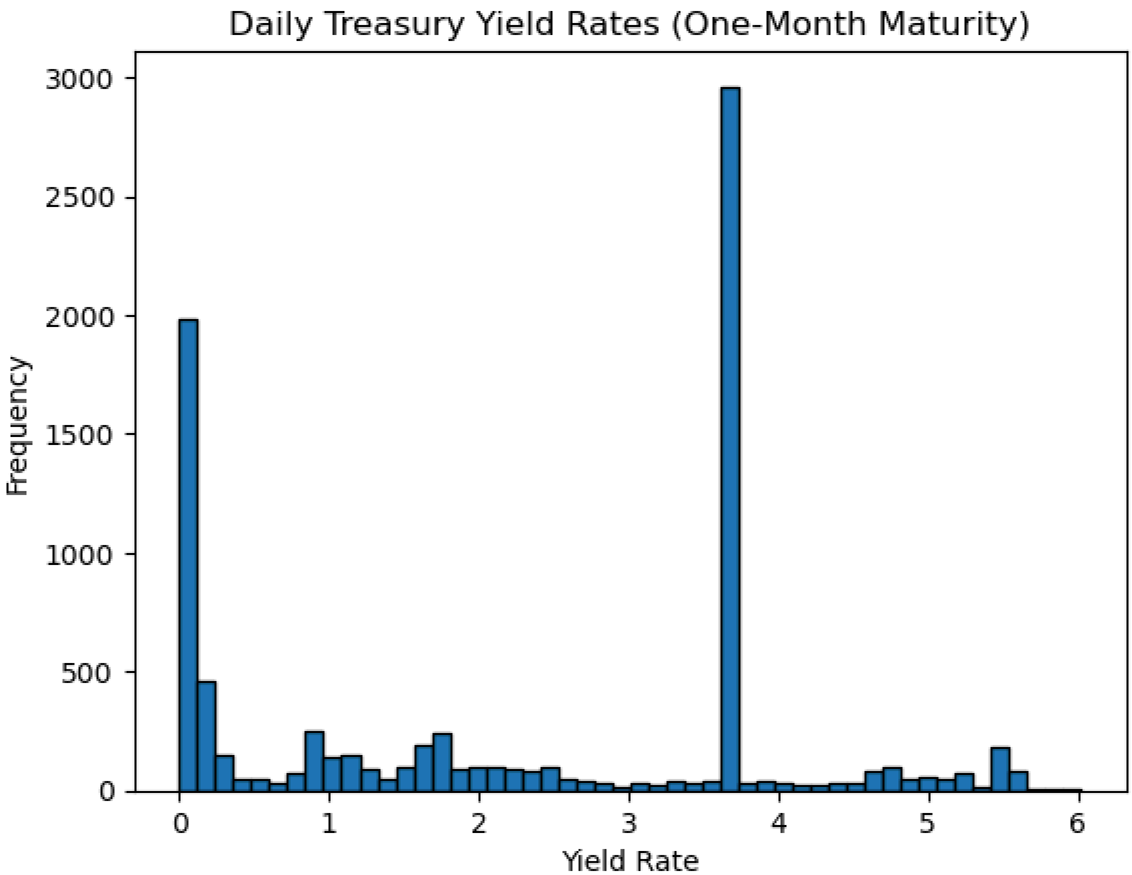
Histograms

In [9]:
```
# Substract S&P 500 closing prices
hist_close = clean_df[' Close']
# Create plot
plt.hist(hist_close, bins=50, edgecolor='black')
# Labels
plt.title("S&P 500 - Historical Closing Prices")
plt.xlabel('Close Price')
plt.ylabel('Frequency')
plt.show()
```

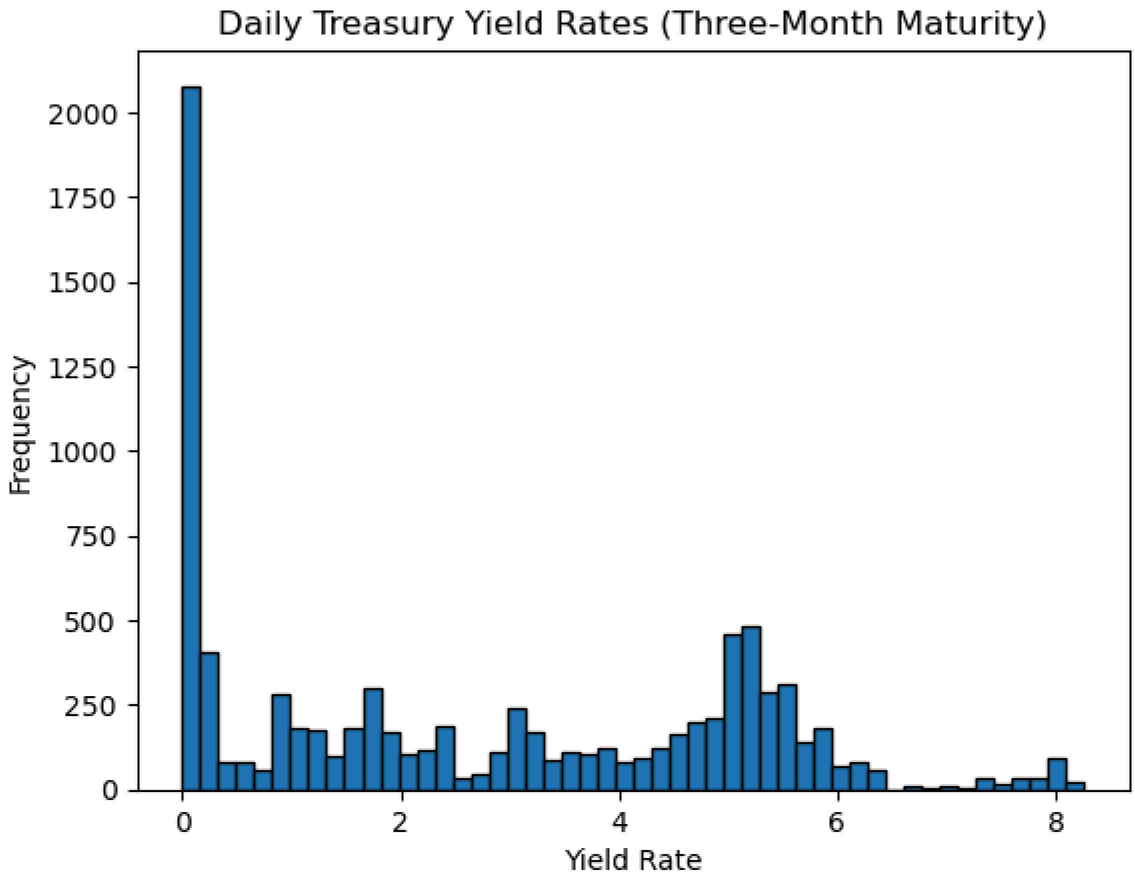


In [10]:
```
# Substract Daily Treasury Yield Rates (One-Month Maturity)
hist_1mo = clean_df['1 Mo']
# Create plot
```

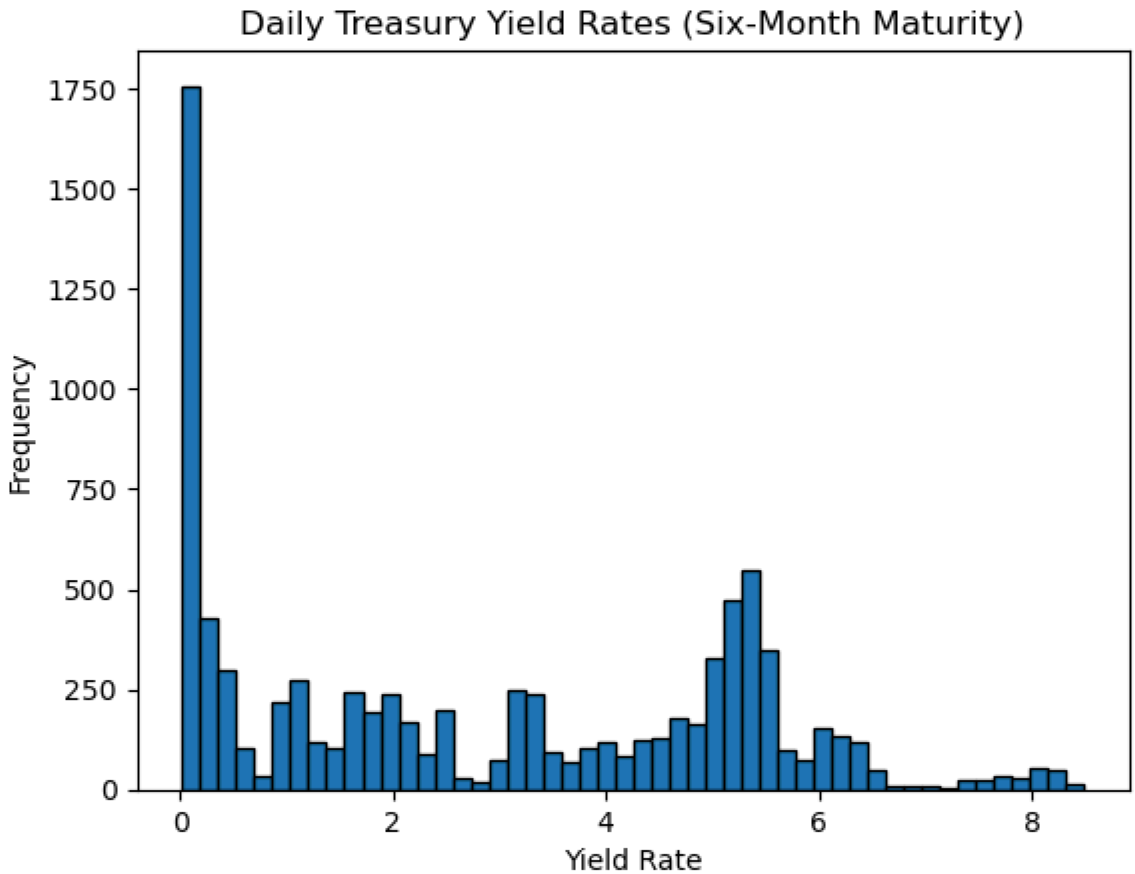
```
plt.hist(hist_1mo, bins=50, edgecolor='black')
# Labels
plt.title("Daily Treasury Yield Rates (One-Month Maturity)")
plt.xlabel('Yield Rate')
plt.ylabel('Frequency')
plt.show()
```



In [11]:
```
# Substract Daily Treasury Yield Rates (Three-Month Maturity)
hist_3mo = clean_df['3 Mo']
# Create plot
plt.hist(hist_3mo, bins=50, edgecolor='black')
# Labels
plt.title("Daily Treasury Yield Rates (Three-Month Maturity)")
plt.xlabel('Yield Rate')
plt.ylabel('Frequency')
plt.show()
```

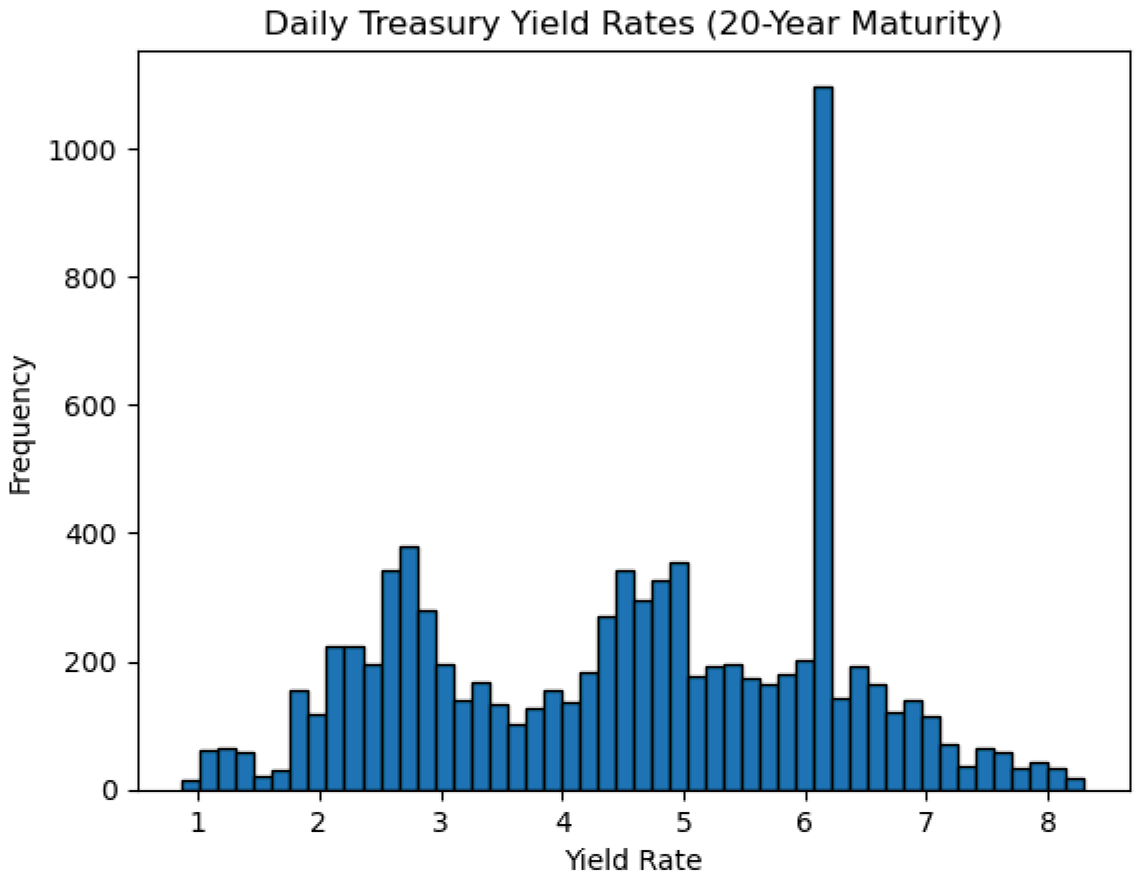


In [12]:
```
# Substract Daily Treasury Yield Rates (Six-Month Maturity)
hist_6mo = clean_df['6 Mo']
# Create Plot
plt.hist(hist_6mo, bins=50, edgecolor='black')
# Labels
plt.title("Daily Treasury Yield Rates (Six-Month Maturity)")
plt.xlabel('Yield Rate')
plt.ylabel('Frequency')
plt.show()
```

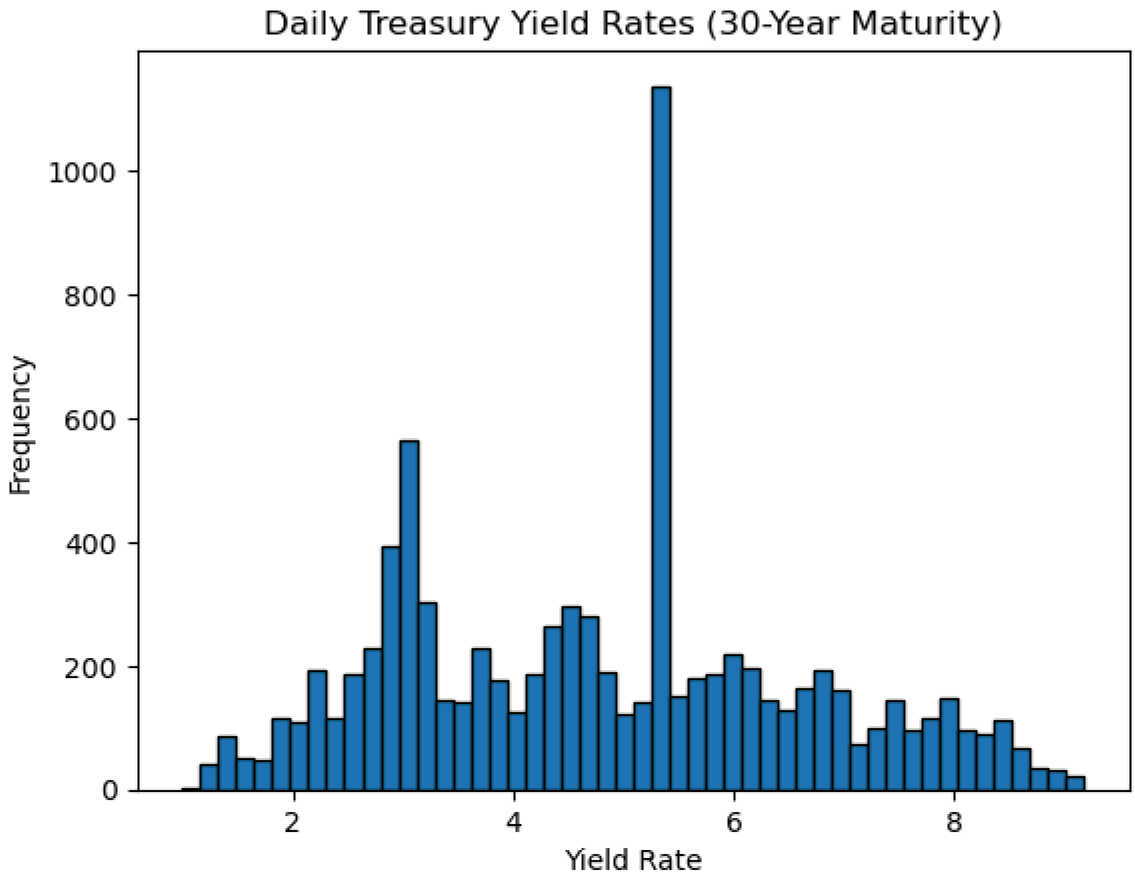


In [13]:

```
# Substract Daily Treasury Yield Rates (20-Year Maturity)
hist_20yr = clean_df['20 Yr']
# Create Plot
plt.hist(hist_20yr, bins=50, edgecolor='black')
# Labels
plt.title("Daily Treasury Yield Rates (20-Year Maturity)")
plt.xlabel('Yield Rate')
plt.ylabel('Frequency')
plt.show()
```



In [14]:
```
# Substract Daily Treasury Yield Rates (30-Year Maturity)
hist_30yr = clean_df['30 Yr']
# Create Plot
plt.hist(hist_30yr, bins=50, edgecolor='black')
# Labels
plt.title("Daily Treasury Yield Rates (30-Year Maturity)")
plt.xlabel('Yield Rate')
plt.ylabel('Frequency')
plt.show()
```



Descriptive Charecteristics About Variables

In [16]:
```
# for loop that calculates descriptive characteristics about the data frame
for column in clean_df.columns:
    if column != 'Date':
        mean = round(clean_df[column].mean(), 2)
        mode = round(clean_df[column].mode(), 2)
        std = round(clean_df[column].std(), 2)
        skewness = round(clean_df[column].skew(), 2)
        kurtosis = round(clean_df[column].kurtosis(), 2)

        print(f"Descriptive statistics for {column}:")
        print(f"Mean: {mean}")
        print(f"Mode: {mode.values}")
        print(f"Standard Deviation: {std}")
        print(f"Skewness: {skewness}")
        print(f"Kurtosis: {kurtosis}")
        print()
```

```
Descriptive statistics for  Close:
Mean: 1642.66
Mode: [1092.54]
Standard Deviation: 1172.76
Skewness: 1.3
Kurtosis: 0.97

Descriptive statistics for 1 Mo:
Mean: 2.22
Mode: [3.67]
Standard Deviation: 1.75
Skewness: 0.03
Kurtosis: -1.43

Descriptive statistics for 3 Mo:
Mean: 2.73
Mode: [0.02]
Standard Deviation: 2.3
Skewness: 0.29
Kurtosis: -1.24

Descriptive statistics for 6 Mo:
Mean: 2.85
Mode: [0.06]
Standard Deviation: 2.33
Skewness: 0.26
Kurtosis: -1.26

Descriptive statistics for 20 Yr:
Mean: 4.56
Mode: [6.12]
Standard Deviation: 1.67
Skewness: -0.12
Kurtosis: -1.0

Descriptive statistics for 30 Yr:
Mean: 4.81
Mode: [5.37]
Standard Deviation: 1.82
Skewness: 0.22
Kurtosis: -0.76
```

Create a PMF

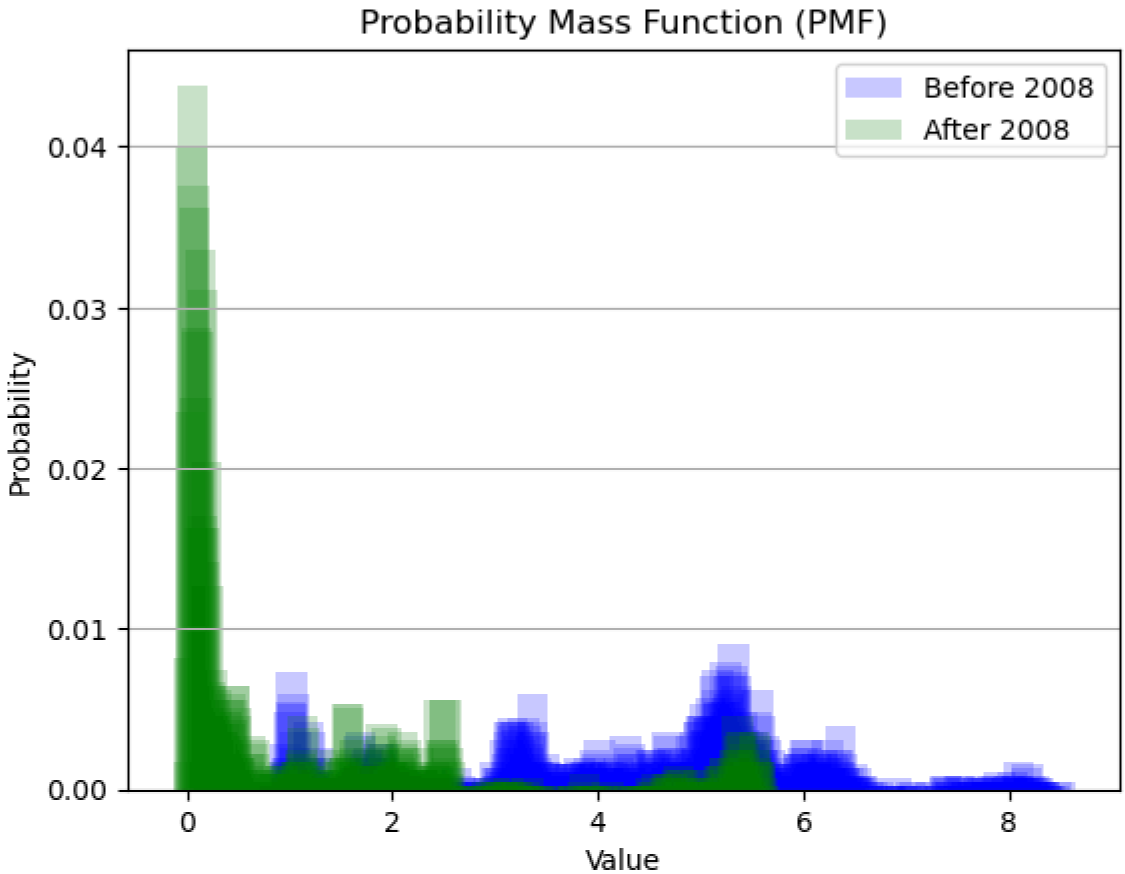
In [18]:

```
# Creating Two Scenarios Before and After the 2008 Financial Crisis
before2008_df = clean_df[clean_df['Date'] < '2008-01-01']
after2008_df = clean_df[clean_df['Date'] > '2008-01-01']
```

In [19]:

```
frequency1 = before2008_df['6 Mo'].value_counts().sort_index()
pmf1 = frequency1 / frequency1.sum()
frequency2 = after2008_df['6 Mo'].value_counts().sort_index()
pmf2 = frequency2 / frequency2.sum()
plt.bar(pmf1.index, pmf1.values, width=0.3, color='blue', alpha=0.2, label="Before 2008")
plt.bar(pmf2.index, pmf2.values, width=0.3, color='green', alpha=0.2, label="After 2008")
plt.xlabel('Value')
plt.ylabel('Probability')
plt.legend()
```

```
plt.title('Probability Mass Function (PMF)')
plt.grid(axis='y')
plt.show()
```

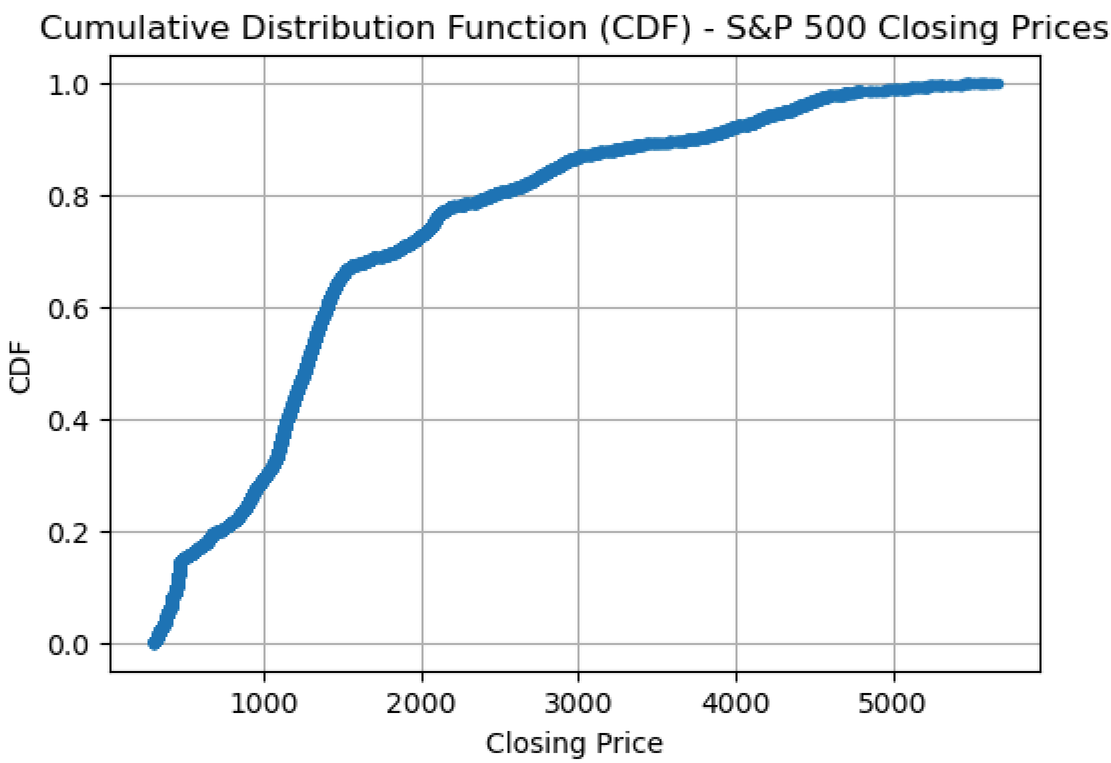


Create a CDF

This CDf shows that the closing price of the S&P 500 has been lower than 2000 points for almost 80% of the time since 1990. This shows that most of the growth of the index occurred recently. Furthermore, as shown in the PMF above, it can be observed that most of the growth of the index occurred after the 2008 financial crisis. Finally, if the closing prices of the S&P 500 are compared with the treasury yields after 2008, an inversed relationship can be observed as the yields have ranged between 0% and 2% for the last 15 years.

In [22]:
```
# Select closing prices
closing_prices = clean_df[' Close']
# Sort the data
sorted_closing_prices = np.sort(closing_prices)
# Calculate the CDF values
cdf_closing_prices = np.arange(1, len(sorted_closing_prices) + 1) / len(sorted_closing_prices)
```

In [23]:
```
# Plot CDF
plt.figure(figsize=(6, 4))
plt.plot(sorted_closing_prices, cdf_closing_prices, marker='.', linestyle='none')
plt.title('Cumulative Distribution Function (CDF) - S&P 500 Closing Prices')
plt.xlabel('Closing Price')
plt.ylabel('CDF')
plt.grid(True)
plt.show()
```



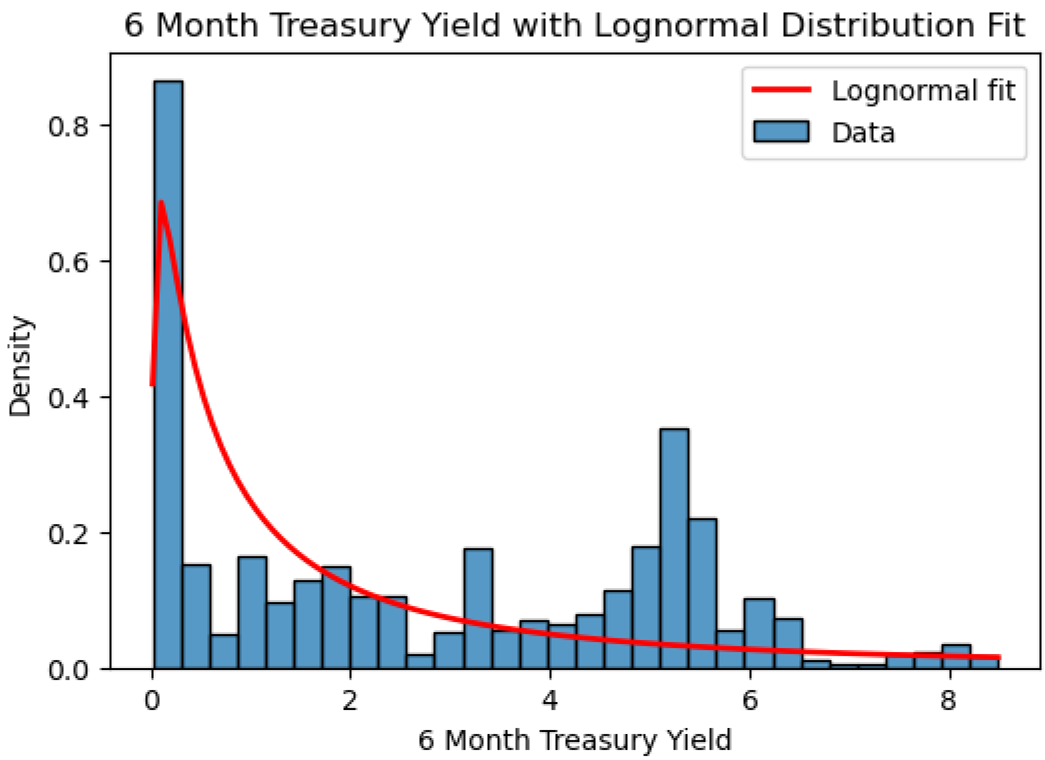
Plot Analytical Distribution

Log-normal distribution is normally used to analyze finances, such as stock prices, as these items cannot be negative and tend to be positively skewed. The log-normal distribution plotted in this analysis shows how the distribution of the 6-month treasury yield is positively skewed. It also shows that in the last 30 years, this yield curve has been relatively low (less than 1%). This might be evidence that not all the treasury yields have the same effect on the S&P 500 index as the 20-30-year rates have been, on average, higher than the 1-month and 6-month rates. A regression analysis might be needed to understand which rate has the strongest impact on the S&P 500.

In [26]:
```
# Fit a lognormal distribution
shape, loc, scale = stats.lognorm.fit(clean_df['6 Mo'], floc=0)
# Generate x values for plotting the PDF
xmin, xmax = clean_df['6 Mo'].min(), clean_df['6 Mo'].max()
x = np.linspace(xmin, xmax, 100)
# Calculate the PDF
pdf = stats.lognorm.pdf(x, shape, loc, scale)
```

In [27]:
```
plt.figure(figsize=(6, 4))
sns.histplot(clean_df['6 Mo'], kde=False, stat="density", bins=30, label='Data')
# Plot the PDF
plt.plot(x,pdf,'r-',lw=2,label='Lognormal fit')
plt.title('6 Month Treasury Yield with Lognormal Distribution Fit')
plt.xlabel('6 Month Treasury Yield')
plt.ylabel('Density')
plt.legend()
plt.show()
```

```
C:\Users\emili\anaconda3\Lib\site-packages\seaborn\_oldcore.py:1119: FutureWarning: use_inf_as_na option is deprecated and will be removed in a future version. Convert inf values to NaN before operating instead.
  with pd.option_context('mode.use_inf_as_na', True):
```

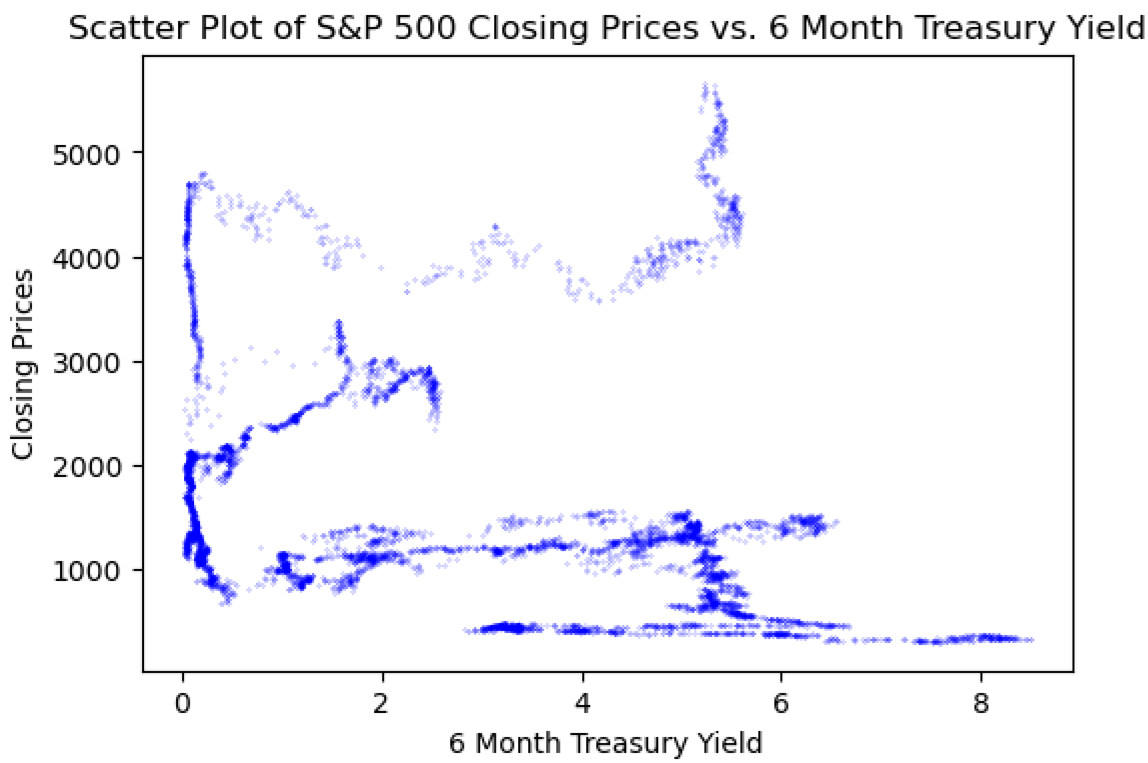


Correlation and Causation

Several methods were used to find and/or explain the correlation and causation between two variables, thesix-month treasury yield and the S&P 500 index. The scatter plot below shows no obvious linear relationshipbetween the six-month treasury yield and the S&P 500 closing prices. The covariance calculation resulted ina value of -716, which might indicate a negative correlation, yet it is hard to interpret. Hence, Pearson'scorrelation was calculated to normalize the relationship between these two variables. Pearson's correlationresulted in -0.26, which could be interpreted as a small to moderate inverse relationship between thesevariables. Finally, a non-linear relationship calculation was used to attempt to model the relationshipbetween these two variables. A polynomial model was used to predict the closing prices of the S&P 500.Unfortunately, the R-squared value demonstrated that this model is not an accurate fit. This analysis provesthat there is no strong relationship between the six-month treasury yield and the closing prices of the S&P500.

Scatter Plot

In [31]:
```
# Create the scatter plot
plt.figure(figsize=(6, 4))
plt.scatter(clean_df['6 Mo'], clean_df[' Close'], color='blue', alpha=0.6, s=.05)
# Labels
plt.xlabel('6 Month Treasury Yield')
plt.ylabel('Closing Prices')
plt.title('Scatter Plot of S&P 500 Closing Prices vs. 6 Month Treasury Yield')
# Show the plot
plt.show()
```



Covariance

In [33]:
```
# Create a covariance matrix
cov_matrix = clean_df[[' Close', '6 Mo']].cov()

# Access the right item from the covariance matrix
cov_close_6mo = cov_matrix.loc[' Close', '6 Mo']
print("Covariance between Closing Price and 6 Month Treasury Yield:", cov_close_6mo)
```

```
Covariance between Closing Price and 6 Month Treasury Yield: -716.4168803910177
```

Pearson's Correlation

In [35]:
```
# Standard deviations
std_close = np.sqrt(cov_matrix.loc[' Close', ' Close'])
std_6mo = np.sqrt(cov_matrix.loc['6 Mo', '6 Mo'])

# Correlation coefficient
correlation = cov_close_6mo / (std_close * std_6mo)

print("Correlation Coefficient between Closing Price and 6 Month Treasury Yield:", correlation)
```

```
Correlation Coefficient between Closing Price and 6 Month Treasury Yield: -0.2623894875108695
```

Non-linear Relationships

In [37]:
```
# Create Variables
x = clean_df[['6 Mo']]
y = clean_df[[' Close']]

# Fit a quadratic model
poly = PolynomialFeatures(degree=2)
model = make_pipeline(poly, LinearRegression())
```

```
model.fit(x, y)

# Predict values
predictions = model.predict(x)

# Mean Squared Error (MSE)
mse = mean_squared_error(y, predictions)
print("Mean Squared Error (MSE):", mse)

# Root Mean Squared Error (RMSE)
rmse = np.sqrt(mse)
print("Root Mean Squared Error (RMSE):", rmse)
```

```
Mean Squared Error (MSE): 1278661.352525973
Root Mean Squared Error (RMSE): 1130.7790909483483
```

Hypothesis Test

The Fisher's Null hypothesis testing was use to test this hypothesis: "Treasury yields affect the S&P 500". This testing measures the correlation of the data and the null hypothesis "Treasury yields do not affect the S&P 500". A regression analysis is needed to use the Fisher's Null Hypothesis method.

Regression Analysis

In [41]:

```
# Plot independent and dependent variables
x = clean_df[['1 Mo', '3 Mo', '6 Mo', '20 Yr', '30 Yr']]
y = clean_df[' Close']

# Add intercept
x = sm.add_constant(x)

# Fit the regression model
model = sm.OLS(y, x).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Close   R-squared:                       0.692
Model:                            OLS   Adj. R-squared:                  0.692
Method:                 Least Squares   F-statistic:                     3920.
Date:                Mon, 24 Mar 2025   Prob (F-statistic):               0.00
Time:                        05:25:46   Log-Likelihood:                -68852.
No. Observations:                8719   AIC:                         1.377e+05
Df Residuals:                    8713   BIC:                         1.378e+05
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       4407.9129     22.951    192.057      0.000    4362.924    4452.902
1 Mo         264.9007     11.191     23.670      0.000     242.963     286.839
3 Mo        -274.6069     54.498     -5.039      0.000    -381.437    -167.777
6 Mo         424.1909     53.281      7.961      0.000     319.748     528.634
20 Yr       -239.5141     13.732    -17.442      0.000    -266.432    -212.596
30 Yr       -565.8167     12.198    -46.386      0.000    -589.727    -541.906
==============================================================================
Omnibus:                     1451.602   Durbin-Watson:                   0.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2981.575
Skew:                           1.000   Prob(JB):                         0.00
Kurtosis:                       5.051   Cond. No.                         97.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Interpretation of the Fisher's Null Hypothesis

The one-month, three-month, six-month, 20-year, and 30-year treasury yields were used to test the effect oftreasury yields on the S&P 500. The regression analysis represented that all coefficients had a p-value lowerthan 0.05. Therefore, it can be concluded that all treasury yields significantly affect the S&P 500 closingprices, and the null hypothesis is rejected.

Sources

Daily Treasury Par Yield Curve Rates: https://home.treasury.gov/interest-rates-data-csv-archive S&P 500 Historical dat : https://www.wsj.com/market-data/quotes/index/SPX/historical-prices