GARCH and Zillow Housing Data

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6/28/2023

Cal State Fullerton

Agenda

Introduction/ EDA

Emilio Vasquez

GARCH Background

Paul (Nik) Lopez

GARCH Housing Data Analysis/Results

Horace Tsai

Conclusion

Lucas Everett

Q & A

All

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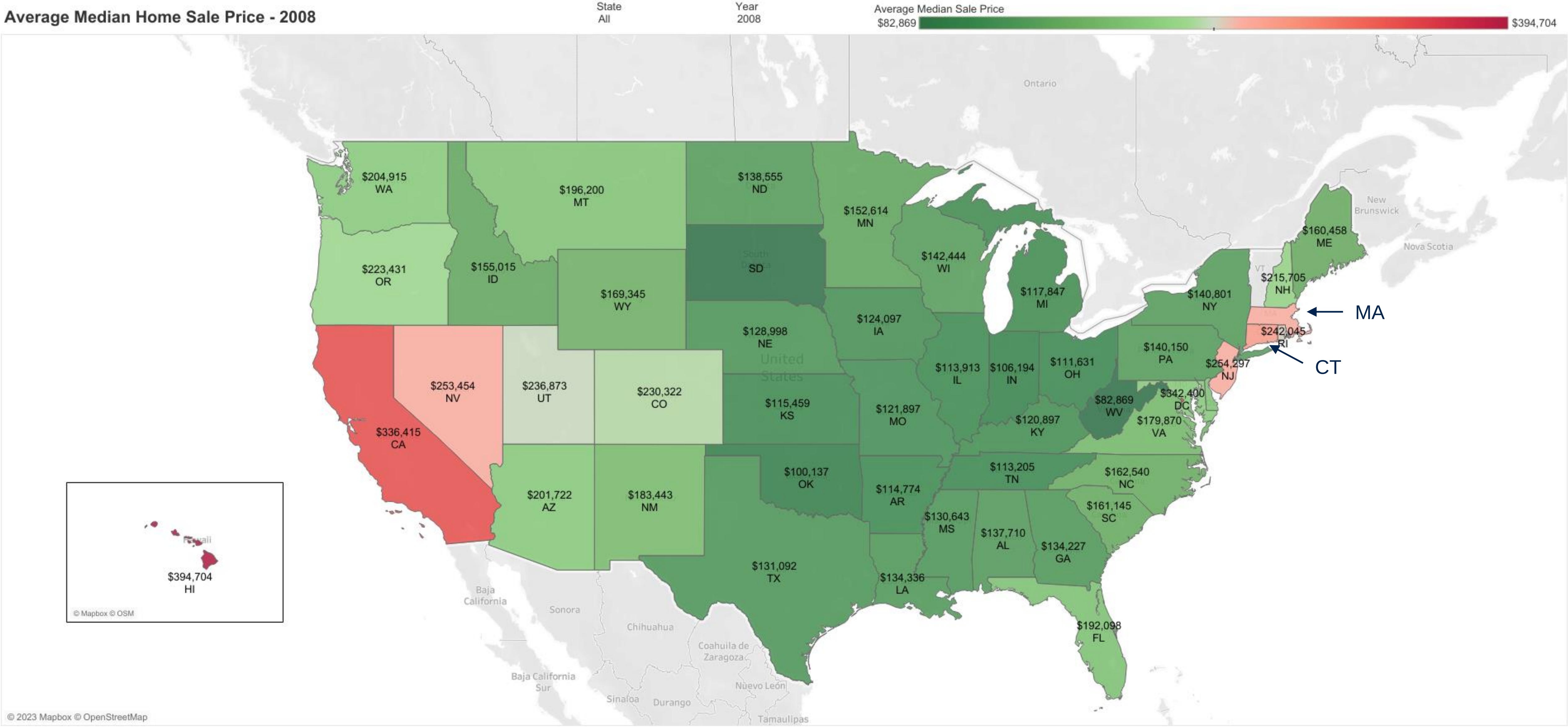
Introduction/EDA

- Emilio Vasquez

Dataset: Median Housing Sale Price from Zillow

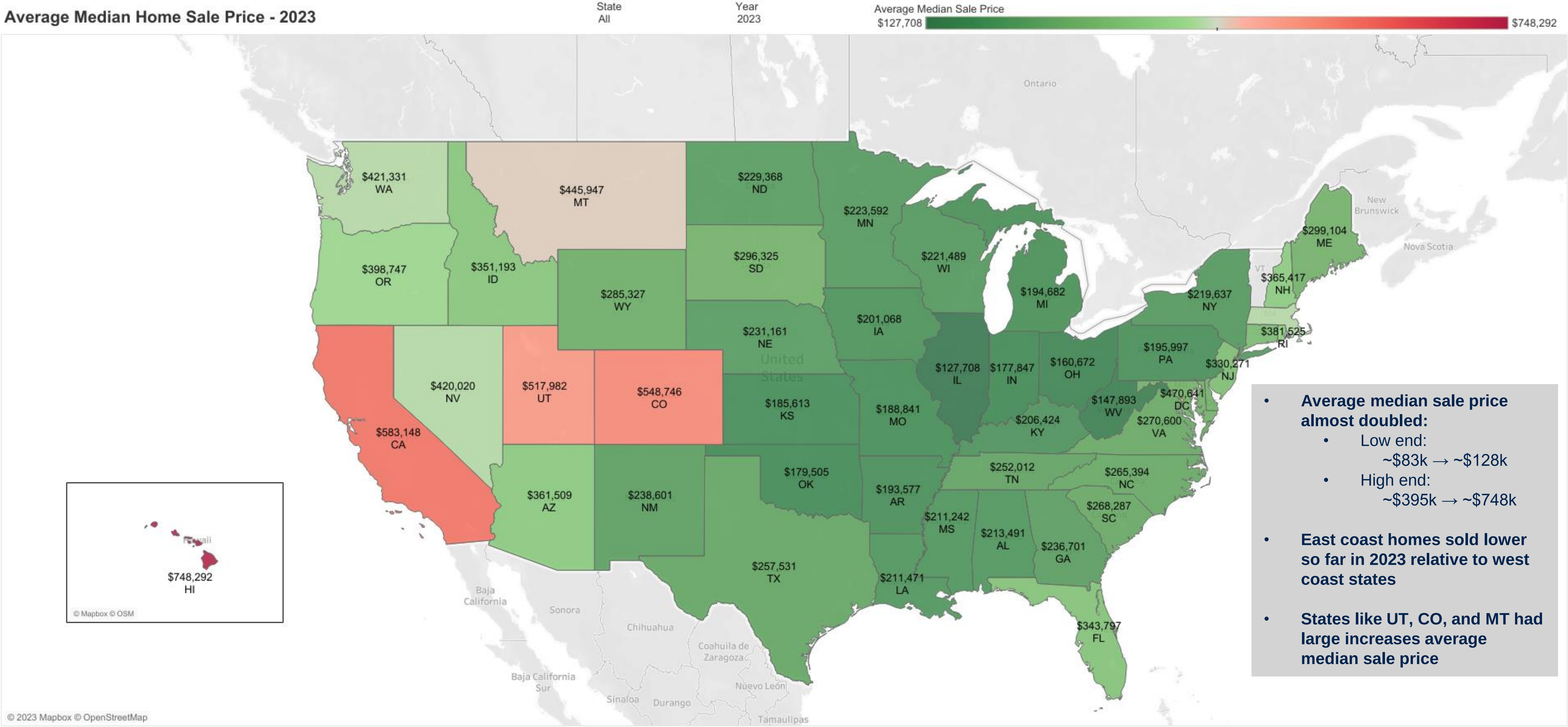
- Downloaded raw monthly median house sale prices from https://www.zillow.com/research/data/
- Spans from 2/29/2008 to 3/31/2023, giving us about 180 datapoints for each city
- Covers 608 different cities across the United States
- Potential bias in the data that could mispresent the true median for any given state

Average Median Home Sale Price - 2008



Dashboard link

Average Median Home Sale Price - 2023



- **Average median sale price almost doubled:**
  - Low end: ~$83k → ~$128k
  - High end: ~$395k → ~$748k
- **East coast homes sold lower so far in 2023 relative to west coast states**
- **States like UT, CO, and MT had large increases average median sale price**

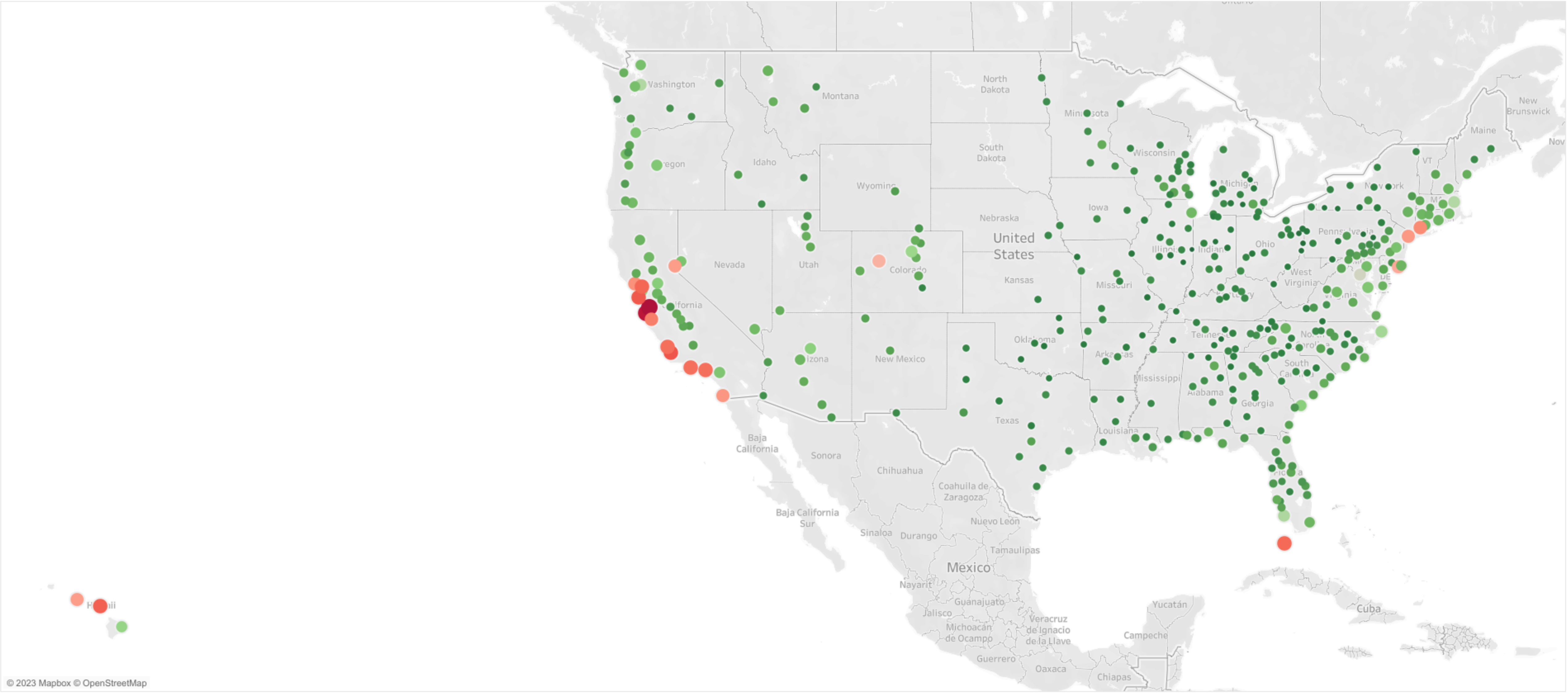
Dashboard link

Average Median Sale Price - All - 2008

State
All

Year
2008

Avg. Price
$0.05M $0.63M



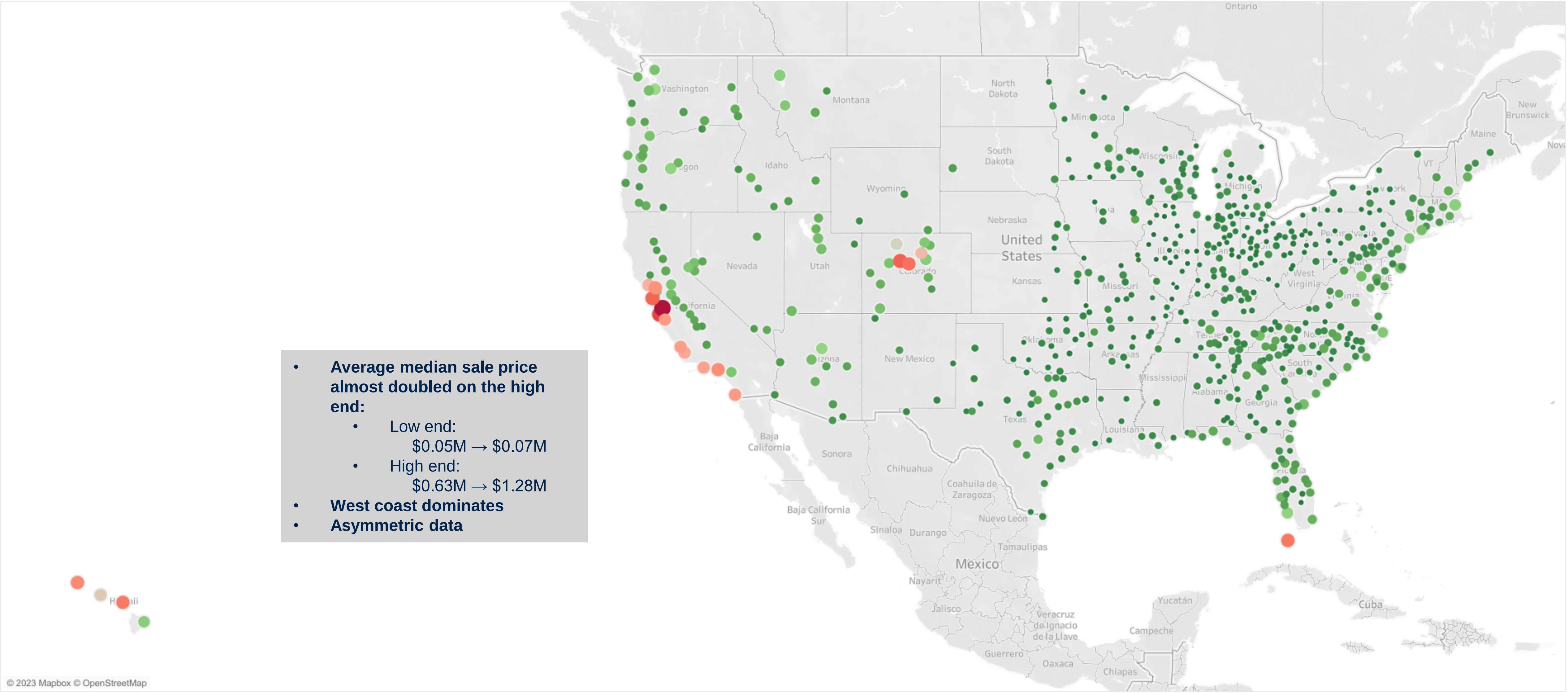
Dashboard link

Average Median Sale Price - All - 2023

State
All

Year
2023

Avg. Price
$0.07M — $1.28M

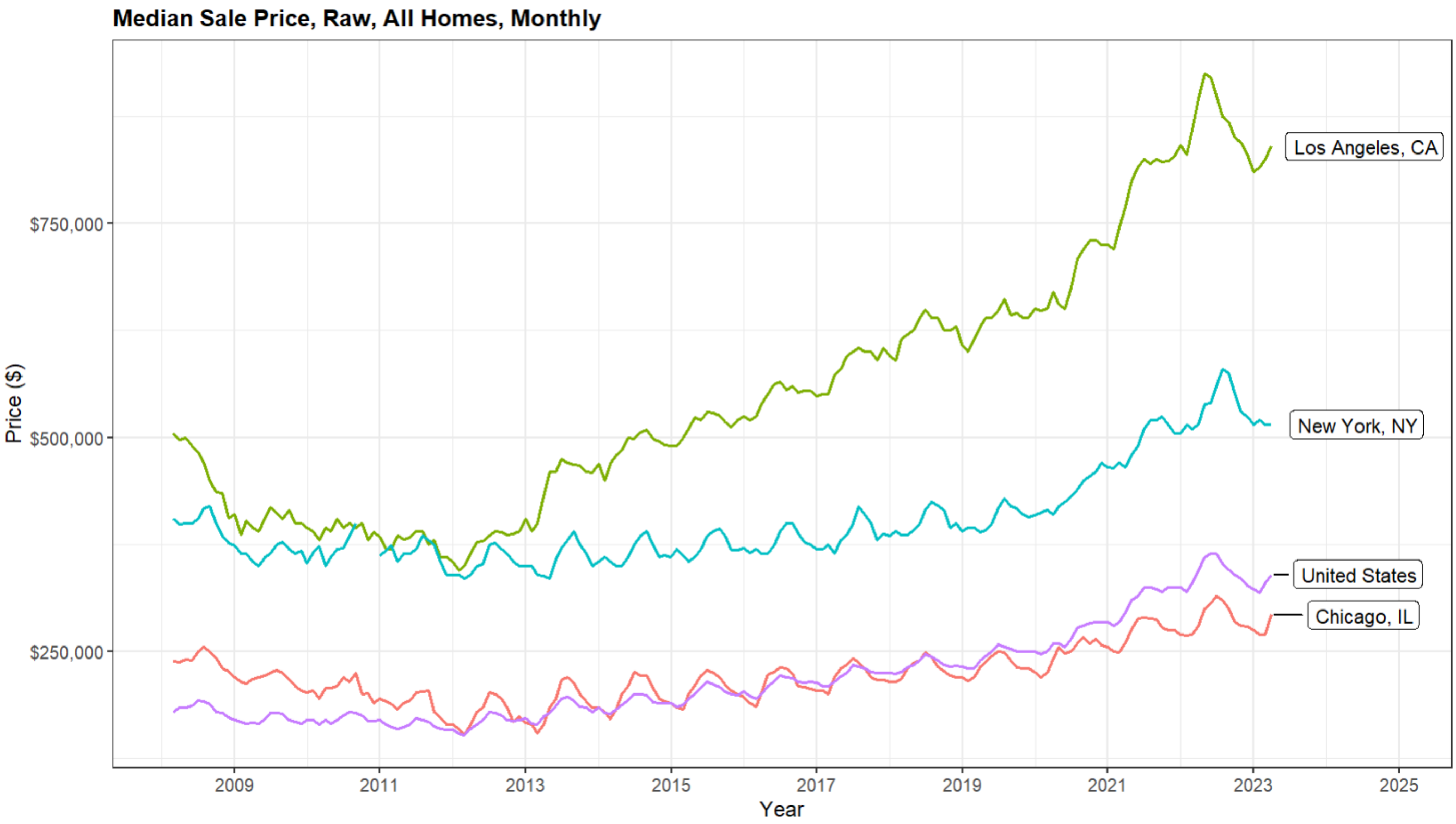


- **Average median sale price almost doubled on the high end:**
  - Low end: $0.05M → $0.07M
  - High end: $0.63M → $1.28M
- **West coast dominates**
- **Asymmetric data**

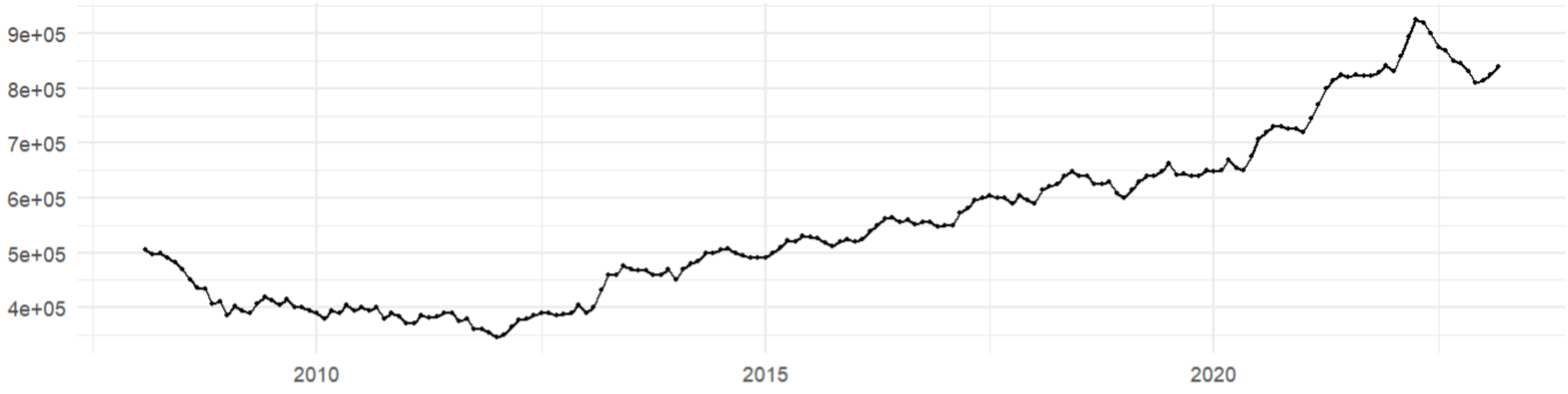
Dashboard link

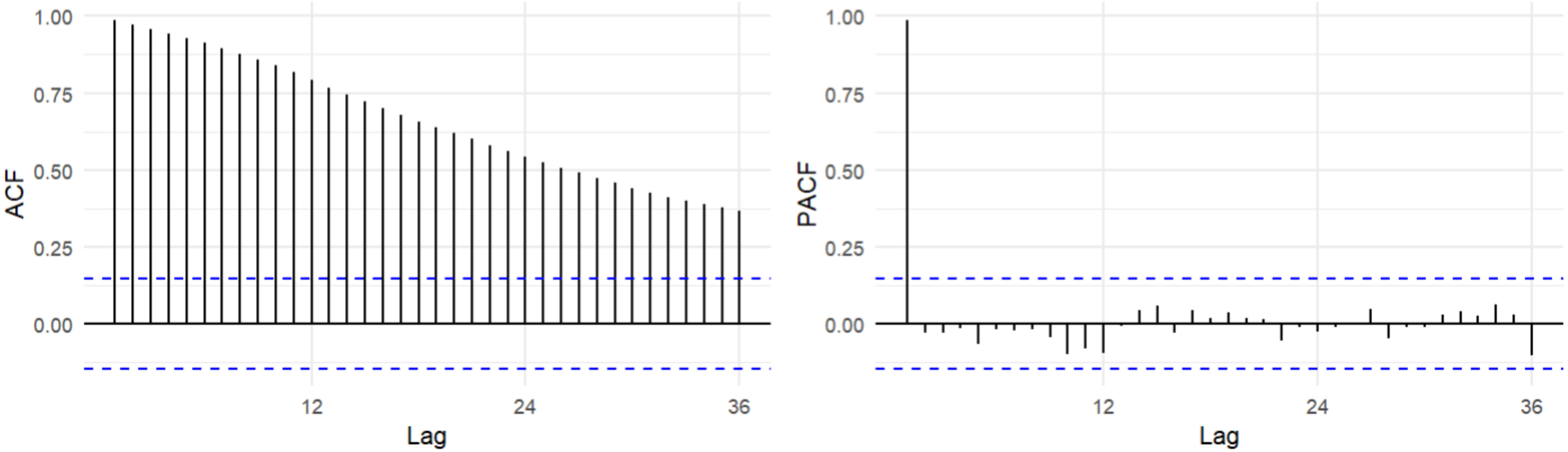
Winners and Losers Since 2008 (Sorted on Increase, by Region):

Dashboard

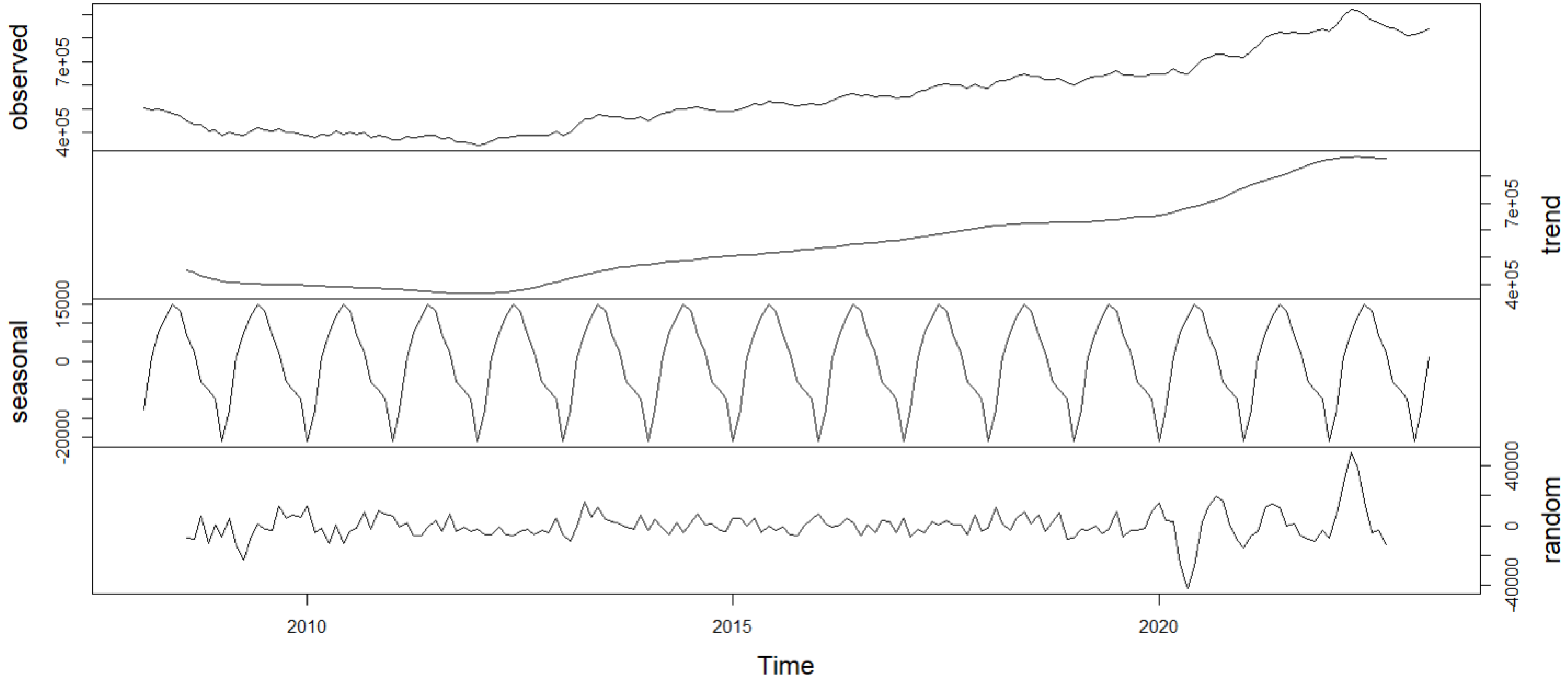

Median Sale Price, Raw, All Homes, Monthly
Price ($)
$750,000
$500,000
$250,000
2009
2011
2013
2015
2017
2019
2021
2023
2025
Year
Los Angeles, CA
New York, NY
United States
Chicago, IL

Time Series EDA for Los Angeles, CA

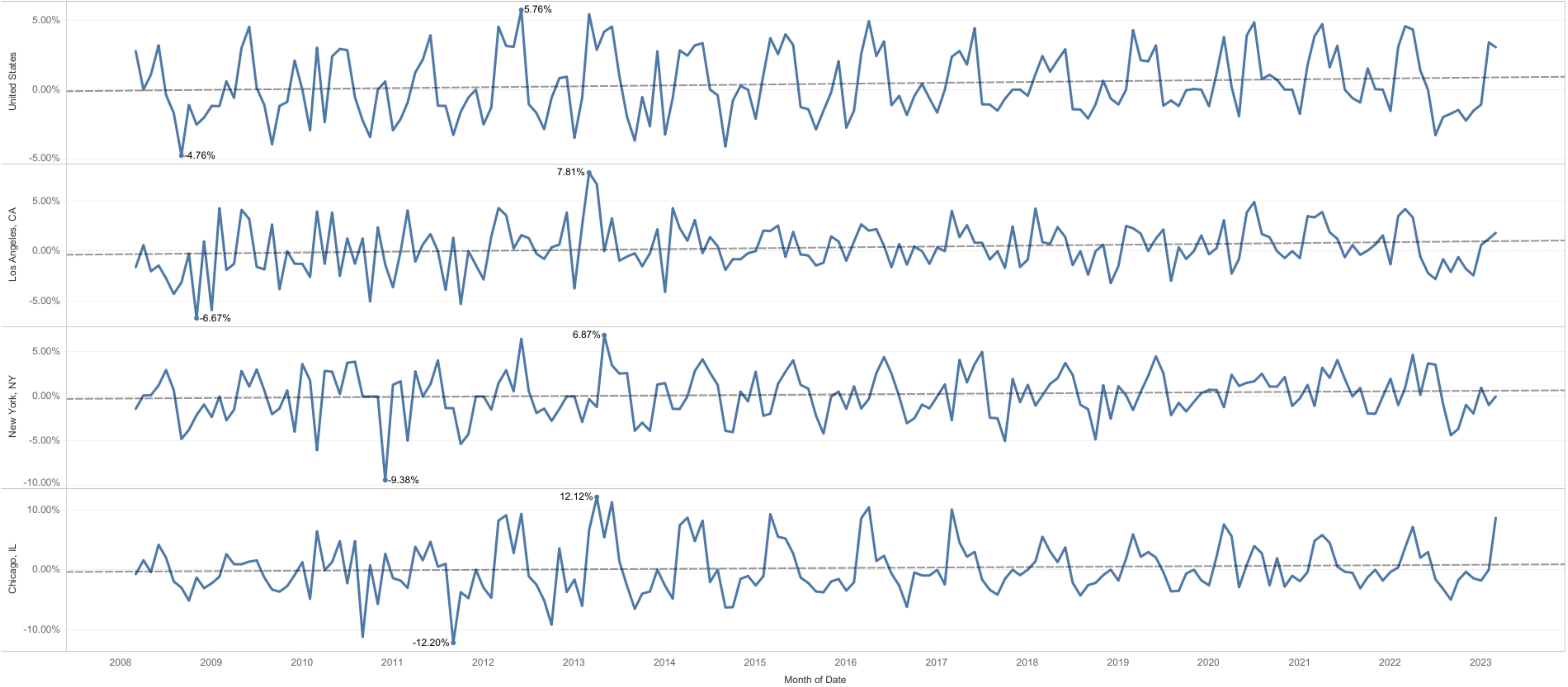




Decomposition of additive time series



Volatility Over Time



Generalized

Auto

Regressive

Conditional

Heteroskedasticity

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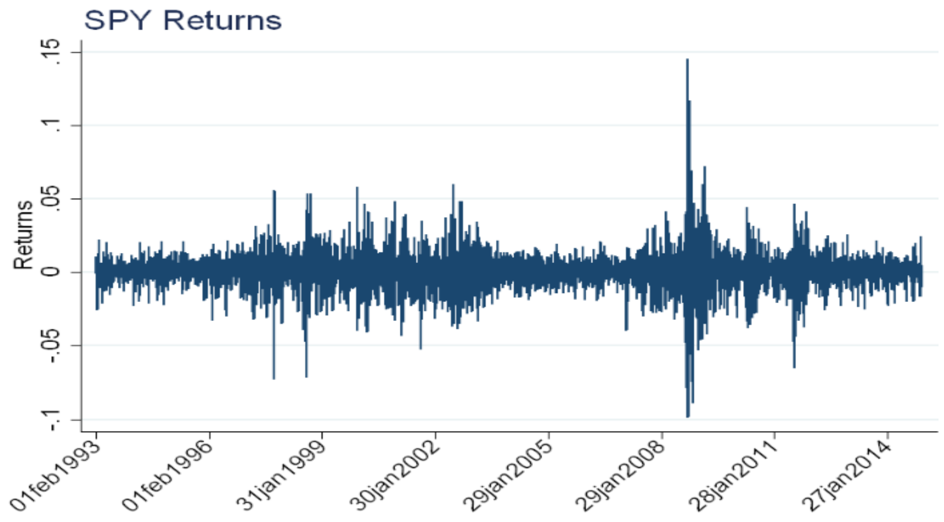
GARCH Background

- Paul (Nik) Lopez

Why are we using GARCH

1. One of the most popular models to forecast volatility in the industry
2. The model can capture volatility clustering that is often observed in time series, particularly in prices and other financial data.
3. The model can capture the fact that volatility can change over time.
4. Model flexibility.
   - The GARCH model has many extensions, such as the EGARCH (Exponential GARCH), that can accommodate asymmetric effects in volatility changes (not pursued here).



Like AR and ARMA, GARCH is an extension of ARCH

<table>
<tr><th>ARMA(1) an extension of AR(1)</th><th>GARCH(1,1) an extension of ARCH(1)</th></tr>
<tr><td>

$$\boldsymbol{AR(1): X_t = \phi X_{t-1} + Z_t}$$

$$where\ t = 0, \pm 1, \dots, and\ Z_t \sim WN(0, \sigma^2)$$

$$\begin{tikzcd} & X_{t-1} \arrow[dl] \\ X_t & Z_{t-1} \arrow[l] \\ & Z_t \arrow[ul] \end{tikzcd}$$

</td><td>

$$\boldsymbol{ARCH(1): X_t = Z_t\ \sigma_t\ = Z_t \sqrt{\alpha_0 + \alpha_1 (X_{t-1})^2}}$$

$$where\ t = 0, \pm 1, \dots, and\ Z_t \sim WN(0, \sigma^2)\ and$$

$$\sigma_t\ is\ std\ dev\ of\ return\ of\ time\ t$$

$$\begin{tikzcd} & Z_t \arrow[dl] & \\ X_t & & \\ & \sigma_t \arrow[ul] & X_{t-1} \arrow[l] \end{tikzcd}$$

</td></tr>
<tr><td>

$$\boldsymbol{ARMA\ (1,1): X_t = \phi X_{t-1} + \theta\ Z_{t-1} + Z_t}$$

$$where\ t = 0, \pm 1, \dots, and\ Z_t \sim WN(0, \sigma^2)$$

$$\begin{tikzcd} & X_{t-1} \arrow[dl] \\ X_t & Z_{t-1} \arrow[l] \\ & Z_t \arrow[ul] \end{tikzcd}$$

</td><td>

$$\boldsymbol{GARCH(1,1): X_t = Z_t\ \sigma_t\ = Z_t \sqrt{\alpha_0 + \alpha_1 (X_{t-1})^2 + \beta \sigma_{t-1}^2}}$$

$$\begin{tikzcd} & Z_t \arrow[dl] & \\ X_t & & X_{t-1} \arrow[dl] \\ & \sigma_t \arrow[ul] & \\ & & \sigma_{t-1} \arrow[ul] \end{tikzcd}$$

</td></tr>
</table>

How we forecast with the GARCH Model

Unlike other models, GARCH models returns versus the prices.

1. Take log returns or percentage changes and fit GARCH to the transformed time series
2. Use ACF and other measures to determine the correct parameters
3. Forecast volatility after the model is fit.
4. Take the last price of our data and reverse the log return or percentage change calculation to get the next predicted price

$$New\ Price = Old\ Price\ * (\,1 + vol\ forecast)$$

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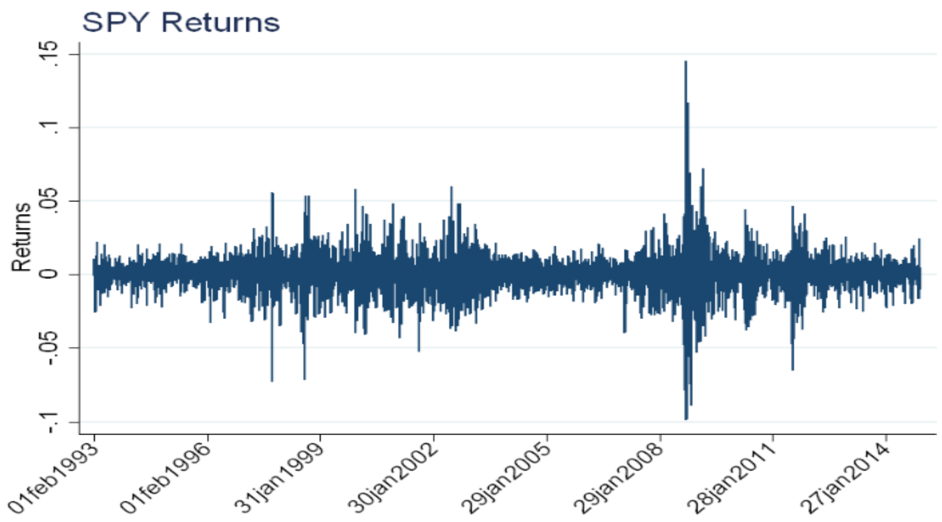
GARCH Housing Data Analysis/Results

- Horace Tsai

When/How to Use GARCH

When to Use GARCH

- Widely used when trying to predict/forecast volatility such as finance or economic data
- Time series data has high points of volatility



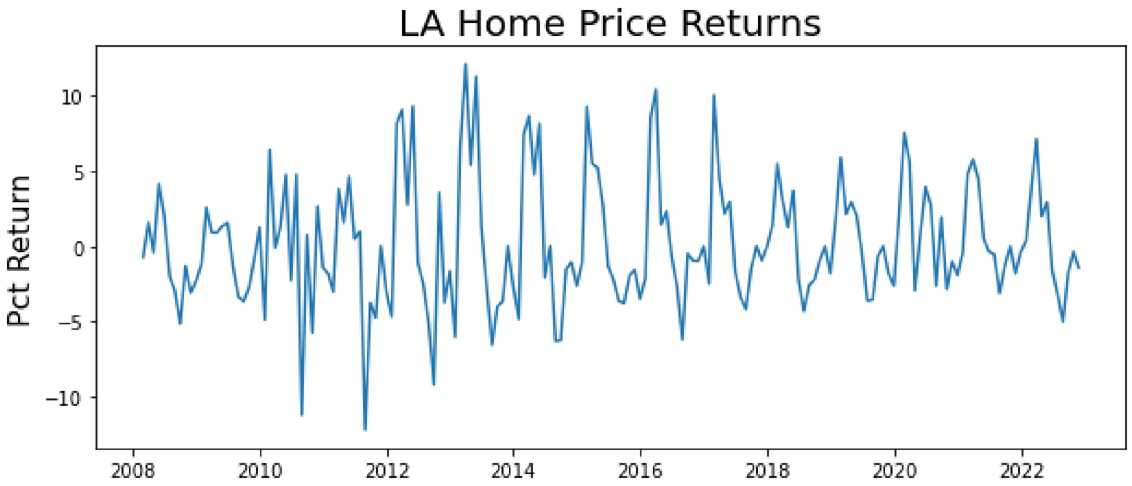
How to Use GARCH

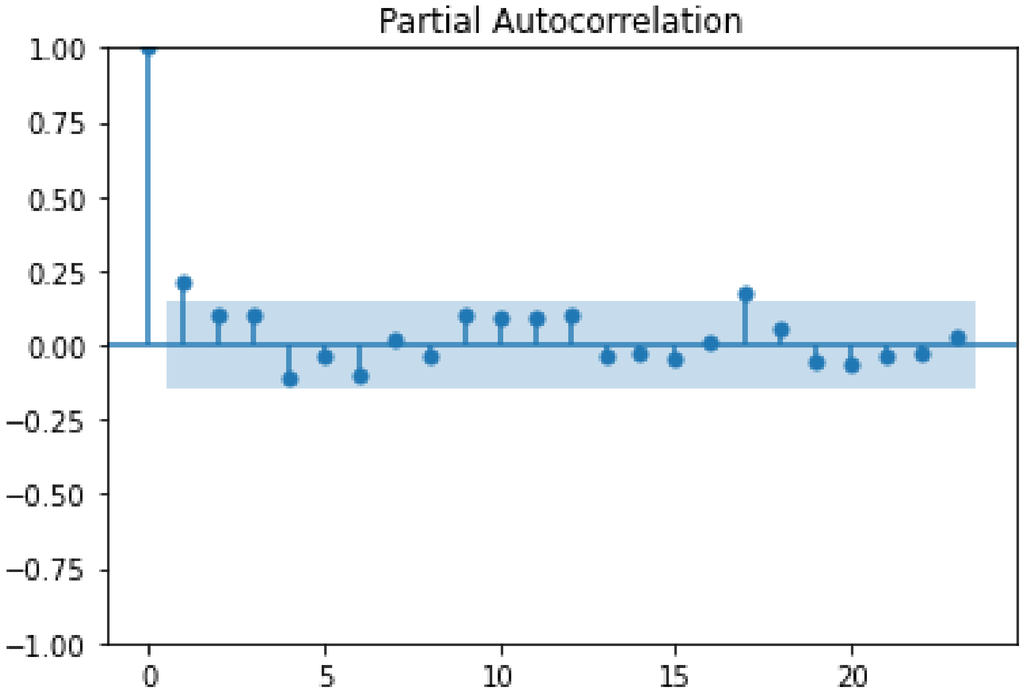
- Plot time series data to make sure data has high points of volatility
- Plot ACF and PACF
- Create GARCH Model based on analysis of ACF and PACF

Analysis/ Results of GARCH

```python
plt.figure(figsize=(10,4))
plt.plot(returns)
plt.ylabel('Pct Return', fontsize=16)
plt.title('LA Home Price Returns', fontsize=20)
```

```
Text(0.5, 1.0, 'LA Home Price Returns')
```





GARCH(2, 2) Fit

$$X_t = Z_t\ \sigma_t \ = Z_t \sqrt{\alpha_0 + \alpha_1 (X_{t-1})^2 + \alpha_2 (X_{t-2})^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2}$$

Vol Model:	GARCH	**Log-Likelihood:**	-395.646
Distribution:	Normal	**AIC:**	803.291
Method:	Maximum Likelihood	**BIC:**	822.382

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.1722	4.885	3.526e-02	0.972	[-9.403, 9.747]
alpha[1]	0.0616	1.837	3.355e-02	0.973	[-3.540, 3.663]
alpha[2]	6.9151e-10	2.791	2.478e-10	1.000	[-5.470, 5.470]
beta[1]	2.2291e-08	2.735	8.150e-09	1.000	[-5.360, 5.360]
beta[2]	0.8925	2.511	0.355	0.722	[-4.028, 5.813]

GARCH(1, 1) Fit

$$X_t = Z_t\ \sigma_t \ = Z_t \sqrt{\alpha_0 + \alpha_1 (X_{t-1})^2 + \beta \sigma_{t-1}^2}$$

Vol Model:	GARCH	**Log-Likelihood:**	-396.589
Distribution:	Normal	**AIC:**	801.178
Method:	Maximum Likelihood	**BIC:**	813.906

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.1081	0.156	0.694	0.487	[-0.197, 0.413]
alpha[1]	0.0325	0.106	0.306	0.760	[-0.176, 0.241]
beta[1]	0.9405	0.127	7.396	1.407e-13	[0.691, 1.190]

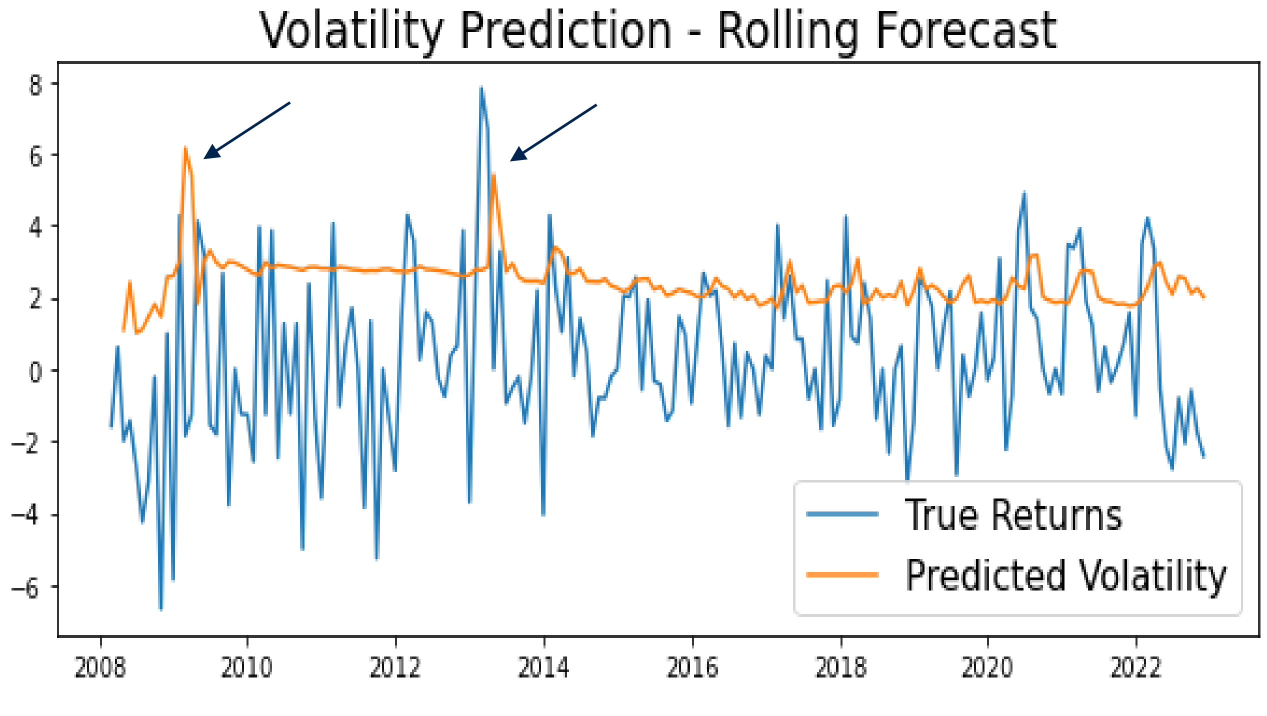
GARCH(2, 0) = ARCH(2) Fit

$$X_t = Z_t\ \sigma_t\ = Z_t\sqrt{\alpha_0 + \alpha_1(X_{t-1})^2 + \alpha_2(X_{t-2})^2}$$

Vol Model:	ARCH	**Log-Likelihood:**	-396.393
Distribution:	Normal	**AIC:**	800.786
Method:	Maximum Likelihood	**BIC:**	813.513

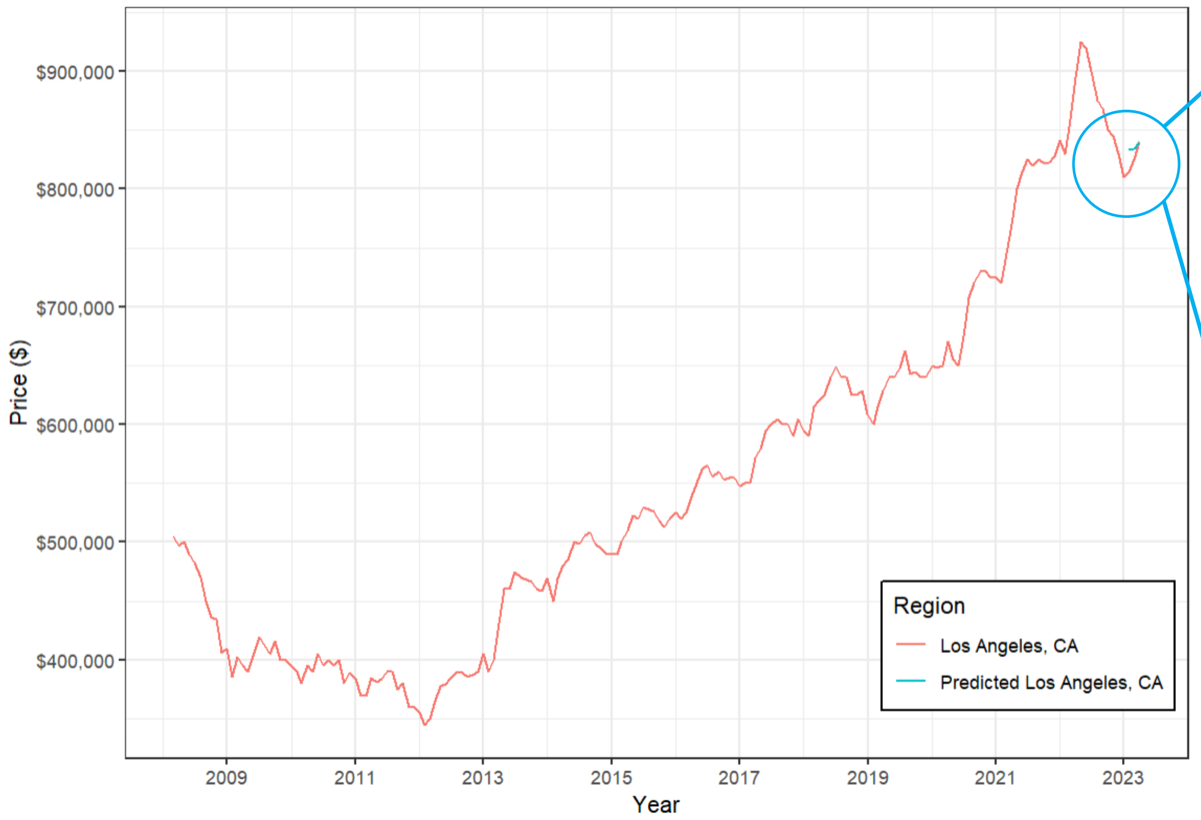
Volatility Model

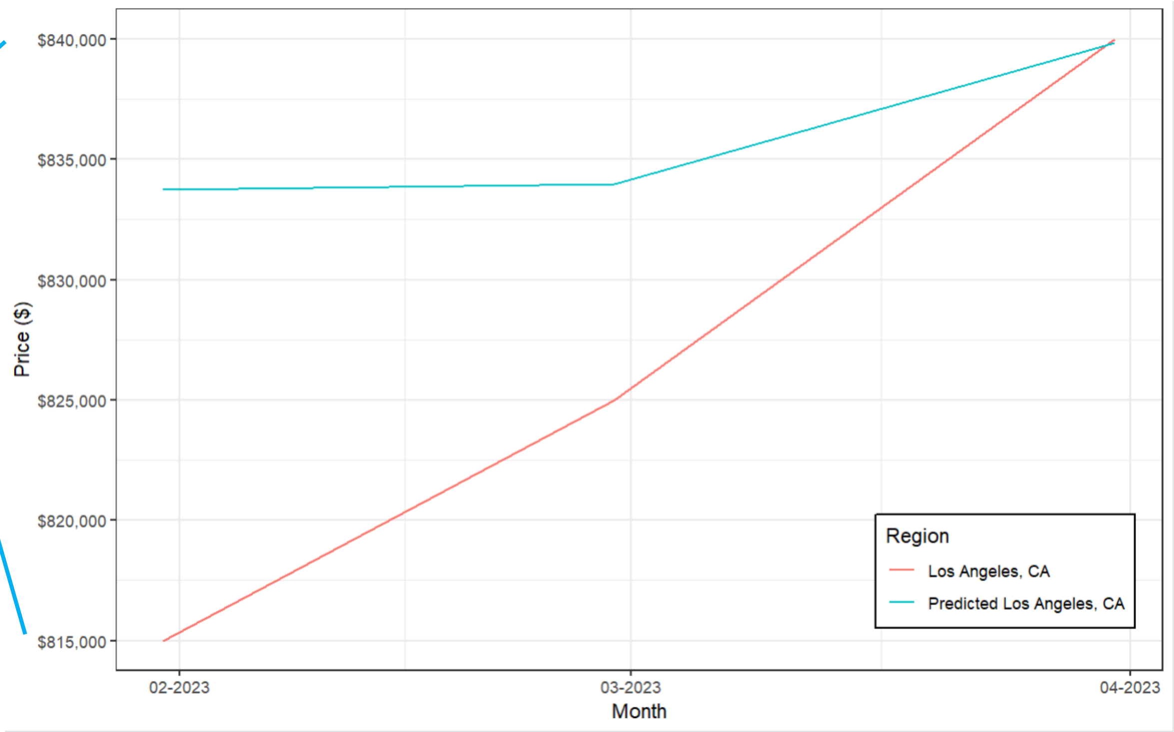
	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.1326	0.640	4.894	9.904e-07	[1.878, 4.387]
alpha[1]	0.1488	8.041e-02	1.850	6.430e-02	[-8.836e-03, 0.306]
alpha[2]	0.2902	0.149	1.953	5.082e-02	[-1.040e-03, 0.581]



Forecast with Garch(2,0)

GARCH(2,0) Price Predictions for First Quarter 2023





Date	Actual	Predicted	Difference
2023-01-31	$815,000	$833,745	$18,745
2023-02-28	$825,000	$833,989	$8,989
2023-03-31	$840,000	$839,850	-$150

Analysis/ Results of Forecast

Model

- Our ARCH(2) fit the best with the lowest AIC and significant values

Model	AIC	Ω (p-value)	α1 (p-value)	α2 (p-value)	β1 (p-value)	β2 (p-value)
GARCH (2,2)	803.29	0.972	0.973	1.00	1.00	0.722
GARCH (1,1)	801.18	0.487	0.760	N/A	1.407e-13	N/A
GARCH (2,0)	800.79	9.904e-07	6.430e-02	5.082e-02	N/A	N/A

Predictions

- Ran our model to forecast volatility for the Months of January, February, and March
- Used the forecasted volatility to compute our new prices by using the following formula:

$$PercentChangeReturns = \frac{newprice - oldprice}{oldprice} * 100$$

Date	Actual	Predicted	Difference
2023-01-31	$815,000	$833,745	$18,745
2023-02-28	$825,000	$833,989	$8,989
2023-03-31	$840,000	$839,850	-$150

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Conclusion

- Lucas Everett

General Shortcomings of the GARCH Model

- The biggest shortcoming is volatility itself

General Shortcomings of the GARCH Model

- Assumes volatility based on prior records and conditional variances
  - Models can be unstable for highly volatile data → incorrect forecasts
- Selecting the correct GARCH model can be cumbersome
- High computational cost due to intensive fitting of complex models to large data sets

Extensions on GARCH

Could have tested other kinds of GARCH models on the data

- IGARCH (Integrated) - restricts the parameters
- NGARCH (Nonlinear) - addresses correlation
- EGARCH (Exponential) - accommodates asymmetric effects

Each distinct GARCH model tries to help access and interpret how the positive and negative changes in the series affect volatility and the return in assets/profits/losses.

Where Can We Go From Here ...

With the data available, there are a lot of different analyses one could zone in on:

- COVID-19 housing market affect
  - Was there a fundamental change in the volatility structure of the time series?
- Median housing prices of states before/after election season
- Try modeling other regions in the dataset
- Implement EGARCH model and other extensions

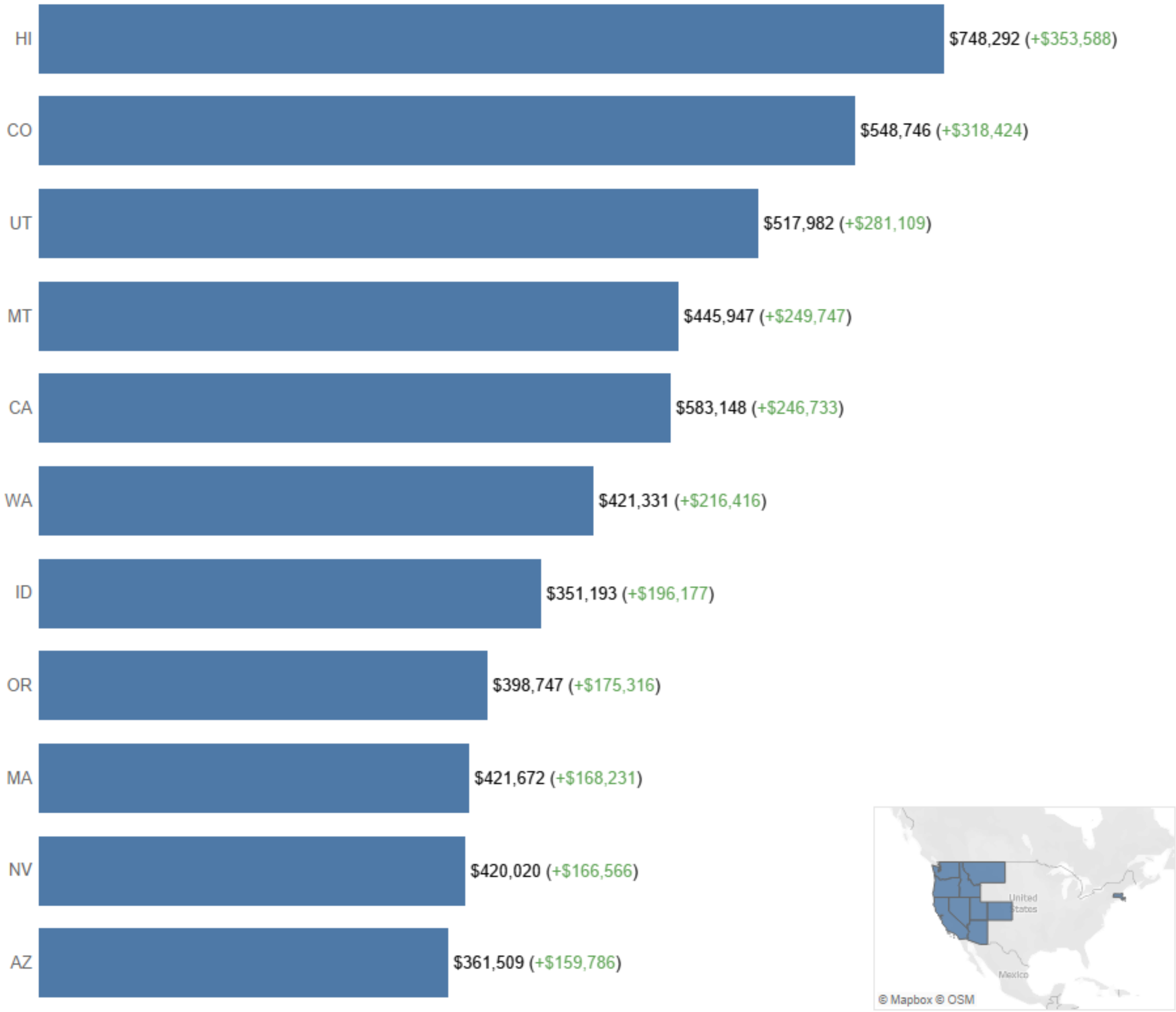
Conclusion

- Certain housing sale prices were shocking/counterintuitive to our initial thoughts after performing EDA
- ARCH(2) worked well for predicting Median Sale Price for Los Angeles, CA
- Small step towards learning more about ARCH/GARCH and becoming the best time series experts on the block!

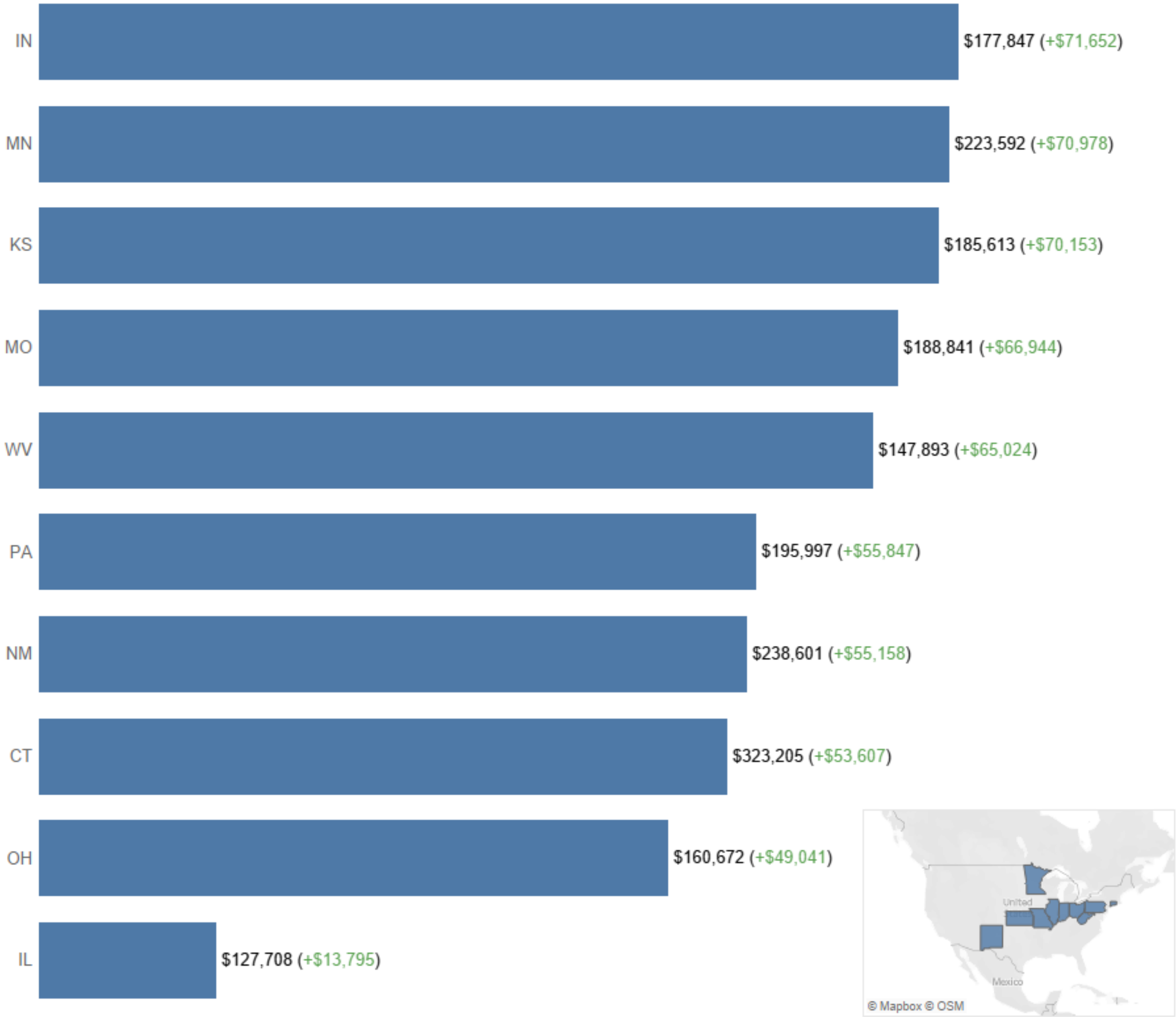
Questions & Answers?

Winners and Losers Since 2008 (sorted on increase):

Top 10 Increases



Bottom 10 Increases



Dashboard link