Macroprudential Policy with Firm Heterogeneity

Emilio Zaratiegui*

Columbia University

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Abstract

I study optimal macroprudential policy when its costs on investment and productivity are taken into account. To do so, I introduce a tractable way of modeling misallocation that generates a link between investment and productivity and can be easily taken to the data. Because macroprudential policies affect investment, they lead to productivity losses. I show that, when the policymaker is constrained in their available instruments, this generates a policy trade-off between financial stability and productivity growth. I derive a formula for the second-best policy that only requires a few sufficient statistics, including its productivity costs. I leverage the tractability of my model to get a range of estimates for the latter using rich firm-level microdata for several European countries. The trade-off is quantitatively relevant: for the baseline crisis probabilities, productivity losses reduce optimal capital controls from 0.5% to a subsidy of almost 0.4%. Productivity losses are also a source of heterogeneity, with capital controls varying as much as 0.35% within the countries in the sample.

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1 Introduction

Macroprudential policies have become a key policy tool in recent years, as shown by the introduction of Basel III and their recognition as a valid policy instrument by the International Monetary Fund (2022) [1]. They are supported by an extensive literature (Bianchi and Mendoza, 2020), which focuses on their role in ensuring financial stability. For small open economies, where foreign borrowing is a significant share source of credit, these policies usually take the form of capital controls.

At the same time, there has been increasing attention on the role of international capital flows in alleviating (Forbes, 2005; Larrain and Stumpner, 2017; Varela, 2017; Bau and Matray, 2023) or exacerbating misallocation (Gopinath et al., 2017; Benigno and Fornaro, 2014). This role introduces an additional consideration to the study of macroprudential policy.

In this paper, I study how optimal macroprudential policy chances when these two forces are present. To do so, I build on the seminal open economy model with borrowing constraints developed by Bianchi (2011) to which I introduce investment and capital misallocation across firms. I introduce a tractable way of modeling misallocation that generates a link between investment and productivity and has a clear mapping to the data. I leverage this feature to estimate the elasticity of productivity to investment for a sample of European countries using firm-level microdata.

Using this model, I first study the determinants of optimal macroprudential policy. On the one hand, pecuniary externalities mean that households do not fully internalize the costs of their consumption, leading them to overborrow. Capital controls, can correct this, as is standard in the literature. On the other hand, by increasing borrowing costs, they reduce investment and productivity. When the government can separately tax or subsidize borrowing and investment, these forces generate no trade-off. I show that the planner implements capital controls to curb borrowing, and subsidizes investment to ensure households invest the optimal amount.

I then show that a trade-off exists when the government can control the total amount of borrowing using capital controls but cannot alter its composition between investment and consumption. In this second-best scenario, I show that the policymaker faces a trade-off between restricting borrowing to reduce consumption, and incentivizing borrowing to increase investment.

I derive a formula for this capital control, which depends on aa sufficient statistic that depends on easily measurable elements. I show how each of these affect optimal policy and

[1] See Cerutti et al. (2017) for a survey of its prevalence across countries

how to measure them using available data, The second-best capital controls depend on the probability of a crisis and the magnitude of the productivity losses that result from capital controls. I use the range of productivity losses that I estimate in the data and find that, when the probability of a crisis is low, the second-best capital controls, implemented as a tax on debt, are as low as -1.25%. As a crisis becomes more likely, the capital controls are as high as 3.4%.

My starting point is the canonical model of macroprudential policy developed by Bianchi (2011). Households derive utility from consuming tradable and non-tradable goods. The former can be traded with the rest of the world, along with non-contingent financial claims. Households own an endowment of non-tradable goods and supply labor and capital to tradable goods producers that produce individual varieties which are aggregated by competitive firms.

Households are subject to a borrowing constraint that depends on their income measured in units of tradable goods. When the constraint binds, households are forced to deleverage by reducing their consumption of tradable goods. This reduces demand for non-tradable goods as well, pushing its price down and reducing the market value of household's income. This further tightens the borrowing constraint, which gives way to another round of deleveraging in a vicious cycle. These dynamics capture the stylized facts of sudden stops, as financial crises with large current account reversals are known.

The pecuniary externality arises because households do not internalize that increasing consumption of tradable goods increases the value of non-tradable and therefore their ability to borrow when the borrowing constraint binds. By increasing consumption during a crisis, macroprudential policies that reduce borrowing during good times can be welfare improving. An example of these policies are capital controls, which increase the cost of foreign borrowing.

Because the model studied by Bianchi (2011) and much of the literature does not consider production economies, they do not capture the effects of macroprudential policy on capital accumulation and productivity. To study the qualitative and quantitative implications of this cost, I add production of tradable goods using labor and capital. To further look at the productivity implications, I consider a setting where firms produce individual varieties which are aggregated with a CES technology. As shown by Hsieh and Klenow (2009), this allows a clear mapping from the distribution of distortions at the firm level to total factor productivity.

I model misallocation by introducing firm-level frictions in the rental market for capital. Domestic banks intermediate between households and the firms. The amount of capital firms can rent from a bank is subject to distortions, which leads to dispersion in the marginal return to capital across firms. I assume that these distortions do not grow proportionally with capital, which implies that an increase in the aggregate amount of capital reduces misallocation in the economy. This is in line with the literature on the link between financial development and growth (Rajan and Zingales, 1998; Levine, 2005) and with the experience of countries that lifted capital controls (Larrain and Stumpner, 2017; Varela, 2017) or restrictions to investment (Bau and Matray, 2023).

I then derive a closed form expression for the link between total factor productivity and aggregate capital, which resembles a production externality. In this setting, capital controls have two adverse effects on output. First, the increase in borrowing cost raises the required rate of return on capital, leading to a decrease in investment. Second, reduced investment, through increased misallocation, negatively affects productivity, which means that the same amount of capital produces fewer goods.

As a benchmark, I first study the problem of a policymaker that is not constrained in its available instruments. Because it can tax borrowing and investment separately, it faces no trade-off. Capital controls address the pecuniary externality and reduce consumption when the economy is not in a crisis, while a subsidy to investment addresses the productivity externality.

I then turn to a more realistic setting, where the policymaker can only control the total level of debt but not its composition between investment and consumption. In this scenario, I find that the planner faces a trade-off between the two objectives: financial stability and productivity growth. The former requires a capital control to reduce consumption during good times, while the latter needs a subsidy on foreign borrowing to encourage investment and productivity growth.

To gain more intuition, I show how the second-best capital control depends on a key sufficient statistic composed of five measurable objects: the probability of a sudden stop, the productivity externality, the marginal propensity to invest in tradable sectors, the marginal propensity to consume, and the elasticity of substitution between tradable and non-tradable goods.

The probability of a sudden stop determines the importance of the macroprudential component. When a crisis is more likely, capital controls should increase to deter borrowing. I consider a range of values for this probability, from the unconditional probability for developed economies (1.7%) to the highest probability in the "red-zones" framework of Greenwood et al. (2022) (37% − 45%). For the former, the policymaker subsidizes borrowing for plausible parameters but imposes strong capital controls for the latter, with interest rates increasing by almost 3.5 percentage points.

The productivity externality is one of the main drivers of the policy trade-off. In one of the key contributions of the paper, I leverage its closed form expression and estimate a range of plausible values using microdata. Using firm-level balance sheet data from Orbis-Amadeus, which has an excellent coverage of European private firms, I measure the dispersion in the marginal returns to capital across firms, which directly informs my estimates. I estimate an elasticity of total factor productivity to investment that ranges between 0.14 and 0.3 for a sample of European economies. At the baseline scenario, where a crisis is not likely, this implies that economies at the top of the range should have a subsidy three times as large as economies at the lower end of the range.

The marginal propensity to invest in tradable production is a key object when the government cannot control the allocation of credit. For low values, most of the increase in credit will finance consumption, which is what the policymaker wants to avoid. Lastly, the marginal propensity to consume and the elasticity of substitution between tradable and non-tradable goods determine the strength of the pecuniary externality. A reduction in debt during good times increases available resources during a sudden stop. The marginal propensity to consume indicates how this translates to tradable consumption. In turn, the elasticity of substitution determines how this increase in consumption will affect the price of non-tradable goods. If either the consumption or the price response is small, this will weaken the externality and the case for macroprudential policy.

I find that the investment and productivity costs have significant effects on the second-best capital controls. Evaluated at the unconditional probability of a sudden stop, my formula predicts a 0.5% tax on foreign borrowing when these effects are not taken into account. Once they are considered, the tax turns into a subsidy of almost 0.4% for the median country in the sample, with almost two thirds of this reduction driven by effects on productivity. Moreover, the latter are a significant source of heterogeneity, generating a maximum difference of 0.35% in capital controls between countries in the sample.

Related Literature. This paper connects two strands of the literature. The first one studies optimal macroprudential policy in a context of pecuniary externalities and financial frictions that make the competitive equilibrium inefficient. The second one studies how frictions in factor markets lead to, through misallocation of resources across firms, reduced aggregate productivity.

Most of the macro-finance literature studying macroprudential policy builds on the insights of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) that showed how financial frictions can amplify the effect of shocks, formalizing an old intuition (Fisher, 1933). The combination of incomplete markets and the role prices play in them lead to the emergence of pecuniary externalities, as studied by Clayton and Schaab (2022); Dávila and Korinek (2017); Gromb and Vayanos (2002); Lorenzoni (2008). In many cases, these externalities lead to inefficiencies, motivating the study of policy interventions. This normative literature, starting with Jeanne and Korinek (2010) and Bianchi (2011), has mostly focused on small open economies.[2]

The main contribution of this paper is to build upon the seminal setup of Bianchi (2011), which has been extensively used (Benigno et al., 2013; Arce et al., 2019; Flemming et al., 2019; Seoane and Yurdagul, 2019; Schmitt-Grohé and Uribe, 2020; Bengui and Bianchi, 2022) to study how effects on productivity affect optimal macroprudential policy.

I make two contributions to this literature. The first one is considering both capital accumulation and endogenous productivity, which increases the costs of macroprudential policy in a second-best setting. In this regard, the model I study resembles the one in Bianchi and Mendoza (2020), which also features capital accumulation, but has exogenous productivity. By analytically characterizing second-best policy in this setting, I show that considering these additional affects optimal macroprudential policy.

This also displays the second contribution to this literature of this paper, which is to characterize both the constrained-efficient and second-best policy, and leverage this characterization to provide a quantification of this trade-off as a function of measurable sufficient statistics. This approach is similar to Dávila and Korinek (2017), who also characterize macroprudential policy using sufficient statistics. Compared to their work, I find a direct mapping to the data which allows me to quantify the trade-off.

In this paper, I also draw on the findings of a recent literature that has studied the effects of capital controls on productivity. Larrain and Stumpner (2017) and Varela (2017) study episodes where restrictions on capital mobility were lifted in Eastern European countries, while Bau and Matray (2023) study the easing of restrictions on foreign equity investments in India. All of them find that easing controls reduces the cost of capital more for constrained firms, which results in increased aggregate productivity through improved allocation of capital. These results are also in line with an earlier literature surveyed by Forbes (2005).

Studying the credit boom in Southern Europe during the introduction of the Euro, Gopinath et al. (2017) find that a decrease in productivity in the manufacturing sector through increased misallocation.[3] This would suggest that capital inflows can have negative effects on productivity, in line with a theoretical literature on credit booms (Benigno et al., 2016; Reis, 2013). While in the quantitative exercise I consider a positive effect of capital inflows on productivity, the paper still provides a framework to consider these effects when studying optimal macroprudential policy.

[2]Noteworthy exceptions are Farhi and Werning (2016), and Elenev et al. (2021), who study the effects of macroprudential policies in a quantitative model of a closed economy.

[3]See also Calligaris et al. (2018); Dias et al. (2016)

These papers are part of a broader literature[4] on how misallocation of factors across firms reduces aggregate productivity. I follow the indirect measuring framework introduced by Hsieh and Klenow (2009), which allows me to link model and data in a tractable manner. This approach was also used by David and Venkateswaran (2019) and David et al. (2021) to decompose the sources of misallocation, finding that technological frictions can explain only a small share of total misallocation between firms. I contribute to this literature by embedding this setup into an optimal policy analysis that takes into account these empirical results.

In that regard, the paper is related to a growing literature that studies the effects on productivity through misallocation of different policies. Baqaee et al. (2024) and González et al. (2024) study how misallocation affects the transmission of monetary policy, and Kurtzman and Zeke (2020) show that quantitative easing can increase misallocation. Lastly, David and Zeke (2024) study how the monetary policy regime shapes the dynamics of TFP and how optimal monetary policy is affected by this mechanism.

Layout The remainder of the paper is organized as follows. Section 2 introduces the model that will guide my analysis. Section 3 defines and characterizes the equilibrium, and discusses the inefficiencies that motivate studying optimal policy. Then, Section 4 sets up and characterizes optimal policy, while Section 5 studies its determinants. In Section 6, I leverage the results from Section 4 to present quantitative results. Lastly, Section 7 concludes.

2 Model

To study the policy trade-off, I build on the canonical model presented in Bianchi (2011) to which I add production in the tradable sector along with financial frictions that differ across firms. Aggregate risk is given by the realization of tradable productivity.

2.1 Environment

I study a small open economy composed of households, firms that produce tradable goods and domestic banks that intermediate between households and firms. Households trade bonds and goods with the rest of the world, while firms can export tradable goods.

[4](see, e.g. Restuccia and Rogerson, 2013; Hopenhayn, 2014)

Households. This block follows closely the setup in Bianchi (2011). Households have preferences over a final consumption good, c_t, described by

$$\mathbb{E}_0\left[\sum_t \beta^t \frac{{c_t}^{1-\sigma}}{1-\sigma}\right] \tag{1}$$

The final good is produced using tradable, $c_{T,t}$ and non-tradable, $c_{N,t}$ goods:

$$c_t = \left[\omega {c_{T,t}}^{\frac{\xi-1}{\xi}} + (1-\omega){c_{N,t}}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{2}$$

To simplify the exposition without significantly affecting the results, I assume that $\sigma = \frac{1}{\xi}$, which removes non-tradable consumption from the marginal utility of tradable goods, and that the household's discount factor equals the world interest rate, $\beta = 1 + r$.

Households are able to borrow, or save, through a one period, risk-free bond d_t denominated in units of tradable goods traded with the rest of the world at an exogenous rate r. Households also accumulate physical capital k_t, which is rented to tradable firms at rate R_t. Because a unit of capital requires one tradable good, its relative price is one. Lastly, they receive a fixed endowment of non-tradable goods, y^N, and are the owners of the tradable firms and the domestic banks.

I assume that the tradable good is the numéraire, so that p_t is the relative price of non-tradable goods. The budget constraint of the households, omitting references to the state of the economy, is then given by

$$c_{T,t} + p_t c_{N,t} + k_{t+1} = p_t y^N + w_t + (1-\delta+R_t)k_t - (1+r)d_t + d_{t+1} + \pi_t \tag{3}$$

where π_t are profits from firms and w_t is the wage rate.

As is common in the macroprudential literature, households also face a borrowing constraint that depends on the market value of their income:

$$d_{t+1} \leq \kappa(w_t + R_t k_t + \pi_t + p_t y^N), \tag{4}$$

where $\kappa < 1$ is a constant. The intratemporal condition follows from combining the first order conditions for tradable and non-tradable goods

$$p_t = \frac{1-\omega}{\omega}\left(\frac{c_{T,t}}{c_{N,t}}\right)^{\frac{1}{\xi}}, \tag{5}$$

which pins down the relative price of non-tradables. Keeping $c_{N,t}$ fixed, p_t is increasing in tradable consumption $c_{T,t}$ with constant elasticity determined by ξ. Intuitively, the more complementary both goods are, which translates to lower ξ, the higher the price response to changes in tradable consumption will be.

The Euler equation for debt d_{t+1}, is given by

$$c_{T,t}^{-\sigma} = \frac{1}{1-\mu_t}\mathbb{E}\left[c_{T,t+1}^{-\sigma}\right], \tag{6}$$

where μ_t is the Lagrange multiplier on the borrowing constraint (4). As usual, the households seek to smooth consumption between periods. When the constraint binds, $\mu_t > 0$ acts like a wedge in this Euler equation. To see this, re-arrange (6) as

$$\mu_t = 1 - \mathbb{E}\left[\left(\frac{c_{T,t+1}}{c_{T,t}}\right)^{-\sigma}\right]$$

Expressed in this way, μ_t measures the distance from the desired consumption smoothing when the borrowing constraint binds. Within the model, I consider periods where the borrowing constraint bind to be a crisis or sudden stop. In this way, μ_t serves both as an indicator of whether a crisis is happening, $\mu_t > 0$, and its severity, measured as the drop in consumption relative to a non-crisis state.

Lastly, combining the Euler equations for debt and capital, I derive an expression for the expected return to capital,

$$\frac{r+\delta+\mu_t(1-\delta)}{1-\mu_t} = \frac{\mathbb{E}\left[c_{T,t+1}^{-\sigma}R_{t+1}(1+\kappa\mu_{t+1})\right]}{\mathbb{E}\left[c_{T,t+1}\right]^{-\sigma}} \tag{7}$$

Ignoring μ_t, this is the standard condition for the expected return to capital, which should be, after discounting, equal to $r+\delta$. The presence of crises has two effects on the capital accumulation decision. First, when the constraint is presently binding, $\mu_t > 0$, this raises the required rate of return on capital, lowering investment. When the constraint is expected to bind in the next period, $\mu_{t+1} > 0$ the effect is the opposite. Because households understand that more income will increase their borrowing ability, they derive an additional benefit from investing an extra unit of capital, given by $\kappa\mu_{t+1}$.

Conditions (5), (6), (7), along with the budget constraint (3), the borrowing constraint (4), and transversality and complementary slackness conditions characterize the solution to the household's problem.

Final Tradable Goods Producers. A competitive firm aggregates differentiated varieties $y_t^T(i)$ to produce a final tradable good according to:

$$y_t^T = \left[\int_0^1 y_t^T(i)^{\frac{\eta-1}{\eta}} \mathrm{di}\right]^{\frac{\eta}{\eta-1}} \tag{8}$$

Solving their cost minimization problem yields a demand for variety i

$$p(i) = \left(\frac{y_t^T}{y_t^T(i)}\right)^{\frac{1}{\eta}}, \tag{9}$$

which depends on total production y_t^T and the price of variety i, $p(i)$. The elasticity of substitution between varieties, η, determines the elasticity of demand.

Intermediate Goods Firms. Firm i uses capital and labor h to produce its variety according to the technology

$$y(i) = A(i)k(i)^{\alpha}h(i)^{1-\alpha} \tag{10}$$

Productivity is determined by an aggregate component A and an invariant firm-specific component $a(i)$ such that, in logs,

$$\log A(i) = \log A + \log a(i), \tag{11}$$

where, across firms, $\log a(i) \sim \mathcal{N}(0, \sigma_a^2)$. While all firms face the same wage w, I assume that they face different costs of capital $R(i)$, which for now I take as given. Internalizing demand for their variety, their problem is given by

$$\max_{k_t(i),h_t(i)} (1+s)\left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} y_t^{T\frac{1}{\eta}} - R_t(i)k_t(i) - w_t h_t(i), \tag{12}$$

where $1+s = \frac{\eta}{\eta-1}$ is a production subsidy financed with a lump-sum tax that corrects the inefficiency arising from the markup.[5]

The following standard result from the misallocation literature allows me to aggregate firms into a representative firm.

Lemma 1 (Firm Aggregation)**.** *Let σ_2^R be the variance of the cost of capital, $R(i)$. Up to second order, tradable production can be represented by an aggregate firm with production*

[5]This subsidy, which is analogous to the commonly used subsidy in New-Keynesian models, ensures that production is at its socially optimal level. Removing this distortion simplifies the analysis of optimal policy by reducing the scope of intervention for the planner with no qualitative implications.

technology

$$y_t^T = \mathrm{TFP}_t \; k_t^{\alpha} h_t^{1-\alpha}, \tag{13}$$

where

$$\log \mathrm{TFP}_t \; = \log A_t + \frac{1}{2}(\eta - 1)\sigma_a^2 - \frac{1}{2} \; \alpha(1 + \alpha(\eta - 1))\sigma_R^2 \tag{14}$$

and the solution to its problem is characterized by,

$$w_t = (1-\alpha)\, \mathrm{TFP}_t \left(\frac{k_t}{h_t}\right)^{\alpha} \tag{15}$$

$$R_t = \alpha\, \mathrm{TFP}_t \left(\frac{h_t}{k_t}\right)^{1-\alpha} \tag{16}$$

This approximation is exact if $R(i)$ is log-normally distributed.

Proof. See A.1 □

This result is very useful to keep the analysis tractable because it shows that it is not necessary to keep track of the entire distribution of $R(i)$, just its variance σ_R^2. As a corollary, it follows that for any variable to have an impact on productivity, it must affect the variance of $R(i)$.

Market Clearing. By definition, the non-tradable goods market clears every period:

$$c_{N,t} = y^N \quad \forall \; t, \tag{17}$$

as do the factor markets,

$$h_t = 1 \quad \forall \; t \tag{18}$$

$$k_t = k_t \quad \forall \; t \tag{19}$$

I now explain in detail the determination of σ_R^2, which will lead to the policy trade-off.

2.2 Capital Misallocation

In this section I introduce the key departure in this model from the existing small open economy models in order to model σ_R^2 and its dynamics.

Capital is owned by households, who decide on $t-1$ how much capital to supply in t. Domestic banks intermediate between the firms and the households. Credit markets are segmented, with each market corresponding to a single firm and bank[6]. Each bank takes deposits $\hat{k}(i)$ from the households, which they use to supply the firm in their market and cover intermediation costs, given by the realization of $f(i)$. As such, capital available to firm i is given by

$$k_t(i) = \hat{k}_t(i) - f(i)$$

where $f(i)$ stands in for both overhead costs such as setting up a branch or a relationship with a firm as well as variable costs.[7]

Given market clearing, this determines the cost of capital faced by the firm, and charged by the bank, which is described by

$$R(i) \propto \left(\hat{k}_t(i) - f(i)\right)^{-\frac{1}{1+\alpha(\eta-1)}}$$

Banks choose $\hat{k}_t(i)$ to maximize their expected profits $R(i)\left(\hat{k}_t(i) - f(i)\right) - R\hat{k}_t(i)$, where R is the rate paid to households. To keep this problem tractable, I make the following two assumptions:

Assumption 1 (Segmented Markets)**.** *After shocks $f(i)$ are realized, banks cannot transfer capital between markets.*

Assumption 2 (Shock Scaling)**.** *Shocks are given by $f(i) = A(i)^{\nu(\eta-1)}\hat{k}(i)^{1-\nu}F(i)$, where $F(i)$ is a random variable and $\nu \geq 0$.*

Assumption 2 provides structure on the dependence on the intermediation costs on $\hat{k}(i)$, which is given by ν. For $\nu < 1$, costs scale less than one by one with supplied capital, which would occur if a part of them are fixed. For $\nu = 1$, costs are fixed and do not depend on intermediated capital. Lastly, $\nu > 1$ would imply that total intermediation costs decrease as capital supply increases. In this paper, I will focus on the case where $\nu \leq 1$.

The bank's problem is to choose the amount of deposits $\hat{k}(i)$ from households. Assumption 1 and 2 ensure that the problem will be symmetric across banks up to the productivity of the firm, as shown by the following Lemma.

Lemma 2. *If costs $F(i)$ are scaled by $A(i)^{\nu(\eta-1)}$, then, in equilibrium, each individual bank takes deposits*

$$\hat{k}(i) = \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)\right]^{\eta-1}}k \tag{20}$$

[6]For simplicity, I assume that these banks are perfectly competitive so that there's no interest rate spread. As with the assumption on the subsidy that eliminates the intermediate good markup, I introduce this solely to reduce the number of dimensions on which the competitive equilibrium differs from the efficient allocation.

[7]$f(i)$ could also stand for unexplained credit demand such as government borrowing.

Proof. See A.5 □

As a result, the market clearing rental rate of capital $R(i)$ faced by firm i is

$$R(i) \propto \left(k_t \left(1 + k_t^{-\nu} F(i)\right)\right)^{-\frac{1}{1+\alpha(\eta-1)}} \tag{21}$$

In broad terms, this formulation captures two empirical regularities: first, firms with the same fundamentals use different amounts of capital; second, these distortions are decreasing in the aggregate amount of capital in the economy. I now go over these two points in more detail.

Firms with positive (negative) draws of $F(i)$ will use more (less) capital and pay lower (higher) rental rates. It follows that the larger the variance of $F(i)$ is, the larger σ_R^2 will be, as the following Lemma shows.

Lemma 3 (Rental Rates Dispersion)**.** *For small distortions $F(i)$, the variance of $\log R(i)$, σ_R^2, is given by*

$$\sigma_R^2 = \left(\frac{1}{1+\alpha(\eta-1)} \frac{1}{k_t^{\nu}}\right)^2 \mathbb{V}ar\left[F(i)\right], \tag{22}$$

Proof. See A.2 □

This closed form solution is an important feature, as it makes very clear the link between the aggregate capital stock and σ_R^2 and allows for an analytical characterization of the elasticity of TFP to aggregate capital, defined as

$$\gamma_t \equiv \frac{\partial \log \mathrm{TFP}_t}{\partial \log k_t}$$

Combining Lemmas 1 and 3, it is easy to see that

$$\gamma_t = \alpha(1+\alpha(\eta-1))\nu\,\sigma_R^2, \tag{23}$$

which captures how an increase in the amount of capital improves its allocation across firms. γ_t is increasing in both the initial level of misallocation in the economy, given by σ_R^2, and its elasticity with respect to k_t, ν. The latter captures how the frictions in the economy, given by the intermediation costs $F(i)$, scale with k_t. For low values of ν, these frictions are almost proportional to k_t, which means that increases in investment do not lead to significant improvements in capital allocation. On the other hand, if ν is high, small increases in k_t greatly decrease misallocation.

Discussion of Formulation. Equation (21) introduces capital misallocation as the result of reduced form intermediation costs. This has the main benefit of being very tractable, as Lemma 3 shows, a feature I will leverage in the quantification section of this article. I argue that this specification is also in line with both empirical facts and the theoretical literature on misallocation.

Regarding the empirical facts, this specification has two main predictions. First, the model predicts that economies with a higher investment rate will experience higher productivity growth. This is consistent with the literature on the deepness of financial markets and growth (Rajan and Zingales, 1998; Levine, 2005).

Second, misallocation of capital across firms increases (decreases) with decreases (increases) in credit. This is line with the literature that looked at the effect of capital controls (Bau and Matray, 2023; Larrain and Stumpner, 2017; Varela, 2017), macroprudential policies (Jiménez et al., 2017), and government spending (Pinardon-Touati, 2024). This qualitative implications are also the same as in the models used by Larrain and Stumpner (2017) and Varela (2017) to interpret their findings.

On the theoretical side, I show in Online Appendix B.1 that equation (21) can be derived from two alternative models that embody commonly used assumptions when studying misallocation: borrowing constraints arising from limited commitment, and heterogeneity in access to equity across firms. In this way, the simplicity of the model is also a feature, as it accommodates a wide class of frictions that can lead to misallocation of capital.

3 Competitive Equilibrium

Having described the environment, I now define the competitive equilibrium of this economy:

Definition (Competitive Equilibrium)**.** *A competitive equilibrium is a set of (state-contingent)*

1. *household allocations* $\{c_{T,t}, c_{N,t}, d_{t+1}, k_{t+1}\}$
2. *firm allocations* $\{k_t, h_t\}$
3. *rental rates dispersion* σ_R^2
4. *prices* $\{w_t, p_t, R_t\}$

such that, given initial allocations $\{k_0, d_0\}_t$*, the non-tradable endowment* y^N*, world interest rate* r*,* A_t*, and* σ_a^2*,*

1. *Given prices, the allocations solve the household problem.*

2. *Given prices, the allocations solve the firm's problem.*

3. *The rental rate dispersion is consistent with profit maximizing by firms.*

4. *Markets clear*

3.1 Characterization

Factor prices are pinned down by the combination of Lemma 1 and market clearing,

$$w_t = (1-\alpha)\,\mathrm{TFP}_t\,k_t^{\alpha} \tag{24}$$

$$R_t = \alpha\,\mathrm{TFP}_t\,k_t^{\alpha-1} \tag{25}$$

Combining factor prices, the budget constraint of the household and non-tradable market clearing yields the resource constraint,

$$c_{T,t} + k_{t+1} = \mathrm{TFP}_t k_t^{\alpha} + (1-\delta)k_t - (1+r)d_t + d_{t+1} \tag{26}$$

The left-hand side represents the use of resources, consumption and investment, while the right-hand side shows the resources available to the economy.

Combining market clearing and the intratemporal condition (5) yields an expression for the relative price of non-tradable goods,

$$p_t = \frac{1-\omega}{\omega}\left(\frac{c_{T,t}}{y^N}\right)^{\frac{1}{\xi}} \tag{27}$$

Because both goods are complements, an increase in tradable consumption pushes up demand for non-tradable goods. Given that supply is perfectly inelastic, this leads to an increase in the price p_t. Because this is a market clearing condition, it depends on aggregate tradable consumption $c_{T,t}$ and households do not internalize this effect of their consumption choices.

To obtain a borrowing constraint that depends on allocations, I use (24), (25) and (27) in 4, which yields

$$d_{t+1} \leq \kappa\left(\mathrm{TFP}_t k_t^{\alpha} + \frac{1-\omega}{\omega}\left(\frac{c_{T,t}}{y^N}\right)^{\frac{1}{\xi}} y^N\right). \tag{28}$$

Households can borrow up to a fraction κ of their income, which is the sum of tradable output and the value in tradable goods of their non-tradable endowment. Aggregate tradable consumption enters this constraint through its effect on the price of non-tradable goods.

I re-state the Euler equation (6) for debt, which remains unchanged, for convenience below,

$$c_{T,t}^{-\sigma} = \frac{1}{1-\mu_t}\mathbb{E}\left[c_{T,t+1}^{-\sigma}\right]$$

Combining the Euler equation for capital (7) and (25) yields

$$\frac{r+\delta+\mu_t(1-\delta)}{1-\mu_t} = \frac{\mathbb{E}\left[c_{T,t+1}^{-\sigma}\alpha\,\mathrm{TFP}_t\,k_t^{\alpha-1}(1+\kappa\mu_{t+1})\right]}{\mathbb{E}\left[c_{T,t+1}^{-\sigma}\right]} \tag{29}$$

Lastly, given Lemmas 1-3, TFP_t can be written as

$$\mathrm{TFP}_t = \mathrm{TFP}(A_t, k_t) \tag{30}$$

3.2 Sources of Inefficiency

Before moving on to analyzing optimal policy, I discuss the two features of the model that make this equilibrium inefficient. First, as it is common in this class of models, the presence of the price of non-tradable goods in the borrowing constraint (28) introduces a pecuniary externality. This is because p_t depends on aggregate tradable consumption, an effect not internalized by households. Suppose that the economy hits the borrowing constraint in period t. Because households are not able to borrow as much as desired, they reduce tradable consumption, as shown by condition (6) when $\mu_t > 0$. Lower tradable consumption decreases the value of the non-tradable endowment, lowering the maximum amount households can borrow.

A reduction in d_{t+1} reduces resources available to households in (26), further reducing consumption and investment. As I will discuss in the next section, a planner that internalizes this will place a higher value on consumption relative to households in periods where the borrowing constraint binds. In this setting, macroprudential policy can be welfare improving by increasing consumption during these episodes, which is achieved by reduced borrowing in previous periods.

The second source of inefficiency is the misallocation of capital across firms, which reduces the productivity of the economy, as indicated by Lemma 1. Ideally, the government would like to set the distortions to zero, increasing productivity. Even if it cannot do so, as I will assume, there's still scope for intervention. As shown by Lemma 3, an increase in investment reduces the weight of these distortions, leading to a better allocation of capital across firms and a lower σ_R^2. The sufficient statistic for this channel will be the elasticity of productivity to capital, γ_t.

While the pecuniary externality calls for restrictions on consumption, the imperfections on the capital market mean that increases in investment can be beneficial. Interestingly, both have opposite effects on borrowing. Given sufficient policy instruments, this poses no problem.[8] If the government, however, is limited to controlling only gross borrowing, then a conflict between objectives arises.

In the following section, I discuss optimal policy and this potential trade-off in mode detail.

4 Optimal Policy

In this section I characterize how a benevolent planner can address the inefficiencies in this economy. I am interested in the problem of a government that internalizes the effect of aggregates but is still subject to the borrowing constraint and cannot control the allocation of capital between firms.[9] I start by considering the solution to this problem when the government has two available instruments, which I call the constrained efficient allocation. This is a useful benchmark and is illustrative of the mechanisms at play.

To keep the analysis tractable, I make a number of assumptions in this section. First, I follow an approach similar to Bianchi and Lorenzoni (2022) and consider a version of the economy where uncertainty, misallocation and borrowing constraints are present only in period $\tilde{t}$. Given these assumptions, the model behaves very similarly to a three-period model. Because the focus of this paper is macroprudential policy, I assume the policymaker takes the equilibrium from $\tilde{t}$ onwards as given.[10]

The relevant input for the policymaker in $\tilde{t}-1$ is the value function of the households in $\tilde{t}$,

$$V_{\tilde{t}}(d_{\tilde{t}}, k_{\tilde{t}}, A_{\tilde{t}}) = \max_{c_{T,\tilde{t}}, k_{\tilde{t}+1}, d_{\tilde{t}+1}} u(c_{T,\tilde{t}}, y^N) + \beta V_{\tilde{t}+1}(k_{\tilde{t}+1}, d_{\tilde{t}+1}) \tag{31}$$

$$\text{s.t. } c_{T,\tilde{t}} + k_{\tilde{t}+1} = \text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha} + (1-\delta)k_{\tilde{t}} - (1+r)d_{\tilde{t}} + d_{\tilde{t}+1} \tag{32}$$

$$d_{\tilde{t}+1} \leq \kappa \left(\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha} + \frac{1-\omega}{\omega} \left(\frac{c_{T,\tilde{t}}}{y^N} \right)^{\frac{1}{\xi}} y^N \right) \tag{33}$$

Note that the production block of the economy depends only on $k_{\tilde{t}}$, and so $\text{TFP}_{\tilde{t}}$ is not affected by the constraint binding or not. Consumption and investment, in turn, are pinned

[8] The open economy assumption does some implicit lifting here. In a closed economy, savings equal investment and so it would not be possible to target both simultaneously. In an open economy, the difference between savings and investment is the current account, which gives the extra degree of freedom to the policymaker.

[9] In other words, the government takes the financial system as as a technological constraint.

[10] In making this assumption, I implicitly ignore ex-post policies, which can also be effective (Benigno et al., 2013).

down by

$$\left(\frac{c_{T,\tilde{t}+1}}{c_{T,\tilde{t}}}\right)^{-\sigma} = 1-\mu_{\tilde{t}}, \tag{34}$$

$$\frac{r+\delta+\mu_{\tilde{t}}(1-\delta)}{1-\mu_{\tilde{t}}} = \alpha \mathrm{TFP}_{\tilde{t}} k_{\tilde{t}+1}^{\alpha-1}, \tag{35}$$

in addition to constraints (32)-(33).

This characterizes implicit policy functions for consumption $c_{T,\tilde{t}}$ and investment $k_{\tilde{t}+1}$ that depend on the state. While a closed form solution is not available, it is easy to see that they depend on whether the economy is in a sudden stop or not. If the economy is not in a crisis ($\mu_{\tilde{t}} = 0$), then it is in a steady state.[11]

When the economy is in a sudden stop, $\mu_{\tilde{t}} > 0$, borrowing is pinned down by (33), while consumption and investment are determined by the Euler equations and the resource constraint (32). In this scenario, both will be lower than their steady state levels, as seen by inspecting conditions (34) and (35).

As previously discussed, when the constraint binds, tradable consumption affects the ability of households to borrow through its effect on the relative price of non-tradables. For any policymaker that seeks to implement macroprudential policy, an important object is how consumption will react to changes in its income or debt. Note that an extra dollar in tradable output or one less dollar in debt has the same effect on household choices[12]. Let $n_t \equiv \mathrm{TFP}_{\tilde{t}} {k_t}^{\alpha} + (1-\delta)k_t - (1+r)d_t$ denote these available resources. I then define the marginal propensity to consume, mpc as

$$\mathrm{mpc}_t \equiv \frac{\partial c_t}{\partial n_t}, \tag{36}$$

which summarizes the response of consumption to changes in income. Importantly, and following from the previous discussion, this is a state dependent object. If $\mu_{\tilde{t}} > 0$, any changes in $n_{\tilde{t}}$ will have limited impact, as in any permanent income model. If the economy is in a sudden stop, however, the households are closer to being hand to mouth and the mpc will be higher.

Before moving on to policy, I characterize the competitive equilibrium for $\tilde{t}-1$, which is a useful benchmark for policy. The household takes as given two policy instruments: a tax on debt τ^d, and a subsidy on the return to capital τ^k.[13] Consumption, investment and

[11] Technically, $c_{T,\tilde{t}}$ and $k_{\tilde{t}}$ are in steady-state, while TFP_T is below its steady state level due to misallocation.

[12] This just follows from inspecting the resource constraint.

[13] Formally, the household pays $(1+\tau^d)(1+r)d_{\tilde{t}}$ and receives $\tau^k k_{\tilde{t}}$ in period $\tilde{t}$.

borrowing are pinned down by,

$$1 = (1+\tau^d)\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right] \tag{37}$$

$$1 = \beta\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\alpha \mathrm{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}\left(1+\kappa\mu_{\tilde{t}}\right)+1-\delta+\tau^k\right)\right] \tag{38}$$

$$d_{\tilde{t}} = c_{T,\tilde{t}-1} + k_{\tilde{t}} - n_{\tilde{t}-1} \tag{39}$$

A tax on debt τ^d increases the effective interest rate paid by households, which increases the cost of present consumption. As a result, if everything else is constant, households consume and invest less. The subsidy on capital τ^k increases the return to investment and will, ceteris paribus, increase investment. Abstracting from changes in the discount factor, this does not alter consumption plans, so the increase in investment is reflected in increased borrowing.

4.1 Constrained Efficient Allocation

I define this allocation as the solution to the problem of a planner that chooses allocations $\{c_{T,\tilde{t}-1}, d_{\tilde{t}}, k_{\tilde{t}}\}$ subject to the borrowing constraint and the process for TFP. Including the latter assumes that the policymaker has no ability to modify the allocation of capital across firms.

More formally, the problem is given by

$$\begin{aligned} V_{\tilde{t}-1}(n_{\tilde{t}-1}) &= \max_{c_{T,\tilde{t}-1}, d_{\tilde{t}}, k_{\tilde{t}}} u(c_{T,\tilde{t}-1}, y^N) + \beta\mathbb{E}_{\tilde{t}-1}\left[V_{\tilde{t}}(k_{\tilde{t}}, d_{\tilde{t}}, A_{\tilde{t}})\right] \\ \text{subject to} &\quad c_{T,\tilde{t}-1} + k_{\tilde{t}} = n_{\tilde{t}-1} + d_{\tilde{t}} \end{aligned}$$

Along with the resource constraint, the solution is characterized by,

$$1 = \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(1+\kappa\mu_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}} \times \mathrm{mpc}_{\tilde{t}}\right)\right], \tag{40}$$

$$\begin{aligned} 1 = \beta\mathbb{E}_{\tilde{t}-1}\Bigg[&\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left((\alpha+\gamma_{\tilde{t}})\mathrm{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}\right)\left(1+\kappa\mu_{\tilde{t}}(1+\mathrm{mpc}_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}})\right) \\ &+(1-\delta)\left(1+\kappa\mu_{\tilde{t}} \times \mathrm{mpc}_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\right)\Bigg], \end{aligned} \tag{41}$$

where $p_{\tilde{t}}$ is given by (5).

Condition (40) pins down optimal borrowing. Compared to the household's condition (37), note that the planner internalizes the effect of its choices on prices when there's a sudden stop. An extra dollar of debt in $\tilde{t}-1$ reduces available resources in $\tilde{t}$ by $1+r$. The marginal propensity to consume, $\text{mpc}_{\tilde{t}}$, shows how this translates into consumption. In turn, the share of non-tradable to tradable consumption times the price elasticity predicts the resulting drop in the value of the non-tradable endowment. Lastly, any drops in the value of income reduces borrowing by κ.

The choice of capital is pinned down by (A.33). Comparing to its competitive equilibrium analog, (38), the planner internalizes two external effects of investment. First, a higher stock of capital mitigates the misallocation across firms, raising $\text{TFP}_{\tilde{t}}$. This is shown by the presence of $\gamma_{\tilde{t}}$, which measures the improvement in productivity that derives from increased investment.

Second, by increasing output in $\tilde{t}$, investment increases available resources to the household. This will, ceteris paribus, increase consumption by the product of $\text{mpc}_{\tilde{t}}$ and the marginal productivity of capital. In turn, this increases the value of the non-tradable endowment and the borrowing capacity of the households.

What are the policy implications of this analysis? Proposition 1 derives the optimal taxes as a function of key sufficient statistics that summarize the channels previously discussed.

Proposition 1 (Constrained Efficient Allocation Implementation)**.** *The constrained efficient allocation can be implemented as a competitive equilibrium with a tax on debt τ^d and a subsidy on investment τ^k that take the following values,*

$$\tau^d = \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1} \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \kappa\mu_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}} \times \text{mpc}_{\tilde{t}}\right] \tag{42}$$

$$\begin{aligned}\tau^k &= \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1} \\ &\times \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\gamma_{\tilde{t}}\text{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right.\right. \\ &\left.\left.+\kappa\mu_{\tilde{t}}\left(\left((\alpha+\gamma_{\tilde{t}})\text{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}+1-\delta\right)\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\times\text{mpc}_{\tilde{t}}\right)\right)\right]\end{aligned} \tag{43}$$

Proof. See A.3 □

The tax on debt, τ^d, captures the difference between the social and the private value of a consumption. As previously discussed, because households do not internalize the aggregate effects of their choices, they consume less than it is socially optimal during a sudden stop. The tax then increases the cost of consumption in $\tilde{t}-1$ so that households internalize this. Note that the tax also depends on the severity and likelihood of a crisis, as reflected by the expectation of the multiplier on the constraint. If $\mu_{\tilde{t}} = 0$ for all possible states in $\tilde{t}$, then the social and private values of consumption are aligned. Likewise, if the sudden stops do not feature large drops in consumption, it follows from (34) that $\mu_{\tilde{t}}$ will be small, pushing down the tax on debt.

The investment subsidy, τ^k, reflects the two sources of inefficiency in this model. The first term captures the additional social value of investment over its private returns, due to its positive effects on productivity, reflected by $\gamma_{\tilde{t}}$. Additional investment also increases the resources available to the household, which increases consumption and thus borrowing capacity. This is captured by the second term.

Because it has two available instruments, the policymaker faces no trade-off in its objectives. This is clear in Proposition 1; The tax on debt τ^d does not depend on the productivity externality $\gamma_{\tilde{t}}$. Likewise, the presence of sudden stops does not reduce the investment subsidy.[14] Thus, in contrast with the literature, this economy does not feature overborrowing in the proper sense of the word; rather, it features overconsumption and underinvestment.

4.2 Second-Best Capital Controls

In this section I study the problem of a more constrained government, who can set capital controls but cannot subsidy investment. Formally, this is equivalent to adding a constraint that $\tau^k = 0$ to the planner's problem. It is clear from inspecting the optimality conditions of the household (37)-(38), that the planner cannot independently implement $k_{\tilde{t}}$ and $c_{\tilde{t}}$ anymore. Reducing consumption requires increasing τ^d to raise the interest rate paid by the households, which disincentivizes investment. To increase investment, the interest rate must decrease, which implies lowering τ^d. In turn, this will lead to an increase in consumption.

It is possible to use this intuition to set up the problem in a tractable way. I assume that households solve their problem in two stages. First, they pick total borrowing $d_{\tilde{t}}$. Second, taking that as given, they allocate resources between consumption and investment. In this stage, they solve[15],

$$\begin{aligned} V_{\tilde{t}-1}(n_{\tilde{t}-1}, d_{\tilde{t}}) = & \max_{c_{T,\tilde{t}-1}, k_{\tilde{t}}} u(c_{T,\tilde{t}-1}, y^N) + \beta V_{\tilde{t}}(k_{\tilde{t}}, d_{\tilde{t}}, A_{\tilde{t}}) \\ & \text{s.t.} \quad c_{T,\tilde{t}-1} + k_{\tilde{t}} = n_{\tilde{t}-1} + d_{\tilde{t}} \end{aligned} \tag{44}$$

This yields policy functions for $c_{\tilde{t}-1}$ and $k_{\tilde{t}}$ that depend on $d_{\tilde{t}}$ and the interest faced by the households, $(1+r)(1+\tau^d)$. The key object is the response of investment to increases in $d_{\tilde{t}}$,

[14] In fact, the subsidy is increasing in $\mu_{\tilde{t}}$.

[15] Technically, the household problem is the one describe in Sections 2-3. I present a version here that is more similar to the planner's problem for brevity.

which I call the marginal propensity to invest, mpi, given by

$$\text{mpi} \equiv \frac{\partial k_{\tilde{t}}}{\partial \bar{d}} \tag{45}$$

In the first stage, households pick $d_{\tilde{t}}$ taking the policy functions as given, with the decision characterized by (37), which I repeat below for convenience.

$$1 = (1+\tau^d)\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right] \tag{46}$$

The planner solves for the optimal $d_{\tilde{t}}$, taking as given the policy functions for $c_{T,\tilde{t}}$ and $k_{\tilde{t}}$, and implements it using τ^d. Given that in this small open economy $d_{\tilde{t}}$ has a close link to the current account, it is also a natural extension when studying capital controls.[16]

After application of the envelope theorem, the first order condition of the planner is given by

$$\begin{aligned} 1 = \beta\mathbb{E}\Bigg[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} & (1+r \\ & +\kappa\mu_{\tilde{t}}\big((1+r) - \text{mpi}\times((\alpha+\gamma_{\tilde{t}})\text{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}+1-\delta)\big)\times \text{mpc}_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} \\ & -\text{mpi}\times\gamma_{\tilde{t}}\text{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\big)\Bigg] \end{aligned} \tag{47}$$

An extra dollar of borrowing increases the resources available to households, the benefit of which is measured by the marginal utility of income. This increase in debt also reduces resources in $\tilde{t}$ by $(1+r)$, which is captured by the first term on the right-hand side. The extra terms correspond to the external effects of household choices.

As previously discussed, by reducing resources available for consumption, borrowing has an additional negative effect during a sudden stop. Unlike in the constrained efficient problem, here borrowing also has effects on investment. An extra dollar of debt leads to a mpi increase in $k_{\tilde{t}}$. Because investment increases available resources, this mitigates the cost of borrowing before a sudden stop. The other external effect of additional borrowing is the productivity increase that follows from investment. The strength of this effect will also depend on how the economy allocates credit, mpi, and the elasticity of $\text{TFP}_{\tilde{t}}$ to investment, $\gamma_{\tilde{t}}$.

These two effects have opposing implications for welfare, implying a policy trade-off. Increases in borrowing allow for additional investment, which has positive effects on productivity. At the same time, this increased leverage is costly during a sudden stop. The

[16] Re-arrange the resource constraint to obtain $-(d_{t+1}-d_t) = (\text{TFP}_t k_t^\alpha - c_{T,t} - (k_{t+1}-(1-\delta)k_t)) - rd_t$. As first term on the RHS is the trade balance, the RHS is the current account ca_t. As a result, $ca_t = -(d_{t+1}-d_t)$.

following proposition shows the second-best capital control, τ^d_{sb}, balances this trade-off.

Proposition 2 (Implementing the Second-Best Allocation)**.** *The tax on debt τ^d_{sb} that implements the second-best allocation is*

$$\tau^d_{sb} = \tau^d - \frac{\text{mpi}}{1+r} \times \tau^k, \tag{48}$$

where τ^d and τ^k, defined in Proposition 1, are the instruments that implement the constrained efficient allocation.

Proof. See A.4 □

Proposition 2 shows that the constrained planner sets a capital control that is a linear combination of the constrained efficient instruments. The weight on the investment subsidy τ^k is given by the marginal propensity to invest, mpi, which determines the response of investment to foreign borrowing. In the next section I explore the determinants of this capital control.

5 Determinants of Optimal Policy

In this section I explore the main determinants of capital controls. To fix ideas, I focus on each source of inefficiency to study how it affects optimal capital controls.

Pecuniary Externality. To help with the intuition, I first consider a case where the borrowing constraint binds with probability equal to 1 in the next period, there are no other sources of uncertainty and tradable production is an endowment $y^T_{\tilde{t}} = y^T$. Under this scenario, the optimal tax is

$$\tau^d_{sb} = \tau^d = \kappa\mu_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}$$

Without investment, this solution matches the constrained efficient allocation, as there's no trade-off for the planner. As all terms are strictly positive, this implies a positive tax on debt that reduces borrowing and, as a result, tradable consumption. Moreover, the tax is increasing in both of the terms.

The second term gives the response of the value of the non-tradable endowment to changes in $c_{T,\tilde{t}}$. It depends on the price elasticity $\frac{1}{\xi}$, which is inversely related to the substitutability between goods, and the share of non-tradable to tradable consumption. The higher this response is, the more effective macroprudential will be, as any increase in $c_{T,\tilde{t}}$ will lead to larger increases in the value of the non-tradable endowment. The amount of income that can be pledged as collateral, given by κ, has a similar effect, as it translates the changes in income to changes in borrowing.

The tax also depends on the severity of the sudden stop, which is given by $\mu_{\tilde{t}}$. Recall from (34) that $\mu_{\tilde{t}}$ reflects the consumption drop during a sudden stop. It follows that capital controls should be higher for countries where crises are more harmful in terms of consumption.

How does the introduction of investment affect the results? I now bring back tradable production with exogenous productivity (i.e. $\gamma_{\tilde{t}} = 0$), so that the pecuniary externality remains the only source of inefficiency. The second-best capital control is now given by

$$\tau^d_{sb} = \kappa \mu_{\tilde{t}} \left(1 - \text{mpi} \times \frac{\alpha \text{TFP}_{\tilde{t}} k^{\alpha}_{\tilde{t}} + 1 - \delta}{1+r}\right) \times \text{mpc}_{\tilde{t}} \times \frac{1}{\xi} \frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}}$$

The introduction of capital means that households now allocate resources between consumption and investment and these choices are an important input for policy. First, an extra dollar in borrowing during $\tilde{t}-1$ leads to mpi additional cents in investment. This has the effect of increasing production, and thus available resources, during $\tilde{t}$. Because macroprudential policy seeks to control consumption, it is possible to think of the mpi as measuring an additional cost of capital controls. Thus, the higher the propensity to invest is, the lower the tax will be.

Second, consumption in $\tilde{t}$ does not change one by one with resources anymore, as households allocate between consumption and investment. The introduction of $\text{mpc}_{\tilde{t}}$ reflects this change. Recall that a fewer dollar in $d_{\tilde{t}}$ through reduced $c_{T,\tilde{t}-1}$ increases available resources by $1 + r$ dollars in $\tilde{t}$. Unlike in the previous example, this does not lead to a $1 + r$ increase in $c_{T,\tilde{t}}$, as the household will allocate the windfall into both consumption and investment. Instead, $c_{T,\tilde{t}}$ will increase by $\text{mpc}_{\tilde{t}} < 1$, dampening the tax relative to the setting without production.

Within the model studied in this paper, $\text{mpc}_{\tilde{t}}$ and mpi are characterized by the optimality conditions and are therefore implicitly determined by the primitives of the model. Framing the discussion in terms of the former, however, allows for more generality. Indeed, one could think of more complex settings, such as models with rich heterogeneity within households, a more detailed production structure or financial system, in which these sufficient statistics still apply.

Misallocation. As previously discussed, the frictions that lead to misallocation result in a positive externality of investment, as a higher capital stock increases total factor productivity. To make the intuition clear, I will keep the perfect foresight assumption for the time being while assuming that there's no borrowing constraint (i.e., $\mu_{\tilde{t}} = 0$). In that economy, the second-best capital control would be

$$\tau^d_{sb} = -\text{mpi} \times \gamma_{\tilde{t}} \times \frac{\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}}{1+r}$$

The second term is the constrained efficient subsidy, that depends on $\gamma_{\tilde{t}}$. Recall from (23), that $\gamma_{\tilde{t}}$ depends in part on the existing level of misallocation in the economy. This has two important implications; the first is that the subsidy will vary across countries. It is an empirical regularity[17] that misallocation is decreasing in the level of development, implying that the subsidy would be higher for less developed economies. The second is that within a country, as the capital stock increases, misallocation will fall and $\gamma_{\tilde{t}}$ will decrease. In this way, the externality resembles a dynamic externality that shrinks as the economy develops.

The size of the subsidy also depends on the efficiency of the government in increasing investment, given by mpi. If the economy mostly redirects borrowing to consumption, that would make the subsidy relatively more expensive, and its optimal level would be higher. An interesting implication is that the subsidy will more effective for economies that either have a naturally higher mpi or can control it in some way. This is consistent, for instance, with the experience of South Korea (Noland, 2007).

So far, I have considered a perfect foresight scenario, where the sudden stop is all but certain. In reality, sudden stops are low probability events[18]. This is important because it introduces a third factor into the trade-off which has asymmetric effects on the two forces at play. While increases in productivity will occur in all states of the world, the borrowing constraint only binds in a few of them.

Uncertainty. First, assume that $\gamma_{\tilde{t}} = 0$. Then, the second-best capital control is

$$\tau^d_{sb} = \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1} \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \kappa\mu_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}}\right]$$

To unpack the role the probability of a sudden stop plays, let π be the probability that $\mu_{\tilde{t}} > 0$. It is possible to write

$$\tau^d_{sb} = \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1} \times \pi \times \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \kappa\mu_{\tilde{t}} \times \frac{1}{\xi}\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}} | \mu_{\tilde{t}} > 0\right].$$

[17] (See, e.g., Hsieh and Klenow, 2009; Buera et al., 2015; Restuccia and Rogerson, 2017)

[18] Bianchi and Mendoza (2020) report a frequency of 2.4% with significant heterogeneity across development levels

As long as $\pi < 1$, the introduction of uncertainty reduces the optimal capital control. This is intuitive, because its benefits only manifest during a sudden stop, while its costs occur in every state of the world.

These implications also carry over to the case where $\gamma_{\tilde{t}} > 0$. Because the benefits to productivity happen in all states of the world, a lower probability of a sudden stop reduces the benefit to capital controls while keeping their costs intact.

What are the overall implications of each of the factors discussed above? The following corollary summarizes the contribution of each.

Corollary. *The second-best capital controls, τ^d_{sb} are*

1. *Increasing in the non-tradable share of consumption $\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}}$, non-tradable price elasticity, $\frac{1}{\xi}$, pledgeable fraction of income κ, sudden stop severity, $\mu_{\tilde{t}}$, sudden stop probability, π, and marginal propensity to consume* $\mathrm{mpc}_{\tilde{t}}$

2. *Decreasing in misallocation, σ^2_R, its elasticity with respect to capital, ν, and the marginal propensity to invest in tradables* mpi

The benefits of capital controls are predominantly given by the strength of the pecuniary externality, which leads the planner to restrict consumption using capital controls. The introduction of investment reduces the effectiveness of capital controls in two ways. First, restricting borrowing leads to decreases in investment, which are costly. Second, because households are not completely hand-to-mouth during a sudden stop, decreases in debt lead to smaller consumption increases. The marginal to propensity to invest and consume, mpi and $\mathrm{mpc}_{\tilde{t}}$ respectively, summarize these two channels.

While investment dampens optimal capital controls, the presence of frictions in the allocation of capital further increase the costs of capital controls. Because these frictions diminish as the stock of capital increases, capital controls reduce productivity. The strength of this channel is summarized by $\gamma_{\tilde{t}}$, which in turn depends on the level of misallocation in the economy, σ^2_R and the elasticity of misallocation with respect to $k_{\tilde{t}}$, ν.

The discussion so far in this paper has been qualitative, highlighting different channels and mechanisms that affect optimal policy. It is not clear, however, what the quantitative implications are. I now leverage the formulation in terms of sufficient statistics to explore this issue.

6 Quantification

In this section, I leverage the results of Proposition 2 to estimate the second-best capital controls and show the quantitative relevance of the channels discussed in the previous section.

While Proposition 2 states the second-best capital controls as a function of a few sufficient statistics and time-series moments, a few assumptions are needed before taking it to the data. I discuss these before describing the empirical work.

Building on the intuition from the previous section, it is possible to write τ_d and τ^k as

$$\tau^d = \pi \times \kappa \frac{1}{\xi} \times \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1} \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \mu_{\tilde{t}} \frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}} \times \text{mpc}_{\tilde{t}} \middle| \mu_{\tilde{t}} > 0\right]$$

$$\tau^k = \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1} \left[\gamma_{\tilde{t}} \times \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \frac{y^T_{\tilde{t}}}{k_{\tilde{t}}}\right] + \right.$$
$$\left. \pi \times \kappa \times \mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \left(\mu_{\tilde{t}} \left(\gamma_{\tilde{t}} \frac{y^T_{\tilde{t}}}{k_{\tilde{t}}} + \left((\alpha + \gamma_{\tilde{t}}) \frac{y^T_{\tilde{t}}}{k_{\tilde{t}}} + 1 - \delta\right) \frac{1}{\xi} \frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}} \times \text{mpc}_{\tilde{t}}\right)\right) \middle| \mu_{\tilde{t}} > 0\right]\right]$$

where π is the probability of a sudden stop in $\tilde{t}$. To simplify the estimation I assume that, conditional on a sudden stop, $\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}}$, $\text{mpc}_{\tilde{t}}$ and $\text{TFP}_{\tilde{t}}$ are constant.

I now explain how I assign values to the parameters and moments that are required.

6.1 Calibrated Parameters, Sufficient Statistics and Time-Series Moments

Table 1: Baseline Calibration of Standard Parameters

Parameter	Value	Source
σ	2	Bianchi (2011)
ξ	0.5	Bianchi (2011)
κ	0.32	Bianchi (2011)
r	0.04	Bianchi (2011)
α	0.3	Standard Value
δ	0.05	Hsieh and Klenow (2009)
η	3	Hsieh and Klenow (2009)

Calibrated Parameters. Table 1 summarizes the calibration of the standard parameters. I keep as close as possible to Bianchi (2011) in calibrating the parameters that pin down the pecuniary elasticity and set $\sigma = 2$ and $\kappa = 0.32$. I slightly depart from his calibration by setting $\xi = \frac{1}{2}$, which follows from the assumption that $\sigma = \frac{1}{\xi}$. While lower, this value is still within the range of feasible estimates considered by Bianchi (2011) and can be considered as close to an upper bound on the pecuniary externality.[19] I also follow Bianchi (2011) in setting $r = 0.04$, as it is common in the literature.

For the parameters governing tradable production, I set $\alpha = 0.3$ and $\delta = 0.05$ as is standard. The elasticity of substitution between varieties, η, determines the effect of misallocation on productivity, as can be seen in Lemma 1. I follow Hsieh and Klenow (2009) and make the conservative choice of $\eta = 3$.

Calibrated Sufficient Statistics. Table 2 lists the sufficient statistics that I calibrate following the literature. To obtain the marginal propensity to consume, mpc during a sudden stop, I consider a consumption elasticity of 1, as reported by Guntin et al. (2023), and multiply it by the average aggregate share of consumption to GDP during sudden stops, which is 62%.[20] This is in the same neighborhood as the estimates of Hong (2023) for Peru, an emerging economy.[21]

To measure the marginal propensity to invest, mpi, I follow Müller and Verner (2023) who find that 10% of the increase in credit during tradable booms is driven by tradable sectors. Thus, I set mpi = 0.1.

I follow two approaches to assign values to π. First, I use the unconditional probabilities described by Bianchi and Mendoza (2020), which are 2.4% for their entire sample, and 2.9% and 1.7% for emerging and advanced economies respectively. Second, a recent literature on financial crises has tried to find predictors of future crises. Greenwood et al. (2022) construct "Red-Zone" indicators, which combines several financial series. Countries in red-zones have a probability of around 14% of experiencing a crisis the next year and between 37-45% within the next three years.

Lastly, I need to assign a value to the ratio of non-tradable to tradable expenditure $\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}}$, which affects the response of $p_{\tilde{t}}$ to changes in $c_{T,\tilde{t}}$ and is therefore important in determining the importance of the pecuniary externality. I follow Bianchi and Mendoza (2020) and set $\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}} = 2$.

Time-Series Moments. I combine data from the World Development Indicators (World Bank, 2021) and Penn World Tables (Feenstra et al., 2015) to obtain a panel containing national accounts data and capital stock. Following Uribe and Schmitt-Grohé (2017), I keep only those countries with at least 30 years of available data, and quadratically detrend all

[19] Bianchi (2011) considers a range 0.4-0.83 and picks the latter as a conservative calibration.

[20] I compute this share by taking the average of consumption over GDP (World Bank, 2021) over the sudden stop episodes in my sample.

[21] Hong (2023) finds a quarterly mpc of 0.20, which translates to an annual mpc in the range of 0.54-0.59, depending on the method used. The quarterly estimates are also in line with the meta-analysis by Sokolova (2023).

Table 2: Calibrated Sufficient Statistics

Sufficient Statistic	Value	Source
$\text{mpc}_{\tilde{t}}$	0.62	Guntin et al. (2023)
mpi	0.1	Müller and Verner (2023)
π	0.017 - 0.45	Bianchi and Mendoza (2020); Greenwood et al. (2013)
$\frac{p_{\tilde{t}} y^N}{c_{T,\tilde{t}}}$	2	Bianchi and Mendoza (2020)

series. To identify sudden stops, I use Korinek and Mendoza (2014) classification. The resulting panel spans 36 countries from 1979 to 2012, and covers middle and high-income economies.[22][23]

Table 3 shows the moments required for the estimation. Due to lack of data for tradable consumption, I use final consumption instead. To estimate $\mu_{\tilde{t}}$, I use (34) and proxy for $\mu_{\tilde{t}} > 0$ using the sudden stop classification in Korinek and Mendoza (2014). Lastly, I obtain the output to capital ratio using data from Feenstra et al. (2015).

Table 3: Time-Series Moments

Moment	Value
$\mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]$	1
$\mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \mu_{\tilde{t}} \mid \mu_{\tilde{t}} > 0\right]$	0.13
$\mathbb{E}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma} \text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}\right]$	0.28
$\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}$	0.25

6.2 Productivity Externality

The key sufficient statistic I need to estimate is $\gamma_{\tilde{t}}$. Recall from (23) that it is given by,

$$\gamma_t = \alpha(1 + \alpha(\eta - 1))\nu\, \sigma_R^2,$$

which requires estimates for σ_R^2 and ν.

[22] I assign to each country its modal WDI classification over the period 1989, when the WDI classification started, and 2012.

[23] Section C.1 gives some additional details.

To estimate σ_R^2, I follow the procedure used by Hsieh and Klenow (2009). Recall that σ_R^2 is the variance of the log of the rental rates, $R(i)$, paid by the firms producing tradable goods varieties. It follows from profit maximization that

$$R(i) = \alpha \frac{p(i)y^T(i)}{k(i)}, \tag{49}$$

Where the denominator is the value added of the firm and the numerator the stock of capital it possesses. Thus, estimating σ_R^2 requires firm-level data that allows to estimate the ratio of these two numbers. The Orbis-Amadeus historical product, which has extensive coverage of private firms in Europe (Kalemli-Özcan et al., 2024), provides exactly that. I now detail the steps I take to measure σ_R^2.

Data Cleaning. To clean the dataset I follow the procedure by Kalemli-Özcan et al. (2024) as closely as possible. The input for the Orbis dataset is income and balance sheet statements submitted annually by firms. To have consistent units, I keep only unconsolidated statements that cover 12 months.

I drop spells with errors in the following way. First, I tag unrealistic changes in assets or sales[24], or negative values for assets, sales, employment or liabilities. I also tag observations that do not report employment or report a number larger than 2 million, or where balance sheet identities don't hold. I drop all tagged observations and only keep the spell after the last identified error.

As it is common in the literature that measures misallocation, I focus on the manufacturing sector[25], defined using the 4-digit NACE classification. Section C.2 presents some summary statistics of the resulting sample which spans 18 countries[26] over the period 1996-2016, covering 1,050,610 unique firms for a total of 9,143,358 observations. Employment by these firms adds up to 11,559,081 employees per year, with the firm size distribution resembling that reported by Eurostat.[27]

Variable Construction. I construct $p(i)y^T(i)$ by subtracting the cost of materials from the operating revenue of the firm and the capital stock $k(i)$ as the sum of tangible and intangible fixed assets. To mitigate potential measurement error, I winsorize both variables at the bottom and top 1% levels. I then estimate $R(i)$ using equation (49) and its log variance

[24] In the order of 10^3

[25] Defined as firms that report NACE codes between 1010 and 3320

[26] I use the same sample as Kalemli-Özcan et al. (2024) and drop the United Kingdom and Greece as some variables required for the analysis are not populated.

[27] See Section C.2 in the Online Appendix.

at the 4-digit industry level. Lastly, I compute the weighted average across industries, where I use total value added as the weight for each industry.[28]

The second column of Table 4 show the average value of σ_R^2 between 2012 and 2016[29] for each country in the sample. The estimates range between 1.30 and 3.36, with a mean of 2.14, indicating a not negligible degree of dispersion.

Table 4: Misallocation Estimates

Country	σ_R^2	$\gamma_{\tilde{t}}$
Austria	1.51	0.16
Belgium	1.53	0.16
Czech Republic	2.43	0.26
Estonia	1.93	0.20
Finland	1.58	0.17
France	1.32	0.14
Germany	1.58	0.17
Hungary	2.69	0.28
Italy	2.12	0.22
Latvia	2.86	0.30
Norway	1.98	0.21
Poland	2.35	0.25
Portugal	1.99	0.21
Romania	2.72	0.29
Slovak Republic	2.68	0.28
Slovenia	2.73	0.29
Spain	2.34	0.25
Sweden	2.32	0.25
Mean	2.15	0.23

Estimates for $\gamma_{\tilde{t}}$. The last step before estimating $\gamma_{\tilde{t}}$ is assigning a value for ν. To calibrate it, I target the TFP loss estimated by Pinardon-Touati (2024). Studying the effect of government spending on private borrowing in France, she finds that a 0.28% decrease in capital leads on average to a 0.04% decrease in TFP, which corresponds to $\gamma_{\tilde{t}} = 0.14$. Given a $\sigma_R^2 = 1.32$ for France, that yields $\nu = 0.22$.

The third column of Table 4 shows the estimated $\gamma_{\tilde{t}}$ using this estimate for ν. A 1% increase in capital leads to increases in TFP between 0.14% and 0.3%. These estimates

[28]Results are robust to using the unweighted measure.

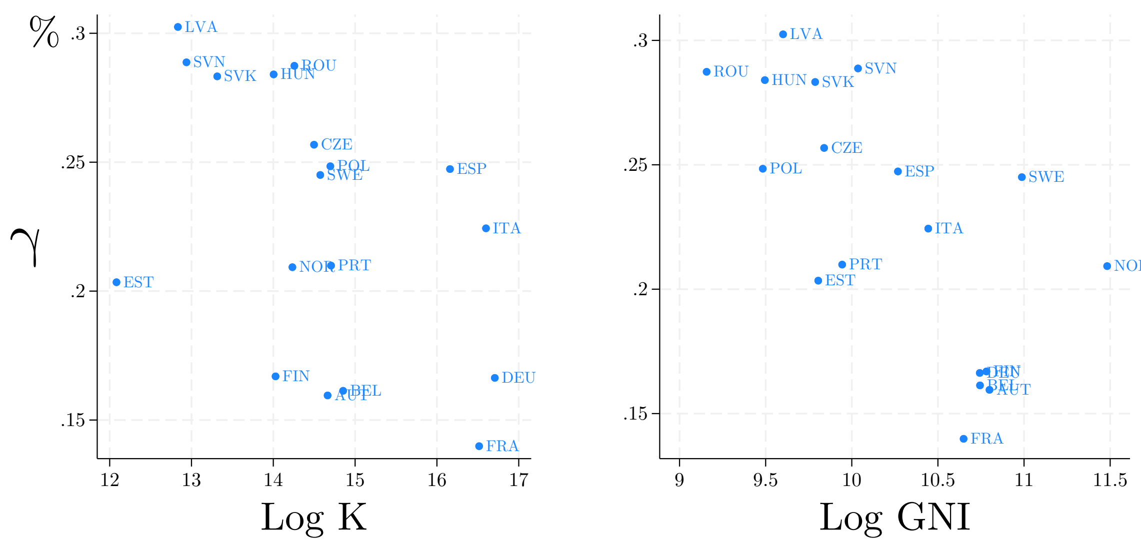
[29]I focus on the latter part of the sample to maximize coverage of manufacturing activity.

are quantitatively in line with the small literature that has studied the response of TFP to changes in capital or interest rates. Jordà et al. (2020), who study the long-run effects of interest rate changes, find that a decrease of 1% in capital is associated with a 0.5% decline in productivity. González et al. (2024), who also study an economy with endogenous productivity, show that a change in interest rates that decrease the capital stock by 1% lead to a decrease in TFP of almost 0.35% for Spain, while my estimate for the same country is 0.25%.

In Appendix B.2, I show that using this value of ν undershoots the effects of a capital control liberalization in India studied by Bau and Matray (2023). As such, I see the value of ν used in this paper as a conservative estimate.

Figure 1 shows the relationship between $\gamma_{\tilde{t}}$ and the stock of capital and Gross National Income (GNI). As predicted by the formulation in Lemma 3, $\gamma_{\tilde{t}}$ is decreasing in the stock of capital. Moreover, it is also decreasing in GNI, suggesting that it is inversely related to the degree of development of an economy.

Figure 1: Correlation of $\gamma_{\tilde{t}}$ with GNI and the capital stock



Notes: The Figure plots the relationship between the estimates of $\gamma_{\tilde{t}}$, and the log of the capital stock (K) and Gross National Income (GNI), respectively, at the country-year level. The capital stock is obtained from Feenstra et al. (2015) and is measured in PPP US dollars. GNI is sourced from World Bank (2021) and is measured in current US dollars using the Atlas method.

To compute the second-best capital controls, I also consider an upper bound using the case of India. Attributing all the dispersion reported in Hsieh and Klenow (2009) in TFPR to MRPK, that would result in $\sigma_R^2 = 5$, which in turn implies $\gamma_{\tilde{t}} = 0.52$.

6.3 Second-Best Capital Controls

It is now possible to quantify the second-best capital controls. Table 5 shows values of τ_{sb}^d for different combinations of $\gamma_{\tilde{t}}$ and π. The former takes values that cover the empirical estimates discussed in the previous section, while the latter spans the values suggested by the literature.

Table 5: Estimates of τ_{sb}^d

		π					
		1.7%	2.4%	2.9%	14%	25%	41%
$\gamma_{\tilde{t}}$	0	0.16	0.22	0.27	1.30	2.32	3.81
	.15	-0.23	-0.17	-0.12	0.90	1.92	3.40
	.23	-0.44	-0.38	-0.33	0.69	1.70	3.18
	.3	-0.62	-0.56	-0.51	0.51	1.52	2.99
	.54	-1.25	-1.18	-1.14	-0.13	0.87	2.33

Notes: The table shows values of the second-best capital control for different values of the crisis probability π and the productivity externality $\gamma_{\tilde{t}}$. The chosen range of values for π follows from the literature and is explained in Section 6. The range of values for $\gamma_{\tilde{t}}$ is explained in Subsection 6.2.

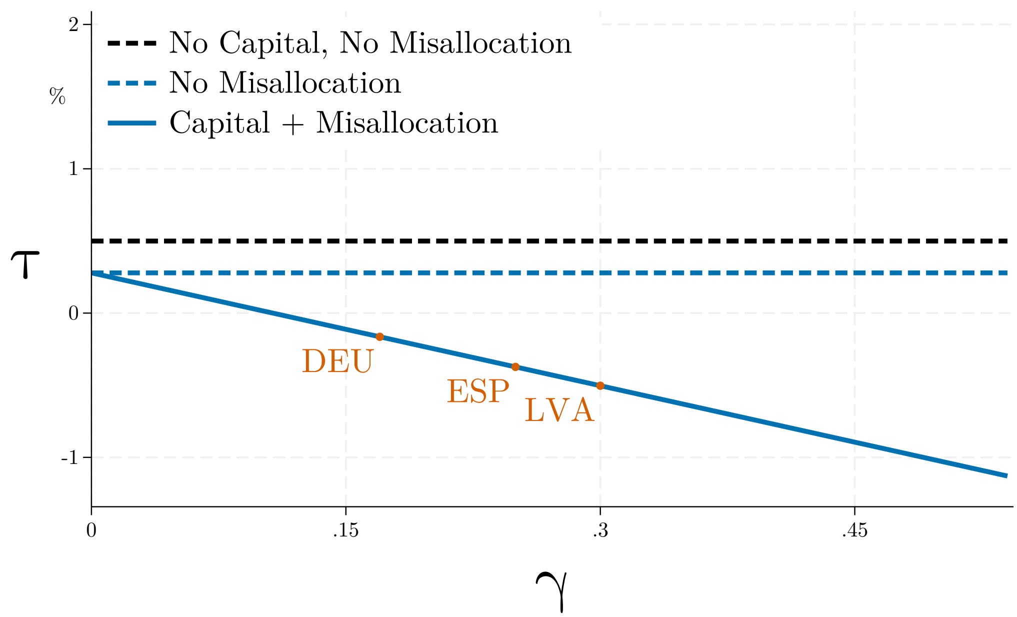
The misallocation channel is quantitatively significant. Compared to the first row, a $\gamma_{\tilde{t}}$ of 0.15, which is at the lower end of the estimates in the sample, implies a capital control that is negative and almost 0.4% lower. Capital controls are negative for crisis probabilities below 3%, suggesting that a borrowing subsidy is optimal during normal times. As the probability of a sudden stop increases, capital controls ramp up to values between 3% and 3.4% for the countries in the sample.

In addition to heterogeneity along π, there's also significant heterogeneity across levels of $\gamma_{\tilde{t}}$. The last row of table 5 shows that economies with a high level of misallocation find it optimal to subsidize borrowing even when the probability of a crisis is substantial. When considering the estimates for $\gamma_{\tilde{t}}$ in Table 4, τ_{sb}^d ranges from -0.12% to -1.02% for $\pi = 2.9\%$.

What are the policy implications of these findings? Figure 2 plots values of τ_{sb}^d for different values of γ for $\pi = 0.03$ and compares it to two counterfactual scenarios. The first one, in dashed black, corresponds to the case where tradable production is an endowment. As a result, there are no negative effects on investment and productivity. For this probability of a crisis, the optimal capital control would be 0.5%. Introducing capital but not allowing for misallocation, dashed blue, reduces it to 0.28%, reflecting the costs on investment. Lastly, the solid blue line plots the τ_{sb}^d that takes into account the costs on both investment and productivity. To put these numbers into context, I show what the taxes would be for

Germany, Spain and Latvia according to their estimated $\gamma_{\tilde{t}}$. In the case of Spain, which has an estimated $\gamma_{\tilde{t}}$ close to the sample median, the tax on borrowing turns into a subsidy of almost 0.4%. Even for Germany, which has the lowest productivity cost out of the three, a subsidy of around -0.15% is optimal. In the case of Latvia, the subsidy would equal 0.5%, showing strong heterogeneity between the countries in the sample.

Figure 2: Comparison of First and Second-Best Taxes



Notes: The figure plots the optimal capital control under different scenarios for different values of γ and $\pi = 0.03$. The dashed black line plots τ^d_{sb} when tradable output is an endowment. The dashed blue line plots τ^d_{sb} for $\gamma = 0$. The solid blue line shows τ^d_{sb} as a function of γ.

The results in this section show that the productivity losses inflicted by capital controls are quantitatively relevant, outweighing the benefit of macroprudential policies for a low probability of crises and plausible values for $\gamma_{\tilde{t}}$. This result hinges on the government not being able to control the allocation of credit, which renders macroprudential policy harmful by deterring capital accumulation.

7 Conclusion

In this paper, I study optimal macroprudential when its effects on productivity are considered. To answer this question, I extend the standard model of macroprudential policy to incorporate production by firms that require physical capital and face heterogeneous frictions in the market for the latter.

I propose a tractable and flexible way of modelling these frictions, which result in capital being misallocated and lower aggregate productivity as a result. Macroprudential policies, such as capital controls, that reduce investment also worsen misallocation, in line with the empirical literature.

Turning to optimal policy, I first study the constrained efficient solution, where the planner has enough available instruments. In this scenario, there's no trade-off, as the planner can tax borrowing to address the pecuniary externality and subsidize investment to improve productivity.

I then study the second-best problem, where the planner can only control total borrowing. I characterize the solution to this problem and show that a trade-off exists. An increase in borrowing reduces welfare if a sudden stop occurs but also leads to increased investment and productivity. I characterize this trade-off using a few sufficient statistics, which allows me to provide intuition into the result and explain the qualitative effects of each of them.

To explore the quantitative relevance of this trade-off, I leverage the formulation of the second-best policy as a function of these sufficient statistics. Along with time-series moments and some calibrated moments from the literature, I exploit the tractable specification for misallocation to calibrate the relevant parameters using detailed firm-level data for a wide selection of European countries.

I find that capital effects have a sizable effect on productivity and that, as a result, the trade-off is quantitatively relevant. For the baseline probability of a crisis, the optimal tax decreases from 0.5% to a subsidy of almost 0.4% for the median estimate of productivity losses in the sample. Moreover, there's strong heterogeneity, with the subsidy varying as much as 0.35% between the countries in the sample.

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Online Appendix

Contents

A Proofs

A.1 Proof of Lemma 1

Proof. I prove this Lemma in two parts. First, I derive the aggregate production function and TFP. Lastly, I obtain the first order conditions of the representative firm.

Aggregate Production Function. Let MC be the marginal cost of the firm, derived from solving the cost minimization problem of the firm, with first order conditions:

$$R(i) = \alpha MC(i) \frac{y(i)}{k(i)} \tag{A.1}$$

$$w = (1-\alpha) MC(i) \frac{y(i)}{h(i)} \tag{A.2}$$

Combining into the production function yields the marginal cost of the firm:

$$MC(i) = A(i)^{-1} \left(\frac{R(i)}{\alpha} \right)^{\alpha} \left(\frac{w}{1-\alpha} \right)^{1-\alpha} \tag{A.3}$$

We can now solve for the problem of the firm as:

$$\max_{y(i)} \frac{\eta}{\eta-1} y(i)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} - y(i)MC(i) \tag{A.4}$$

with solution:

$$y(i) = (MC(i))^{-\eta}\, y \tag{A.5}$$

Plugging back into the factor demands, we get:

$$k(i) = A(i)^{\eta-1} y \left(\frac{\alpha}{R(i)}\right)^{1+\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)} \tag{A.6}$$

$$h(i) = A(i)^{\eta-1} y \left(\frac{\alpha}{R(i)}\right)^{\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{\alpha+(1-\alpha)\eta} \tag{A.7}$$

Then, we can obtain aggregate production as

$$y = \left[\int y(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} \tag{A.8}$$

$$= \frac{\left[\int y(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}}{\left(\int k_t(i)di\right)^{\alpha} \left(\int h_t(i)di\right)^{1-\alpha}} {k_t}^{\alpha} {h_t}^{1-\alpha} \tag{A.9}$$

$$= \frac{\left[\int A(i)^{\eta-1} R(i)^{-\alpha(\eta-1)} di\right]^{\frac{\eta}{\eta-1}}}{\left(\int (A(i))^{\eta-1} \left(R(i)^{-1}\right)^{1+\alpha(\eta-1)} di\right)^{\alpha} \left(\int (A(i))^{\eta-1} \left(R(i)^{-1}\right)^{\alpha(\eta-1)} di\right)^{1-\alpha}} {k_t}^{\alpha} {h_t}^{1-\alpha} \tag{A.10}$$

Assuming either log-normality or up to second order, we can write:

$$y = TFP k^{\alpha} h^{1-\alpha} \tag{A.11}$$

where[30]

$$TFP = \left(\int A(i)^{\eta-1}\right)^{\frac{1}{\eta-1}} \exp\left[-\frac{1}{2}\alpha(1+\alpha(\eta-1))\sigma_R^2\right] \tag{A.12}$$

taking logs concludes the first part of the proof.

First order conditions. Aggregate labor and capital demand at the firm level,

$$k = y\alpha^{1+\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)} \int A(i)^{\eta-1} R(i)^{-(1+\alpha(\eta-1))} \tag{A.13}$$

$$h = y\alpha^{\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{\alpha+(1-\alpha)\eta} \int A(i)^{\eta-1} R(i)^{-\alpha(\eta-1)} \tag{A.14}$$

[30] Note that the covariance between $A(i)$ and $R(i)$ drops out due to properties of the log-normal distribution between (A.10)and (A.12)

divide by each other,

$$\frac{k}{h} = \alpha \frac{w}{1-\alpha} \exp\left(-\mathbb{E}\left[\log R(i)\right] + \frac{1}{2}(1+2\alpha(\eta-1))\sigma_R^2\right) \tag{A.15}$$

plug back into (A.14),

$$1 = TFP^{\eta} \left(\frac{k}{h}\right)^{\alpha\eta} \left(\frac{1-\alpha}{w}\right)^{\eta}, \tag{A.16}$$

solving for w,

$$w = (1-\alpha) TFP \left(\frac{k}{h}\right)^{\alpha} \tag{A.17}$$

In the same way, substitute $\frac{1-\alpha}{w}$ in (A.13)

$$\exp\left(\mathbb{E}\left[\log R(i)\right]\right) = \alpha \mathbb{E}\left[A(i)^{\eta-1}\right]^{\frac{1}{\eta-1}} \left(\frac{h}{k}\right)^{(1-\alpha)} \exp\left(\frac{1}{2}(1-3\alpha+\alpha^2+2\alpha\eta-\alpha^2\eta)\sigma_R^2\right) \tag{A.18}$$

Let $R \equiv \frac{\int R(i)k(i)}{\int k(i)}$,

$$R = \frac{\int A(i)^{\eta-1} y \alpha^{1+\alpha(\eta-1)} R(i)^{-\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}}{\int A(i)^{\eta-1} y \left(\frac{\alpha}{R(i)}\right)^{1+\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}} \tag{A.19}$$

$$R = \exp\left(\mathbb{E}\left[\log R(i)\right] - \frac{1}{2}\left(1+2\alpha(\eta-1)\right)\sigma_R^2\right) \tag{A.20}$$

$$R = \frac{\exp\left(\mathbb{E}\left[\log R(i)\right]\right)}{\exp\left(\frac{1}{2}\left(1+2\alpha(\eta-1)\right)\sigma_R^2\right)} \tag{A.21}$$

$$R = \frac{\alpha \mathbb{E}\left[A(i)^{\eta-1}\right]^{\frac{1}{\eta-1}} \left(\frac{h}{k}\right)^{(1-\alpha)} \exp\left(\frac{1}{2}(1-3\alpha+\alpha^2+2\alpha\eta-\alpha^2\eta)\sigma_R^2\right)}{\exp\left(\frac{1}{2}\left(1+2\alpha(\eta-1)\right)\sigma_R^2\right)} \tag{A.22}$$

$$R = \alpha \mathbb{E}\left[A(i)^{\eta-1}\right]^{\frac{1}{\eta-1}} \left(\frac{h}{k}\right)^{(1-\alpha)} \exp\left(\frac{1}{2}\alpha(1+\alpha(\eta-1))\sigma_R^2\right) \tag{A.23}$$

$$R = \alpha TFP \left(\frac{h}{k}\right)^{(1-\alpha)} \tag{A.24}$$

□

A.2 Proof of Lemma 3

Proof. Start from capital demand (A.6) and solve for $R(i)$ as a function of $k(i)$,

$$R(i) = \alpha \left(k(i)^{-1} A(i)^{\eta-1} y \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}\right)^{\frac{1}{1+\alpha(\eta-1)}} \tag{A.25}$$

Combining with (??),

$$R(i)=\alpha\left(k^{-1}\left(1+\frac{F(i)}{k^{\nu}}\right)^{-1}y\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}\right)^{\frac{1}{1+\alpha(\eta-1)}}, \tag{A.26}$$

taking logs,

$$\log R(i)=\log\alpha+\frac{1}{1+\alpha(\eta-1)}\left(-\log k-\log\left(1+\frac{F(i)}{k^{\nu}}\right)+\log y+\log\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}\right), \tag{A.27}$$

Lastly, using the assumption of small $F(i)$,

$$\log R(i)=\log\alpha+\frac{1}{1+\alpha(\eta-1)}\left(-\log k-\frac{F(i)}{k^{\nu}}+\log y+\log\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}\right). \tag{A.28}$$

It follows that

$$\mathbb{V}ar\left[\log R(i)\right]=\left(\frac{1}{1+\alpha(\eta-1)}\frac{1}{k^{\nu}}\right)^{2}\sigma_F^2 \tag{A.29}$$

□

A.3 Proof of Proposition 1

Proof. I need to show that conditions (37)-(39) hold. The resource constraint (39) holds trivially. Set τ^d so that (37) hold,

$$(1+\tau^d)\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]=\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(1+\kappa\mu_{\tilde{t}}\times\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\times\mathrm{mpc}_{\tilde{t}}\right)\right] \tag{A.30}$$

$$\tau^d\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]=\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\kappa\mu_{\tilde{t}}\times\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\times\mathrm{mpc}_{\tilde{t}}\right)\right] \tag{A.31}$$

$$\tau^d=\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1}\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\kappa\mu_{\tilde{t}}\times\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\times\mathrm{mpc}_{\tilde{t}}\right)\right] \tag{A.32}$$

and τ^k so that (38) holds,

$$\begin{aligned}&\beta\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\alpha\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}\left(1+\kappa\mu_{\tilde{t}}\right)+1-\delta+\tau^k\right)\right]\\&=\beta\mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left((\alpha+\gamma_{\tilde{t}})\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}\right)\left(1+\kappa\mu_{\tilde{t}}(1+\mathrm{mpc}_{\tilde{t}}\times\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}})\right)\right.\\&\left.+(1-\delta)\left(1+\kappa\mu_{\tilde{t}}\times\mathrm{mpc}_{\tilde{t}}\times\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\right)\right],\end{aligned} \tag{A.33}$$

$$\tau^k \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]$$
$$= \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha}(\gamma_{\tilde{t}}(1+\kappa\mu_{\tilde{t}}) + (\alpha+\gamma_{\tilde{t}})\kappa\mu_{\tilde{t}} \times \text{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}) + (1-\delta)\kappa\mu_{\tilde{t}}\text{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}}\right)\right] \tag{A.34}$$

$$\tau^k = \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\right]^{-1}$$
$$\times \mathbb{E}_{\tilde{t}-1}\left[\left(\frac{c_{T,\tilde{t}}}{c_{T,\tilde{t}-1}}\right)^{-\sigma}\left(\gamma_{\tilde{t}} + \kappa\mu_{\tilde{t}}\left(\left((\alpha+\gamma_{\tilde{t}})\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1} + (1-\delta)\right)\frac{1}{\xi}\frac{p_{\tilde{t}}y^N}{c_{T,\tilde{t}}} + \gamma_{\tilde{t}}\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}\right)\right)\right] \tag{A.35}$$

□

A.4 Proof of Proposition 2

Proof. The first order condition of the planner with respect to $\bar{d}$ is,

$$c_{T,\tilde{t}-1}^{-\sigma} =$$
$$\beta\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left(1+r+\kappa\mu_{\tilde{t}}\left((1+r) - \text{mpi}((\alpha+\gamma_{\tilde{t}})\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1} + 1 - \delta)\right)\text{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} - \right.\right. \tag{A.36}$$
$$\left.\left.\text{mpi} \times \gamma_{\tilde{t}}\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right)\right]$$

The analogous condition for the household is

$$c_{T,\tilde{t}-1}^{-\sigma} = \beta\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left((1+r)(1+\tau^d)\right)\right] \tag{A.37}$$

Putting them together,

$$\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left(1+r+\kappa\mu_{\tilde{t}}\left((1+r) - \text{mpi}((\alpha+\gamma_{\tilde{t}})\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha} + 1 - \delta)\right)\text{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} - \text{mpi} \times \gamma_{\tilde{t}}\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right)\right]$$
$$= \mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left((1+r)(1+\tau^d)\right)\right] \tag{A.38}$$

$$\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left(\kappa\mu_{\tilde{t}}\left((1+r) - \text{mpi}((\alpha+\gamma_{\tilde{t}})\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha} + 1 - \delta)\right)\text{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} - \text{mpi} \times \gamma_{\tilde{t}}\text{TFP}_{\tilde{t}} k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right)\right]$$
$$= \tau^d(1+r)\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\right] \tag{A.39}$$

$$\tau^d \mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\right]$$
$$= \beta \mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left(\kappa\mu_{\tilde{t}}\left((1+r) - \mathrm{mpi}((\alpha+\gamma_{\tilde{t}})\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha} + 1 - \delta)\right)\mathrm{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} - \mathrm{mpi}\times\gamma_{\tilde{t}}\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right)\right] \tag{A.40}$$

$$\tau^d = \beta \mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\right]^{-1}\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left(\kappa\mu_{\tilde{t}}\left((1+r) - \mathrm{mpi}((\alpha+\gamma_{\tilde{t}})\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha} + 1 - \delta)\right)\mathrm{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} - \mathrm{mpi}\times\gamma_{\tilde{t}}\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right)\right] \tag{A.41}$$

$$\tau^d = \mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\right]^{-1}\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\kappa\mu_{\tilde{t}}\mathrm{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}}\right] -$$
$$\beta\times\mathrm{mpi}\times\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\right]^{-1}\mathbb{E}\left[c_{T,\tilde{t}}^{-\sigma}\left(\kappa\mu_{\tilde{t}}\left((\alpha+\gamma_{\tilde{t}})\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha} + 1 - \delta\right)\mathrm{mpc}_{\tilde{t}}\frac{1}{\xi}\frac{p_{\tilde{t}y^N}}{c_{T,\tilde{t}}} + \gamma_{\tilde{t}}\mathrm{TFP}_{\tilde{t}}k_{\tilde{t}}^{\alpha-1}(1+\kappa\mu_{\tilde{t}})\right)\right] \tag{A.42}$$

Using the definitions (42) and (43) completes the proof. □

A.5 Proof of Lemma 2

Proof. To simplify the proof, I will solve the problem of a benevolent planner that must allocate capital to firms. This is analogous to the structure described in the main body because banks are assumed to be competitive. I solve this problem by backward induction. First, firms choose labor given capital.

$$\max_{h(i)} \left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} - wh(i) \tag{A.43}$$

with first order condition,

$$(1-\alpha)\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} h(i)^{\frac{\alpha-\alpha\eta-1}{\eta}} = w \tag{A.44}$$

$$h(i) = \left(\frac{(1-\alpha)\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} \tag{A.45}$$

Which means the value of the firm $V(A(i), k(i))$, taking capital and productivity as given, is

$$V(A(i), k(i) = \left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}}\left(\left((1-\alpha)\frac{\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{\eta}{1+\alpha(\eta-1)}}\right)^{(1-\alpha)\frac{\eta-1}{\eta}}$$
$$- w\left(\frac{(1-\alpha)\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{\eta}{1+\alpha(\eta-1)}}$$

$$= \left((A(i)k(i)^\alpha)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} \right)^{\frac{\eta}{1+\alpha(\eta-1)}} w^{-\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \left((1-\alpha)\frac{\eta-1}{\eta} \right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}}$$

$$- w^{-\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \left((1-\alpha)\frac{\eta-1}{\eta} (A(i)k(i)^\alpha)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{1+\alpha(\eta-1)}}$$

$$= \left((A(i)k(i)^\alpha)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} \right)^{\frac{\eta}{1+\alpha(\eta-1)}} w^{-\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \left(\left((1-\alpha)\frac{\eta-1}{\eta} \right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} - \left((1-\alpha)\frac{\eta-1}{\eta} \right)^{\frac{\eta}{1+\alpha(\eta-1)}} \right)$$

$$= \left((A(i)k(i)^\alpha)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} \right)^{\frac{\eta}{1+\alpha(\eta-1)}} \left(\frac{(1-\alpha)\frac{\eta-1}{\eta}}{w} \right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \frac{1+\alpha(\eta-1)}{\eta}$$

$$V(A(i), k(i)) = k(i)^{\frac{\alpha(\eta-1)}{1+\alpha(\eta-1)}} A(i)^{\frac{\eta-1}{1+\alpha(\eta-1)}} \Gamma,$$

where Γ collects all the terms that do not depend on $k(i)$ or $A(i)$.

Uncertainty Reveal. After choosing k and before choosing labor, $F(i)$ is revealed, such that

$$k(i) = \hat{k}(i) - A^{\nu(\eta-1)} F(i) \hat{k}(i)^{1-\nu} \tag{A.46}$$

where $\hat{k}(i)$ is the capital chosen by the banks. Then, the value ex-ante V_e is given by

$$V_e(\hat{k}(i), A(i)) = \mathbb{E}\left[\left(\hat{k}(i) - A^{\nu(\eta-1)} F(i) \hat{k}(i)^{1-\nu} \right)^{\frac{\alpha(\eta-1)}{1+\alpha(\eta-1)}} A(i)^{\frac{\eta-1}{1+\alpha(\eta-1)}} \Gamma \right] \tag{A.47}$$

Capital Choice. The allocation of capital is a solution to

$$\max_{k(i)} \int V_e(k(i), A(i)) \quad \text{s.t.} \quad \int k(i) = k \tag{A.48}$$

with first order condition:

$$\begin{aligned} &\hat{k}(i)\mathbb{E}\left[\left(1 - \frac{A^{\nu(\eta-1)}F(i)}{\hat{k}(i)^\nu} \right)^{\frac{1}{1+\alpha(\eta-1)}} \left(1 - (1-\nu)\frac{A^{\nu(\eta-1)}F(i)}{\hat{k}(i)^\nu} \right) \right]^{-(1+\alpha(\eta-1))} \\ &= A(i)^{\eta-1} \left(\frac{1+\alpha(\eta-1)}{\alpha(\eta-1)} \Gamma\lambda \right)^{-(1+\alpha(\eta-1))} \end{aligned} \tag{A.49}$$

Dividing by the same condition for firm j,

$$\begin{aligned} &\frac{\hat{k}(i)}{\hat{k}(j)} \left(\frac{\mathbb{E}\left[\left(1 - \frac{A(i)^{\nu(\eta-1)}F(i)}{\hat{k}(i)^\nu} \right)^{\frac{1}{1+\alpha(\eta-1)}} \left(1 - (1-\nu)\frac{A(i)^{\nu(\eta-1)}F(i)}{\hat{k}(i)^\nu} \right) \right]^{-(1+\alpha(\eta-1))}}{\mathbb{E}\left[\left(1 - \frac{A(j)^{\nu(\eta-1)}F(j)}{\hat{k}(j)^\nu} \right)^{\frac{1}{1+\alpha(\eta-1)}} \left(1 - (1-\nu)\frac{A(j)^{\nu(\eta-1)}F(j)}{\hat{k}(j)^\nu} \right) \right]^{-(1+\alpha(\eta-1))}} \right)^{-(1+\alpha(\eta-1))} \\ &= \left(\frac{A(i)}{A(j)} \right)^{\eta-1} \end{aligned} \tag{A.50}$$

I guess and later verify that $k(i) = \bar{A}A(i)^{\eta-1}k$, where $\bar{A}$ is a constant.

$$\left(\frac{A(i)}{A(j)}\right)^{\eta-1}\left(\frac{\mathbb{E}\left[\left(1-\frac{F(i)}{(\bar{A}k)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{F(i)}{(\bar{A}k)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}}{\mathbb{E}\left[\left(1-\frac{F(j)}{(\bar{A}k)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{F(j)}{(\bar{A}k)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}}\right)^{-(1+\alpha(\eta-1))} \tag{A.51}$$

$$= \left(\frac{A(i)}{A(j)}\right)^{\eta-1}$$

$$1 = 1 \tag{A.52}$$

To find $\bar{A}$, plug into the resource constraint,

$$\int k(i) = k \tag{A.53}$$

$$\int \bar{A}A(i)^{\eta-1}k = k \tag{A.54}$$

$$\bar{A}k\int A(i)^{\eta-1} = k \tag{A.55}$$

$$\bar{A} = \mathbb{E}\left[A(i)^{\eta-1}\right]^{-1} \tag{A.56}$$

□

B Capital Misallocation Block

In this section I explore some alternative microfoundations and show how to relate the empirical findings of Bau and Matray (2023) to the model.

B.1 Possible Microfoundations

I know explore three possible microfoundations that span three commonly mentioned sources of misallocation: Credit shocks, borrowing constraints, and equity constraints.

B.1.1 Credit shocks

In this model, the frictions $F(i)$ represent shocks to the credit supply of firms, as in the models of Chodorow-Reich (2013); Herreño (2023); Pinardon-Touati (2024). These shocks can be overhead costs of the bank, monitoring or operating costs, additional sources of demand that reduce available credit to firms such as government credit demand or changes in the balance sheet of banks.

No matter the source, I assume that these shocks $S(i)$ take the form

$$S(i) = A^{\eta-1}F(i)k^{\gamma} \tag{B.1}$$

Where $F(i)$ is a random variable that represents different realizations in these shocks across firms. For instance, a firm that is more geographically remote or more opaque will imply higher costs for the bank. Likewise, different firms might be served by banks with different balance sheets, or they might be located in regions with different government spending. What is important is that the magnitude of these shocks do not scale perfectly with the total capital available to the bank. This is achieved by $0 < \gamma < 1$.

Let $\nu = 1 - \gamma$, then, as a result, the amount banks can lend is given by

$$k(i) = \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)\right]^{\eta-1}} k \left(1 - \frac{F(i)}{\mathbb{E}\left[A(i)^{\eta-1}\right]^{\nu} k^{\nu}}\right) \tag{B.2}$$

B.1.2 Borrowing Constraints

In this model, I assume that the household in charge of the bank can divert a fraction of the credit. If they do that, the other households can seize a fraction of their capital k. I follow the same timing as Bianchi and Mendoza (2018) and assume that this diversion happens before any other trading occurs. Once the capital is seized, I assume that the household buys it again.

After diverting the credit, the household can hide a fraction $A(i)^{\nu(\eta-1)}F(i)k(i)^{-\nu}$ of its assets, where $F(i)$ is the idiosyncratic component. While in this case it is a characteristic of the household, it can also be interpreted as describing the firm level of opaqueness or perceived riskiness.

It follows that, to avoid any diversion in equilibrium, the following incentive compatibility must hold,

$$k(i) \leq \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)\right]^{\eta-1}} k \left(1 - \frac{F(i)}{\mathbb{E}\left[A(i)^{\eta-1}\right]^{\nu} k^{\nu}}\right) \tag{B.3}$$

In equilibrium, (B.3) holds with equality.

B.1.3 Internal and External Finance

In this setup, firms assemble capital by combining bank credit and equity $E(i)$ in the following way,

$$k(i) = \left(\theta k_b(i)^{\frac{\rho-1}{\rho}} + (1-\theta)(A(i)^{\eta-1}E(i))^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{B.4}$$

The CES specification is a reduced form way of modelling the imperfect substitutability between both sources of financing, given by ρ in this case.

For simplicity, I assume that Equity is fixed, although it would be possible to have it be a choice of the firm. Then, total capital can be written as

$$k(i) = \theta k_b(i) \left(1 + \frac{1-\theta}{\theta}\left(\frac{A(i)^{\eta-1}E(i)}{k_b(i)}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{B.5}$$

Using Lemma 2,

$$k(i) = \theta \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)^{\eta-1}\right]} k \left(1 + \frac{1-\theta}{\theta}\left(\frac{E(i)}{\frac{k}{\mathbb{E}[A(i)^{\eta-1}]}}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{B.6}$$

$$\tag{B.7}$$

Let $F(i) \equiv \left(\frac{1-\theta}{\theta}\mathbb{E}\left[A(i)^{\eta-1}\right] E(i)\right)^{\frac{\rho-1}{\rho}}$ and $\nu \equiv \frac{\rho-1}{\rho}$ to write

$$k(i) = \theta \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)^{\eta-1}\right]} k \left(1 + \frac{F(i)}{k^{\nu}}\right)^{\frac{1}{\nu}}, \tag{B.8}$$

which yields the desired result.

B.2 Comparison to Bau and Matray (2023)

In this section I show how the effects on productivity predicted by my model relate to the identified empirical estimates by Bau and Matray (2023)

This paper exploits a set of reforms in India that made it easier for foreign capital to access firms. To identify their effects, they leverage the staggered liberalization across industries and also the difference between firms with high and low marginal revenue productivity of capital (mrpk). I focus on two estimates, the effect on the average mrpk and the difference in mrpk changes across low and high mrpk firms.

Model Derivation. I now show how changes in aggregate capital relate to these two measures in the model. Recall the assumption that:

$$k(i) = k\left(1 - \frac{F(i)}{k^{\nu}}\right)$$

Taking capital as given, the firm solves

$$\max_{h(i)} (1+s)\left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} - h(i)w \tag{B.9}$$

With first order condition,

$$(1+s)\frac{\eta-1}{\eta}(1-\alpha)\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} h(i)^{\frac{\alpha-1-\alpha\eta}{\eta}} = w \tag{B.10}$$

and solution

$$h(i) = \left(\frac{(1+s)\frac{\eta-1}{\eta}(1-\alpha)\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} \tag{B.11}$$

Define mrpk as mrpk $\equiv \frac{\eta-1}{\eta}\alpha\frac{(A(i)k(i)^{\alpha}h(i)^{1-\alpha})^{\frac{\eta-1}{\eta}}}{k(i)}$ and use the optimal $h(i)$ to find

$$\text{mrpk}(i) = (1+s)\frac{\eta-1}{\eta}\alpha\frac{\left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}}}{k(i)} \tag{B.12}$$

$$\text{mrpk}(i) = (1+s)\frac{\eta-1}{\eta}\alpha A(i)^{\frac{\eta-1}{\eta}}k(i)^{\frac{\eta(\alpha-1)-\alpha}{\eta}}h(i)^{\frac{(1-\alpha)(\eta-1)}{\eta}} \tag{B.13}$$

$$\text{mrpk}(i) = (1+s)\frac{\eta-1}{\eta}\alpha A(i)^{\frac{\eta-1}{\eta}}k(i)^{\frac{\eta(\alpha-1)-\alpha}{\eta}}\left(\left(\frac{(1+s)\frac{\eta-1}{\eta}(1-\alpha)\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{\eta}{1+\alpha(\eta-1)}}\right)^{\frac{(1-\alpha)(\eta-1)}{\eta}} \tag{B.14}$$

$$\text{mrpk}(i) = (1+s)\frac{\eta-1}{\eta}\alpha A(i)^{\frac{\eta-1}{\eta}}k(i)^{\frac{\eta(\alpha-1)-\alpha}{\eta}}\left(\frac{(1+s)\frac{\eta-1}{\eta}(1-\alpha)\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \tag{B.15}$$

$$\text{mrpk}(i) = \alpha\left((1+s)\frac{\eta-1}{\eta}\right)^{\frac{\eta}{1+\alpha(\eta-1)}}\left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}}A(i)^{\frac{\eta-1}{1+\alpha(\eta-1)}}k(i)^{\frac{-1}{1+\alpha(\eta-1)}} \tag{B.16}$$

Take logs and focus on terms dependent on k,

$$\log \text{mrpk}(i) = -\frac{1}{1+\alpha(\eta-1)}\log k(i) + t.i.k. \tag{B.17}$$

And using the assumption on how $k(i)$ is determined,

$$\begin{aligned}
\log \text{mrpk}(i) &= -\frac{1}{1+\alpha(\eta-1)}\log k(1-\frac{F(i)}{k^{\nu}}) + t.i.k. && \text{(B.18)}\\
&= -\frac{1}{1+\alpha(\eta-1)}\left(\log k + \log(1-\frac{F(i)}{k^{\nu}})\right) + t.i.k. && \text{(B.19)}\\
&= -\frac{1}{1+\alpha(\eta-1)}\left(\log k - \frac{F(i)}{k^{\nu}}\right) + t.i.k., && \text{(B.20)}
\end{aligned}$$

where the third line uses the assumption that $F(i)$ is small. Taking expectation of (B.20) and using the assumption that $\mathbb{E}[F(i)] = 0$ yields

$$\mathbb{E}[\log \text{mrpk}(i)] = -\frac{1}{1+\alpha(\eta-1)}\log k + t.i.k. \tag{B.21}$$

With log-derivative

$$\frac{\partial \mathbb{E}[\log \text{mrpk}(i)]}{\partial \log k} = -\frac{1}{1+\alpha(\eta-1)} \tag{B.22}$$

It follows that changes in $\mathbb{E}[\text{mrpk}(i)]$ can be approximated as,

$$\Delta\mathbb{E}[\log \text{mrpk}(i)] = -\frac{1}{1+\alpha(\eta-1)}\Delta\log k \tag{B.23}$$

To obtain the relative change between firms, take the difference of (B.20) for firms i and j,

$$\log \mathrm{mrpk}(i) - \log \mathrm{mrpk}(j) = \frac{1}{1+\alpha(\eta-1)} \frac{F(i) - F(j)}{k^\nu} \tag{B.24}$$

and take the derivative with respect to $\log k$,

$$\frac{\partial \log \mathrm{mrpk}(i)}{\partial \log k} - \frac{\partial \log \mathrm{mrpk}(j)}{\partial \log k} = \frac{\nu}{1+\alpha(\eta-1)} \left(\frac{F(j) - F(i)}{k^\nu} \right) \tag{B.25}$$

$$\frac{\partial \log \mathrm{mrpk}(i)}{\partial \log k} - \frac{\partial \log \mathrm{mrpk}(j)}{\partial \log k} = -\nu \left(\log \mathrm{mrpk}(i) - \log \mathrm{mrpk}(j) \right) \tag{B.26}$$

which links the relative changes in mprk to changes in total capital. Then,

$$\Delta \log \mathrm{mrpk}(i) - \Delta \log \mathrm{mrpk}(j) = -\nu \left(\log \mathrm{mrpk}(i) - \log \mathrm{mrpk}(j) \right) \times \Delta \log k \tag{B.27}$$

Results Comparison. Equations (B.23) and (B.27) make predictions about the average change and relative change in mrpk in response to changes in k. To compare these with the results in Bau and Matray (2023), I use the same calibration as in the main text, setting $\alpha = 0.3, \eta = 3$ and $\nu = 0.22$.

Bau and Matray (2023) report[31] a 32% increase in capital and a 19% decrease in $\mathbb{E}\left[\mathrm{mrpk}(i)\right]$. Given that increase in capital, equation (B.23) predicts

$$\Delta \mathbb{E}\left[\log \mathrm{mrpk}(i)\right] = -0.62 \times 0.32 = -0.2, \tag{B.28}$$

which is remarkably close.

Bau and Matray (2023) also show[32] that firms with high mrpk reduced their mrpk by 32% more than low mrpk firms. They also mention that the former originally had a mprk 160% higher than the latter. Combining these, the prediction by (B.27) is

$$\Delta \log \mathrm{mrpk}(i) - \Delta \log \mathrm{mrpk}(j) = -0.22 \times 1.6 \times 0.32 = -0.11 \tag{B.29}$$

Given that Bau and Matray (2023) report a 33% difference the model can be considered as predicting a lower bound[33] for the effects of investment on productivity. Of note is also the fact that they identify the effect of foreign capital entering into the market, which potentially improves the ability of the financial system. In the model, this would translate into an increase in ν, which here I take as a primitive.

[31] See Table 4 (Bau and Matray, 2023).
[32] See Table 5 (Bau and Matray, 2023).
[33] This value is slightly larger than the upper endpoint of the 95% confidence interval for their estimate.

C Data Appendix

In this section I provide more details into the data work on Section 6.

C.1 Country Panel Data

Table C.1 shows summary stats for the sample of countries used to calculate the time series moments. The sample has 19 middle-income and 17 high-income economies.

C.2 Orbis

I access the Orbis dataset through the Orbis Project at the CBS grid, where it was made available through the efforts of Kochen (2022).

Table C.2 presents summary statistics for the 18 countries selected in the sample, which follow the sample from Kalemli-Özcan et al. (2024). Table C.3 shows the distribution of output and employment across the firm-size distribution. The results are similar to those reported by Kalemli-Özcan et al. (2024)[34], who compare it to aggregate statistics from Eurostat and find that Orbis provided a representative sample.

[34] Table D.2.5

Table C.1: Summary Statistics for Panel of Countries

Country	First Year	Last Year	σ_Y	σ_C/σ_Y	$\sigma_{CA/Y}$	Sudden Stops
Middle-Income						
Argentina	1979	2012	9.75	1.19	3.05	3
Brazil	1979	2012	9.73	1.38	2.01	1
Chile	1979	2012	11.43	1.18	3.37	2
Colombia	1979	2012	5.74	1.48	2.84	1
Dominican Republic	1979	2012	7.40	0.86	3.11	1
Ecuador	1979	2012	7.58	1.37	3.00	2
Egypt	1979	2012	3.98	0.75	3.35	0
El Salvador	1979	2012	8.17	1.64	2.45	1
Hungary	1979	2012	13.95	0.57	3.35	0
Indonesia	1979	2012	7.01	0.81	2.38	1
Malaysia	1979	2012	5.63	1.53	7.59	1
Mexico	1979	2012	6.44	1.14	2.32	2
Morocco	1979	2012	4.19	1.12	2.85	4
Peru	1979	2012	13.75	0.84	2.98	2
Philippines	1979	2012	10.13	0.48	2.74	1
South Africa	1979	2012	9.56	0.91	2.67	2
Tunisia	1979	2012	6.23	2.10	2.38	1
Ukraine	1979	2012	25.39	1.13	4.88	0
Venezuela	1979	2012	8.96	1.83	6.64	1
High-Income						
Canada	1979	2012	3.99	0.91	2.10	1
Czech Republic	1979	2012	5.64	0.95	1.78	1
Denmark	1979	2012	3.04	0.97	1.54	1
Finland	1979	2012	6.45	0.82	3.44	0
France	1979	2012	2.44	1.09	1.05	1
Germany	1979	2012	2.42	1.00	2.26	0
Iceland	1979	2012	7.32	1.18	5.99	2
Italy	1979	2012	2.58	1.01	1.72	1
Netherlands	1979	2012	4.34	1.39	1.60	1
Norway	1979	2012	3.60	1.36	4.04	2
Portugal	1979	2012	4.84	1.49	4.86	1
South Korea	1979	2012	7.18	1.38	3.49	1
Spain	1979	2012	5.42	1.22	2.84	0
Sweden	1979	2012	4.48	1.02	2.34	1
Switzerland	1979	2012	2.54	0.58	2.44	0
United Kingdom	1979	2012	5.07	1.17	1.35	1
United States	1979	2012	3.01	1.10	1.30	2

Notes: This table presents summary statistics for the sample of countries used to estimate time-series moments in Section 6. σ_Y, σ_C and $\sigma_{CA/Y}$ are the standard deviations of log GDP per capita, log consumption per capita and the current account over GDP. All three variables are quadratically detrended. Sudden stops refer to the number of Sudden Stops as identified by Korinek and Mendoza (2014).

Table C.2: Summary Statistics for Countries in Orbis Sample

Country	N. Firms	N. Obs	Total Employees
Austria	9,283	88,004	72,401
Belgium	23,074	255,545	424,162
Czech Republic	42,064	314,829	823,799
Estonia	7,278	61,552	60,523
Finland	13,570	122,259	199,776
France	133,919	1,312,707	1,449,405
Germany	69,432	622,434	1,711,145
Hungary	144,485	1,084,401	483,002
Italy	204,584	1,659,439	2,042,621
Latvia	9,467	71,653	92,640
Norway	16,938	116,000	45,704
Poland	29,400	199,855	776,776
Portugal	55,038	462,410	352,406
Romania	62,621	539,698	807,445
Slovak Republic	22,344	136,706	190,646
Slovenia	20,231	138,929	116,640
Spain	158,426	1,634,164	1,520,580
Sweden	28,456	322,773	389,409

Notes: The first column shows the number of unique firms per country in the data. The second column and third column show the average number across years of observations and total employees respectively.

Table C.3: Size Distribution in the Manufacturing Sector: Total Sample, 2006

Panel A: Gross Output	AT	BE	CZ	DE	EE	ES	FI	FR	HU	IT	LV	NO	PL	PT	RO	SE	SI	SK
1 to 19 employees	0.05	0.03	0.07	0.01	0.14	0.20	0.07	0.09	0.04	0.13	0.03	0.16	0.02	0.20	0.13	0.21	0.12	0.16
20 to 249 employees	0.38	0.40	0.45	0.24	0.61	0.60	0.41	0.45	0.62	0.60	0.51	0.51	0.33	0.63	0.48	0.42	0.44	0.45
250+ employees	0.57	0.57	0.49	0.75	0.25	0.20	0.52	0.46	0.34	0.27	0.46	0.32	0.66	0.17	0.39	0.37	0.45	0.39
Panel B: Employment																		
1 to 19 employees	0.16	0.11	0.05	0.01	0.12	0.23	0.09	0.09	0.03	0.11	0.13	0.23	0.01	0.24	0.11	0.16	0.09	0.08
20 to 249 employees	0.38	0.40	0.39	0.30	0.56	0.47	0.41	0.33	0.31	0.50	0.54	0.48	0.34	0.53	0.35	0.33	0.39	0.38
250+ employees	0.45	0.50	0.55	0.69	0.32	0.30	0.49	0.58	0.65	0.38	0.33	0.29	0.65	0.22	0.54	0.51	0.52	0.53

Notes: This table presents the firm size distribution of output and employment for the countries in the sample. I use total operating revenue as the definition of revenue at the firm level. The year 2006 is chosen for comparability to Kalemli-Özcan et al. (2024). The countries are Austria (AT), Belgium (BE), Czech Republic (CZ), Germany (DE), Estonia (EE), Spain (ES), Finland (FI), France (FR), Hungary (HU), Italy (IT), Latvia (LV), Norway (NO), Poland (PO), Portugal (PT), Romania (RO), Sweden (SE), Slovenia (SI) and (Slovakia).