Algorithms and Computability

Some Further Comments on Complexity Theory

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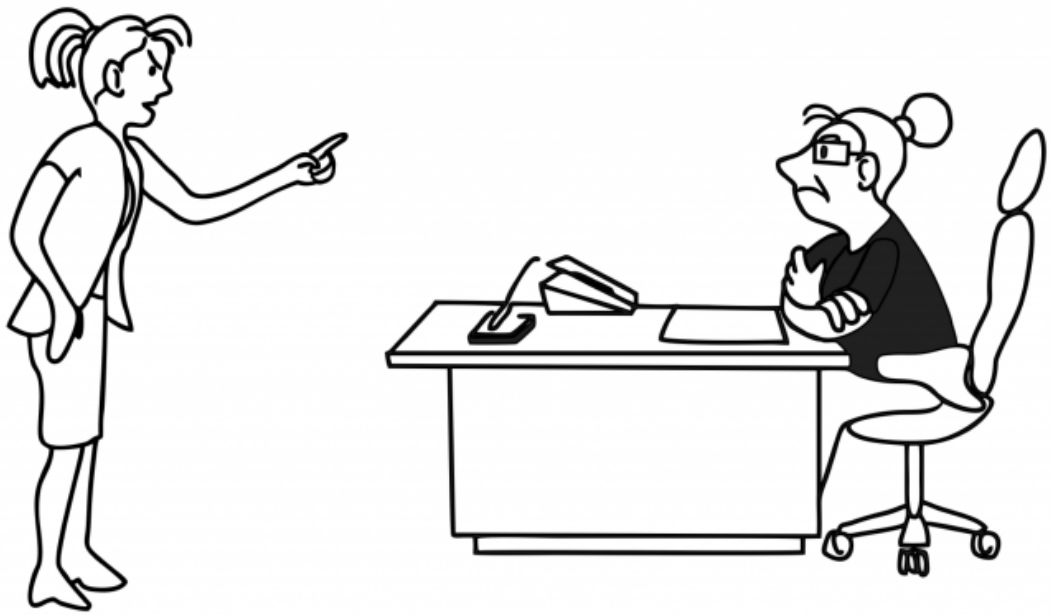
When You Cannot Find a Good Algorithm

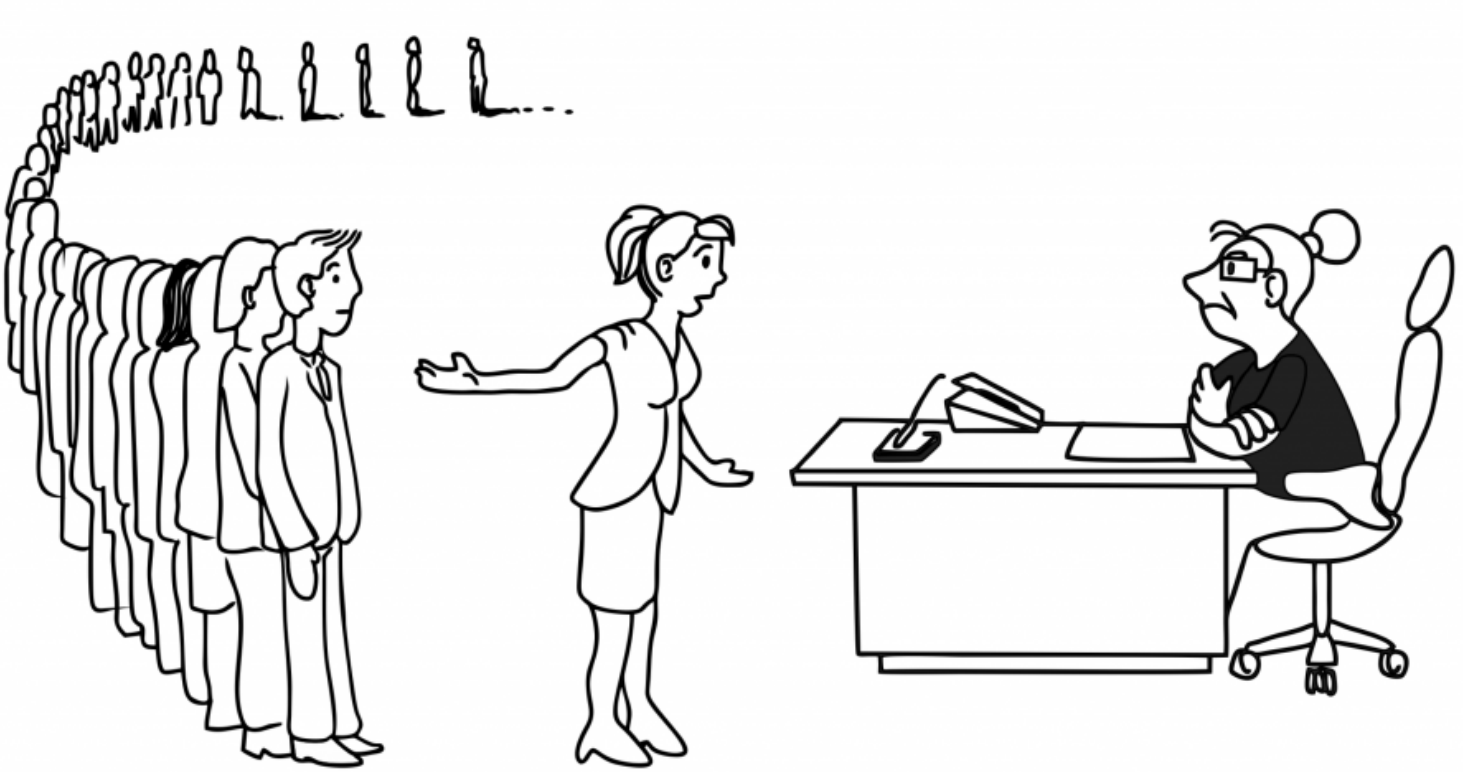
"I can't find an efficient algorithm, I guess I'm just too dumb."

When You Cannot Find a Good Algorithm



"I can't find an efficient algorithm, because no such algorithm is possible!"

When You Cannot Find a Good Algorithm



"I can't find an efficient algorithm, but neither can all these famous people."

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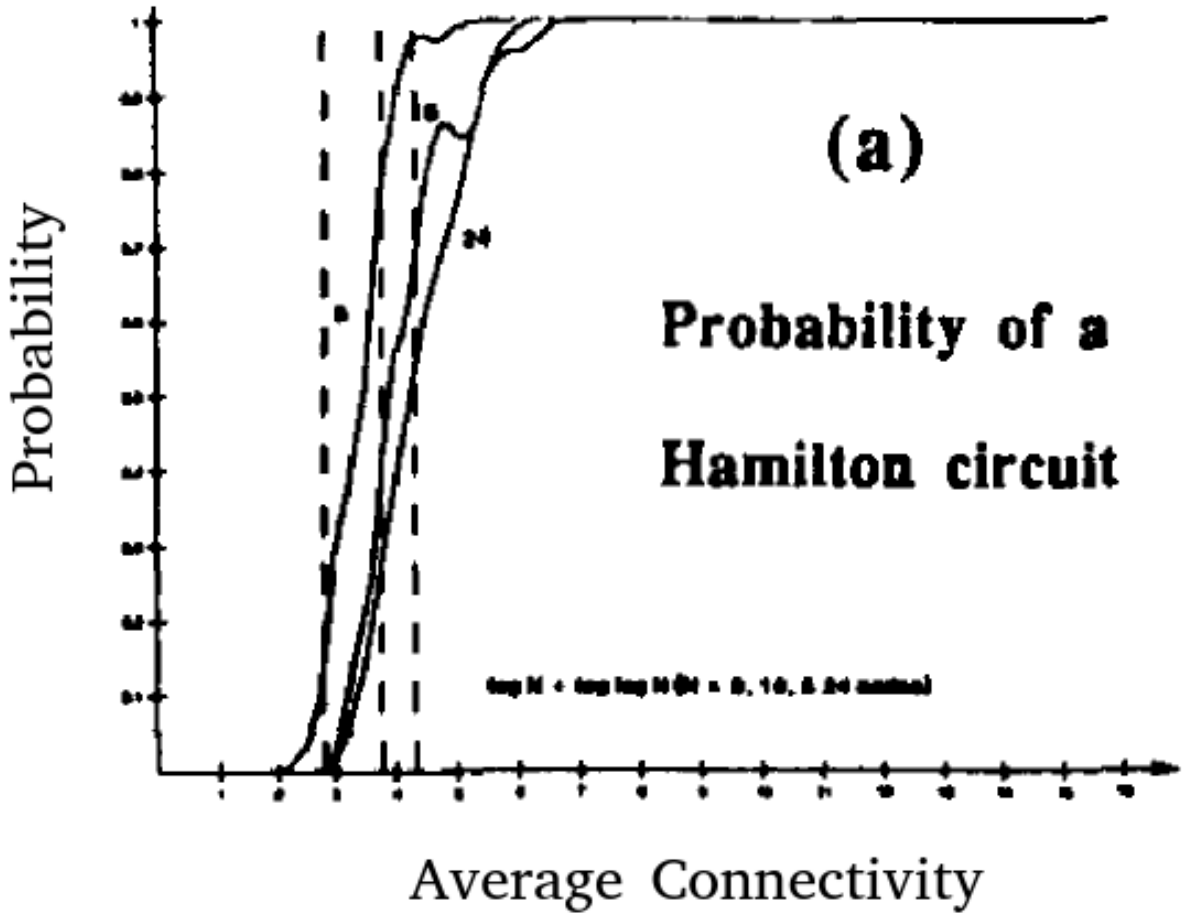
  - That makes it infeasible to solve even for modest problem sizes

What Does NP-Complete Mean in Practice?

- There are *some* problem instances for which any known algorithm requires exponential running time
  - That makes it infeasible to solve even for modest problem sizes
- However, we may still be able to solve many (even most!) problem instances efficiently
  - Often, practical instances have reasonable structure
  - Good heuristics exist for many problems
  - For instance, modern SAT solvers show a quadratic running time in most practical instances, and can solve practical instances with millions of constraints and hundreds of thousands of variables

What Does NP-Complete Mean in Practice?

- Probability of Hamiltonian circuits in random graphs



[Cheeseman et al. '91]

Deal With NP-Complete Problems in Practice

- Heuristics
- Randomized algorithms ("guess and check")
- Restrict structure (may make a problem become easier, e.g., 2SAT, DNF-SAT)
- Approximation algorithms (find a possibly suboptimal solution)
  - Example algorithm for vertex cover:
    - Find an uncovered edge in the graph
    - Add both vertices to the cover
    - Remove all adjacent edges
    - Repeat until done
    - At most $2\times$ as large as the best minimal cover
  - The above is optimal under "unique games conjecture"
  - Knapsack has ε-close approximation algorithm

Is There More to Complexity Theory?

- Oh yes!
- `https://complexityzoo.net/Complexity_Zoo` lists 550 classes

Is There More to Complexity Theory?