A74, Dartmouth College
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A74 EXERCISES: Electronic populations (9.5, 10.6)

1. The Hydrogen Balmer lines are transitions from the first excited state ($n = 2$ level). Consider Hα, the transition to/from $n = 1$ to $n = 2$.

   (a) Write the expression for the relative populations of these two energy levels as a function of temperature.

   (b) For the Sun and for an A-type star ($T = 10,000$K), what are the relative population of the two levels? What does this imply about the relative strengths of the Hα lines in these stars?

   (c) As T increases, what does the Boltzmann equation suggest should happen to the Hα line for very hot (O & B) stars?

   (d) Calculate the ionization fraction for Hydrogen for the Sun and the A-type star. Use a number density of electrons of $2 \times 10^{17}\text{cm}^{-3}$.

   (e) What does this imply about the Hydrogen lines for very hot stars?

2. Consider a line profile ϕ_ν. We'll use this result on Wednesday when thinking about evaporating planetary atmospheres. I'm using both frequency ν and velocity v in this problem–sorry.

   (a) Approximate the line profile ϕ_ν as a top hat with width $\Delta\nu$; here we are saying that we'll consider a narrow band of frequencies to all contribute equally to absorption while anything outside of that band does not contribute at all. It may help to draw the functions.

   (b) Now translate into a velocity width Δv. Use λ_0 to represent the center of the line, noting that the change in $\Delta\nu$ is small compared to ν_0 and λ_0. (This is straightforward, don't overthink it).

   The reason for translating to velocity v is that frequently in astronomy, line broadening is given in terms of a velocity width rather than a frequency or wavelength width. The center of the line is considered to be $v = 0$ and the generic relationship between frequency, wavelength, and velocity allows you to switch from ν from v.