A74, Dartmouth College
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**A74 EXERCISES: Relativity (4)**

1. Cosmic rays from space collide with the nuclei of atoms in Earth's upper atmosphere, producing elementary particles called muons. Muons are unstable and decay after an average lifetime of $\tau = 2.2\mu$s, as measured in a laboratory where the muons are at rest. The number of muons as a function of time should be: $N(t) = N(0)e^{-t/\tau}$. At the top of Mt Washington (in the White Mountains of New Hampshire), a detector counted 563 muons/hr, moving downward with a speed $u = 0.9952c$. At sea level, 1907 m lower, another muon detector counted 408 muons/hr.

   (a) Ignoring relativity, how many muons should have been counted at sea level?

   (b) How does time dilation come into play?

   (c) Accounting for time dilation, how many muons should have been counted at sea level?

2. In its rest frame (but the moving reference frame as viewed from Earth!), the quasar SDSS 1030+0524 produces Ly$\alpha$ emission at 121.6 nm. On Earth (our observer's rest frame), this emission is observed at $\lambda = 885.2$ nm.

   (a) Calculate the apparent velocity of the quasar, starting from Eq. 4.12b. Is this directed towards or away from you? Note this velocity results from the expansion of the Universe.

   (b) Calculate the redshift of the quasar, where $z = \Delta\lambda/\lambda$. This corresponds to a time when the Universe was about 0.9 Gyr old.

   (c) Ly$\alpha$ emission corresponds to the $n = 2$ to $n = 1$ transition, with the $n = 1$ state corresponding to neutral hydrogen. If there is neutral hydrogen in the Universe at a redshift of around $z = 5.5$, this would result in a Ly$\alpha$ absorption feature in the observed quasar spectrum. At what wavelength would this appear?

   This is the physics behind the "Ly$\alpha$ forest": a series of closely-packed absorption lines, each corresponding to the light from the source (usually a quasar) being absorbed by neutral hydrogen at different redshifts along the line of sight.

Both problems adapted from Carrol & Ostlie.