A74, Dartmouth College
Prof. Emily Boudreaux

A74 EXERCISES: Synchrotron (6)

Caution on unit systems: the textbook uses CGS, if you're using SI make sure all of your units *and constants* are correct.

1. Electrons of a certain energy can be considered to emit all their energy as photons with energy equal to the critical frequency ω_c, which depends on γ and B. For an ultrarelativistic gas, the electron temperature is related to the energy according to $E = 3kT_e$.

   (a) Determine T_e in terms of ν and B, plugging in for all the constants.

   (b) Assume you are looking at a self-absorbed synchrotron source. As in lecture, when considering the self-absorbed synchrotron spectrum, you may assume $T_b \approx T_e$ because we are at low frequencies where the material is opaque. You make a measurement at $\nu = 1$ GHz and find that the brightness temperature $T_b = 10^{11}$ K. What is the magnetic field strength of this source?

2. What might make the Real Life (TM) synchrotron spectrum of a radio galaxy differ from the idealized sources we've been discussed?

3. Like for other emission sources, we can calculate the lifetime for synchrotron sources.

   (a) Derive and equation for the cooling time for a single electron, using the "isotropic distribution of electrons" assumption since we don't have a preferred direction α.

   (b) The Crab Nebula emits synchrotron emission. Consider a typical electron with $\gamma = 4 \times 10^7$, radiating with energy 20 keV, or 4.8×10^{18} Hz, and assume the magnetic field strength is about 10^{-4} Gauss. What is the cooling time for such electrons?

   (c) Comparing the cooling time you calculated to the age of the Crab Nebula (about 900 years), what does this imply about the nebula? What role might the pulsar play (think back to HW2 Q4)?

Adapted from lectures by S. Ransom and C. Flynn.