Astronomy 74
Assignment 4

Your homework should be legible and comprehensive. This is great practice for presenting written material to scientists, supervisors, and customers! To be specific, you should:

1. write and draw diagrams neatly, or use a computer
2. clearly describe the steps to your solution, not just show the math
3. include a written narrative about the problem, your approach, and your conclusions
4. leave a half-page separation between ending one problem and starting the next, and use only one column

Note: You are allowed to make simplifying assumptions (though please state the assumption and the reason it's valid). This will be very useful for some problems! =

1. *Estimate* the approximate size of a protogalaxy which can collapse under the weight of its own self-gravity. To do so, present an *order-of-magnitude* derivation of the maximum radius R of an object that can collapse out of virialized clump of fully ionized hydrogen, assuming that the only way the gas cools is through bremsstrahlung radiation. In general, objects can only collapse if their cooling time is shorter than their gravitational collapse time. The cooling time of a gas cloud is the time it takes to lose of order unity (say 0.5) of its thermal energy. The collapse time is the time the cloud takes to shrink its radius by order unity. Recall that if a gas is virialized, its kinetic temperature is uniquely related to the depth of its gravitational potential well. Clearly state any other assumptions you make while making your derivation.

   Express your result in kpc, and comment on its validity.

2. Exploring bremmstrahlung from ionized hydrogen spheres. Part a) is intended to help you think about parts b) and c) so you should do it first.

   (a) Consider two spheres of ionized hydrogen plasma, which emits bremsstrahlung radiation. Both have a mass M and a uniform temperature T. The first sphere has radius r_1 and is optically thin. The second sphere has radius r_2 and is optically thick. The rough physical scenario this represents is the isothermal collapse of a cloud of gas that is cooled by bremsstrahlung. What are the luminosities of the spheres in terms of their masses, radii, and/or temperatures?

   (b) An HII region (composed of ionized hydrogen) produces a radio emission from bremmstrahlung. At $\nu = 0.3$ GHz, the flux is 0.1 Jy. $\tau = 1$ is at $\nu = 1$ GHz, and the flux there is 1 Jy. Sketch the radio spectrum of the HII region, labeling the slopes in each regime and indicating where $\tau >> 1$ and $\tau << 1$.

   (c) The HII region from part b) has a temperature $T = 10^4$ K. Calculate the angular diameter of the HII region, and its linear diameter assuming it is at a distance of $d = 10$ kpc.

3. Cosmic rays are ultra-relativistic, and therefore ultra-high energy, particles that originate outside our Solar System. They can be electrons, protons, neutrons, or the nuclei of heavier elements. The origins of cosmic rays are uncertain (as we'll see in this problem) but they have both galactic (supernovae blast waves) and extragalactic (from AGN or gamma ray bursts... or maybe some new physics entirely!) origins. Cosmic rays propagating through the Milky Way do so in the presence of the Milky Way's magnetic field, which we'll approximate as a 3μG uniform field in the plane of the Galaxy.

   (a) Consider a shower of extragalactic cosmic rays with energies E, entering the Galaxy perpendicular to the disk (this means the initial path of the cosmic ray is perpendicular to the Galactic magnetic field). The particles follow a curved path due to the presence of the magnetic field.

   First, calculate the gyroradius (r_g) – this is the radius of the curved path the particle takes (and the origin of synchrotron radiation; see the set up in Sec 6.1).

   Next, calculate the angular deviation between where we would have observed the cosmic ray to enter the Galaxy in the absence of the magnetic field, and where it was actually observed in after executing gyration. To do this, consider the maximum angular offset from the particle's entry point ti the Galactic disk and the disk midplane (which is where we would eventually make our observation). (You've got a triangle here: one leg is h and the other r_g). Take the scale height h of the Galaxy to be $h = 300$pc. Put your answer in terms of particle energy.

   (b) Continuing with part a), calculate the angular deviation for particles of 10^{18} and $10^{20.5}$eV, which we'll consider the be the low and high of the energies in the cosmic ray shower. Considering that, in reality, the particle may not have completed a full gyration, what does this mean for localizing the source of extragalactic cosmic rays?

   (c) Find an expression for the cooling time of cosmic rays in terms of m, Z, B, γ, and α. Then, average over pitch angles α. You should be starting from Eq. 6.5a.

   (d) Calculate the cooling time for electrons, protons and an iron nucleus, all starting with energies of 10^{18}eV. Why do we keep talking about electrons when we discuss cooling?

4. We observe synchrotron emission all along the sky from the Milky Way. According to Beuermann et al. (1985, A&A, 153, 17), the brightness temperature within 3 degrees of the Galactic disk, at a frequency of 408 MHz, is typically 150 K, and has a spectrum $F_\nu \propto \nu^{-1}$ near this frequency. Assume that a typical line of sight goes through a path length of 10 kpc of a region with a uniform magnetic field $B = 3\mu$G and a number density of relativistic electrons $\eta(\gamma)d\gamma = c_o\gamma^{-p}d\gamma$.

   (a) What is the value of p of the energy distribution of the electrons?

   (b) What is the γ factor of the electrons that contribute most of the emission at the frequency of 408 MHz?

   (c) How long does it take for the electrons with this γ factor to slow down?

   (d) Calculate the value of c_o, assuming isotropic emission. The approach to do this is very non-intuitive in my opinion, and the first thing to recognize is that the power equation in 6.36 contains the same C as 6.20, which in this problem we've called c_0. Then, think back to Chapter 1 and how we can manipulate our basic radiative transfer equations.

What is the energy density in relativistic electrons with γ factor larger than the value computed in part (b)? Compare that energy density to the magnetic energy density. (Think about how to apply equation 6.36; assume a typical value of $\sin\alpha = 1/\sqrt{2}$.)