Astronomy 74
Assignment 3

Your homework should be legible and comprehensive. This is great practice for presenting written material to scientists, supervisors, and customers! To be specific, you should:

1. write and draw diagrams neatly, or use a computer
2. clearly describe the steps to your solution, not just show the math
3. include a written narrative about the problem, your approach, and your conclusions
4. leave a half-page separation between ending one problem and starting the next, and use only one column

Note: You are allowed to make simplifying assumptions (though please state the assumption and the reason it's valid). This will be very useful for some problems!

1. The cosmic microwave background is observed to have an anisotropy of $\delta T = 3.346 \pm 0.017$ mK (where $2\delta T$ is the min-to-max difference; Bennett *et al.* 2003). The dipole is interpreted to be the result of the Doppler shift caused by the solar system motion relative to the nearly isotropic blackbody field. Derive the angular distribution of temperature T seen by an observer who is moving with a velocity v with respect to a blackbody field. Use this result to determine the motion of the solar system with respect to the cosmic microwave background. To do so, you will need to search the astronomical literature to find an accurate temperature for the CMB. Be precise in your answer.

2. Find the optical depth for Thomson scattering for the following systems:

   (a) The disk of the Galaxy, in the directions perpendicular and parallel to the disk,. The warm ionized interstellar medium (ISM) of our Galaxy, which can be approximated by a disk of half-thickness 500 pc, 20 kpc radius, electron density of about 0.1 cm^{-3}, and electron temperature $T \sim 10^4$ K.

   (b) Intergalactic space, over the distance of the Hubble radius, assuming that the electron density is 10^{-5} cm^{-3}. The Universe is about 14 billion years old and the Hubble radius is the distance light travels in this time. Ignore the expansion of the universe.

   (c) The Sun, assuming it is a sphere of completely ionized Hydrogen (70%), Helium (28%) and Oxygen (2%) with a uniform density. The percentages give the fraction of the total mass.

   (d) Discuss the results, i.e. which is objects are optically thick and which optically thin and what this implies for observations.

3. A quasar emits jets in opposite directions at relativistic speeds. Consider a blob emitted along the jet with Doppler parameter β at an angle θ relative to the observer. Assume that the quasar is far from the observer, and ignore any effects of cosmology–this makes this treatment not very representative of a quasar, but it does capture the behavior of high mass x-ray binaries.

(a) Calculate the *apparent* Doppler parameter ($\beta_{\rm app}$) of the jet across the sky (the transverse velocity) as a function of β and θ. Instead of using the Lorentz transforms, think about the radiation emitted by the blob as it moves relativistically through space, which is then received by the stationary observer.

(b) Find the angle for which $\beta_{\rm app}$ is maximum, and calculate the maximum $\beta_{\rm app}$. For $\gamma = 10$ (which seems to be a typical Lorentz factor of quasars as far as I can figure though they can be even larger; Hovatta et al. 2009), how many times faster than the speed of light could the jet appear to be moving?

(c) Calculate the ratio between the emitted intensity of the blob (I_0) and the observed intensity of the blob ($I_{\rm obs}$), and then the ratio of the intensity jet moving away from us to the jet moving towards us (assume the jets are emitted in exactly the opposite directions). Use your result to explain why we often observe only one jet from a source. Assume that the intensity in the blob rest frame $I_\nu \propto \nu^{-\alpha}$.

(d) Quasars are often observed to have one, very bright jet with no visible counter-jet. On the other hand, radio galaxies often exhibit less-luminous jets and counter-jets. How can quasars and radio galaxies with jets be explained as a single type of object?

4. In astrophysics, we frequently argue that a source of radiation which undergoes a fluctutation of duration Δt must have a physical diameter of order $D < c\Delta t$.

(a) Consider a pulsar, our neutron star with the rapid "lighthouse" flashes (we considered the spin-down of a pulsar in the previous homework). Imagine that the pulse involves the entire pulsar brightening and dimming, instantaneously and for an infinitesimal amount of time. Draw a diagram and argue that the duration of the pulse gives the equation stated for the maximum diameter of the pulsar D. (To arrive at exactly the above, you'll have to assume you can see through the neutron star, which is obviously false, but you can at least see to one limb, which is $D/2$ and we're only getting a rough upper limit anyways).

If the duration of the pulse is 1/1000th of a second, what's the maximum diameter of the emitting object? Compare this to the size of the Sun, a white dwarf, and a neutron star.

(b) Now suppose instead that the source is an optically thick spherical shell of radius powered by a stationary point at the center. This might, for example, be the active region surrounding the supermassive black hole in a quasar. The shell has radius $R(t)$, expanding relativistically with $\beta \sim 1, \gamma >> 1$. Show that if the observer sees a fluctuation from the shell of duration Δt, the source can actually be much larger than $D \sim c\Delta t$. Calculate the upper limit on R in terms of γ, c, and Δt.

In the rest frame of the shell's surface, each surface element can be treated as an isotropic emitter. You want to consider the relativistic beaming of the emission from each of these surface elements. You'll likely want to draw a diagram to consider the geometry of the emission.