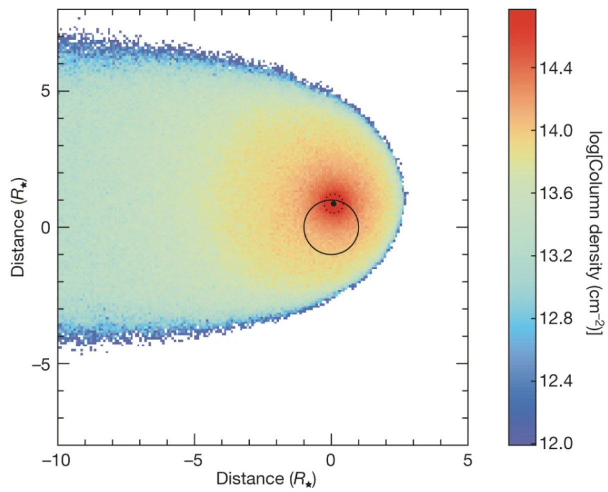
A74, Dartmouth College
Prof. Emily Boudreaux

A74 EXERCISES: Einstein coefficients (1.6)

1. The exoplanet Gl 436 b has a large evaporating atmosphere. In the paper by Ehrenreich et al[1] they model the planet's extended, neutral hydrogen envelope. The $\log_1 0$ of the column density ranges from 14.5 near the core of the planet to 13 in the tail, as shown in the figure below (which illustrates their model). The transit of the evaporating atmosphere can be observed at Lyα wavelengths. You don't need a calculator for this worksheet.



a. In the case of an envelope comprised only of neutral hydrogen (n_1 would represent neutral hydrogen in this scenario in Equation 1.76), show that τ_ν can written in terms of the Einstein coefficient for absorption B_{12}:

$$\tau_\nu = \frac{h\nu}{4\pi} B_{12}\phi(\nu)N \quad (1)$$

$$= \sigma_{\nu 0}\phi(\nu)N \quad (2)$$

where N is the column density of, in this case, neutral hydrogen. Start by considering Equation 1.76 for the case of absorption only, and consider both the definition for absorption coefficient α_ν (pg 10) and τ_ν (Eq 1.26). Assume the distribution of hydrogen is uniform. Eq 1.74 will also get you there in fewer steps and it may help to look at it, but I want you to think a bit about the definitions and how the parts fit together.

b. What is τ_ν in terms of $\sigma_{\nu 0}$, N, λ_0, and Δv, assuming you are observing near the line center? You'll need to use the result from the last worksheet that we can approximate the line profile as: $\phi(\nu) = \lambda_0/\Delta v$.

c. The factor $\sigma_{\nu 0} = \frac{h\nu}{4\pi} B_{12}$ can be calculated from the Einstein A coefficient, A_{21}, after using the Einstein relations (Eq 1.72) to swap out the B_{12} for the A_{21}. The reason for the swap

[1] https://ui.adsabs.harvard.edu/abs/2015Natur.522..459E/abstract

is that I found the value for A_{21} for Ly$-\alpha$ but not B_{21}! I'll save you the math (check the solutions) and tell you the answer is $\sigma_{v0} = 1.1 \times 10^{-2}$ cm^2/s. λ_0 = 1216 Angstroms, and a decent choice for δv is 10 km/s. What is the optical depth in the tail of Gl 436b's evaporating atmosphere?

d. Approximately how much of the Lyα light emitted from the star is absorbed in the tail of the atmosphere?

e. What is the optical depth of the interstellar medium towards Gl 436b in the core of the Lyα line? The log column density of the ISM is about 18 for Gl 436, that is, $N = 10^{18}$.

This makes the center of the Lyα line completely unobservable due to attenuation by the ISM. However, the stellar Lyα line is very broad, and in the tail of the evaporating atmosphere, the planetary H atoms are actually be accelerated away from the star. The planetary H atoms therefore absorb *blueshifted* Lyα light (away from the line center that is attenuated by the ISM) and the transit of the planetary atmosphere can therefore actually be observed.

In the figure below, the vertical gray region is all absorbed by the ISM and only the wings of the Lyα line are observable. The transit however is seen from -40 to -120 km/s, where the in-transit spectra (red and green) show an approximately 50% decrease in light relative to the out of transit spectra. This matches what you calculated pretty well!