Astronomy 74
Assignment 1

Your homework should be legible and comprehensive. This is great practice for presenting written material to scientists, supervisors, and customers! To be specific, you should:

1. write and draw diagrams neatly, or use a computer
2. clearly describe the steps to your solution, not just show the math
3. include a written narrative about the problem, your approach, and your conclusions
4. leave a half-page separation between ending one problem and starting the next, and use only one column

Note: You are allowed to make simplifying assumptions (though please state the assumption and the reason it's valid). This will be very useful for some problems!

10 pts

1. Consider the **perfectly** flat, blackbody disk encircling a blackbody star with radius R_* at its midplane. Assume the disk perfectly absorbs light form the star, that there is no heat transport within the disk, and that the disk perfectly re-emits all the energy it absorbs. This means each annulus of the disk is effectively independent. Another consequence of the lack of heat transfer is that the top side of the disk only absorbs light from the top half of the star.
   (a) Calculate the temperature of the disk, T_D as a function of distance away from the star (disk radius, r). You'll be considering the flux absorbed at a single point on some annulus with disk radius r, you'll need to integrate over the surface of the star. Drawing a diagram will help you to think about the component of the flux from the star that is relevant for absorption. You can assume that $r >> R_*$, but only after doing the integral (this will involve a Taylor expansion).
   (b) This is a good first approximation of a protoplanetary disk. Consider a disk that consists of two narrow rings, one at r_1 and one at r_2. Sketch the approximate flux as a function of wavelength (the units are arbitrary, but indicate the meaning of the features in the sketch).

10 pts

2. Consider the **perfectly** flat, blackbody disk encircling a blackbody star from the previous question. The spectral energy distribution, or SED, is νF_ν, which is the flux radiated by an object per logarithmic frequency interval.
   (a) How does νF_ν relate to λF_λ? Demonstrate the relationship quantitatively.
   (b) Why is νF_ν, rather than F_ν the quantity of interest if we want to understand, overall, how different energies are represented in a spectrum? (For example, if we want to ask whether the spectrum is dominated by energy emitted at x-rays, or at gamma rays, or in the optical?)
   (c) Let's continue Q1, where we have a disk with a certain radial temperature profile, $T(r)$, where every annulus radiates as a blackbody of that temperature. For an observer very far away from this source (such that the source is a point source, and not resolved), what

is νF_ν? You'll need to integrate over the solid angle presented by the disk, which will include considering the inclination of the disk. Set up the integral with the variables ν, $B_\nu(T(r))$, D (the distance from the observer to the disk), r (the incremental disk radius), and i (the disk inclination, where $i = 0$ is a face-on disk).

3. A perfect blackbody at a temperature T has a shape of an oblate ellipsoid, it's surface being given by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$$

10 pts with $a > b$. This might be a simple model for a rapidly rotating star, where its rotation makes it oblate.

   (a) Would the measured luminosity (total power emitted) of the blackbody be isotropic? Why or why not?

   (b) Consider an observer at a distance d from the blackbody with $d >> a$ (and b). What is the direction to the observer from the blackbody, relative to the z-axis, for which the maximum amount of flux will be observed (at a fixed distance d)? Calculate the maximum flux.

   (c) Repeat question **b** but for the minimum flux.

   (d) If the two observers who see the maximum and minimum flux from distance d can resolve the blackbody, what is the surface brightness (specific intensity) that each one will measure?

   (e) Now, consider a simple model for an elliptical galaxy, a perfectly oblate ellipsoid also described the equation at the beginning of this problem. Instead of being a blackbody, it contains a large number (N) of stars. For simplicity, assume that all stars have the same radius R, the same surface temperature T and that $NR^2 << ab$ (think about what this means). Are there any differences from the case of a blackbody? Explain. Answer questions (a – d) for the galaxy.

4. The Eddington limit is the maximum luminosity an object can have and not spontaneously eject layers. The value depends on the mass absorption coefficient, κ. We'll show in this problem that the Eddington limit for a fully ionizing hydrogen gas with Thompson scattering off free electrons is:

$$L_{EDD} = 1.25 \times 10^{38} \text{ergs s}^{-1} (M/M_\odot) = 3.3 \times 10^4 (M/M_\odot) L_\odot$$

It comes up in a wide range of astrophysics. In the context of stellar astrophysics, it is the maximum luminosity a star can have and remain in hydrostatic equilibrium. Because stellar mass and luminosity are related, this sets an upper limit for stability around $100 M_\odot$. Another area of application is active galactic nuclei (AGN); in this context, the luminosity comes from the accretion of gas surrounding the supermassive black hole at the center of the

10 pts galaxy.

Note that this derivation will assume spherical symmetry and that the source is in equilibrium, which isn't necessarily true and some objects can be found to be radiating at "super-Eddington" luminosities.

(a) A black hole is surrounded by a spherically symmetric cloud of gas. The accreting black hole has mass M and luminosity L. Consider an optically thin shell a distance r away from the black hole. Find the condition such that shell is not ejected by radiation pressure in terms of M/L. In other words, find the equation of the form $M/L > X$, where X will include a dependence on κ, the mass absorption coefficient, which you should assume is independent of ν.

(b) The minimum κ is for Thomson scattering off free electrons, assuming the cloud is completely composed of ionized hydrogen gas. Use this to solve for L_{EDD}, the maximum luminosity, in terms of the Thompson scattering cross-section σ_T rather than κ.

(c) The Thomson cross section is $\sigma_T = 6.65 \times 10^{-25}$ cm^2. Use this to demonstrate that L_{EDD} is given by the equation given at the beginning of the problem.

(d) Quasars are distant AGN that outshine their host galaxies, such that they appear as a point source. A Seyfert galaxy's host is still visible, with a dimmer AGN at the center. $L_{QSO} = 10^{13} L_{\odot}$, and $L_{Sey} = 10^{11} L_{\odot}$. What's the minimum mass of the black holes at the center of these galaxies? How does that compare to the mass of the black hole at the center of the Milky Way?