Astronomy 74
Assignment 5

Your homework should be legible and comprehensive. This is great practice for presenting written material to scientists, supervisors, and customers! To be specific, you should:

1. write and draw diagrams neatly, or use a computer
2. clearly describe the steps to your solution, not just show the math
3. include a written narrative about the problem, your approach, and your conclusions
4. leave a half-page separation between ending one problem and starting the next, and use only one column

Note: You are allowed to make simplifying assumptions (though please state the assumption and the reason it's valid). This will be very useful for some problems!

15 pts

1. This example takes an in depth look at synchrotron emission from a source that has energy injected into it. The Crab Nebula is one example of such a source.

   (a) *Derive the timescale for synchrotron cooling for electrons an isotropic distribution of velocities (averaged over the pitch angle α). Put your answer in seconds and also in years, as a function of γ and B. Note whatever units of B you use.*

   Note that R & L say on page 167 that $\frac{\partial}{\partial t}(\gamma m e_c^2) = q\vec{v}\cdot\vec{E} = 0$, and from there conclude that $\gamma =$ constant because $\vec{E} = 0$. They are referring to the fact that the *external* E-field is 0. The electron, however, also feels its own E-field, and so the total $\vec{E} \neq 0$, and hence γ is not actually constant. It is, however, very nearly constant over 1 gyro-period (except near ultra-strong magnetic fields, $\sim 10^{15}$ G), and the dynamics over 1 gyro-period is all that concerns R&L on page 167. The electron feeling its own E-field is the radiation reaction discussed in R&L §3.5.

   (b) The power-law distribution of electrons in astrophysical sources are maintained against synchrotron losses by continuous energization by central engines (a.k.a. injection). The injection (input) spectrum of electrons is modified by synchrotron losses to produce a steady-state (output) distribution. Call $\eta(E,t) = dN/dE$ the differential energy spectrum of electrons . Continuity of electrons in energy space reads

   $$\frac{\partial \eta(E,t)}{\partial t} + \frac{\partial}{\partial E}[\dot{E}\eta(E,T)] = I$$

   where I is the rate of injection of electrons with some input distribution and $\dot{E}$ is the rate of energy loss of a single electron by synchrotron radiation. This equation merely describes how the number of electrons in a given energy bin changes with time, taking into account a flux divergence (the second term on the left- hand-side) and a source term (the right-hand-side). One generally assumes a steady-state distribution of electron energies for which $\eta \propto E^p$. Given p, how must I scale with E? Give only the scaling and forget about the numerical coefficients. As with most scaling problems, you don't have to solve anything in detail; work to order of magnitude.

(c) Electrons having a given energy must wait a characteristic time before synchrotron losses become important. Before this time elapses for all such electrons, how does η scale with E?

(d) The spectral index ($\alpha = d\ln F_\nu / d\ln \nu$) of radiation in a fixed frequency range from a radio jet flattens with increasing distance from the central galaxy. That is, $\alpha = -0.5$ at the remote edge of the jet (the "hot spot"), and $\alpha = -1$ closer in. Given the understanding you developed in the previous parts of this problem where are the "freshest" electrons located, i.e., those newly injected into the energy spectrum? Are they at the end of the jet, or are they closer in? In other words, where is the principal site of particle acceleration? Explain your reasoning.

(e) Sketch several profiles of η vs. E at various times, assuming I is constant in time.

5 pts

2. A spherical cloud of ionized hydrogen with uniform density and of radius R is emitting Bremsstrahlung radiation in the X-rays. The total observed X-ray surface brightness (intensity), integrated over frequency, at the center of the cloud is $S_x = 10^{-12}\,\mathrm{erg\,cm^{-2}s^{-1}arcmin^{-2}}$. The X-ray spectrum has the Bremsstrahlung shape with temperature $kT = 7\,\mathrm{keV}$. The microwave background radiation is observed through the cloud at a frequency of 90 MHz (well into the Rayleigh-Jeans regime), and at the cloud center it has an intensity lower than the average CMB intensity by a fractional amount $\Delta T/T = 5\times10^{-5}$. The cloud is observed to have an angular radius on the sky of 3 arc minutes.

   Find the radius R and the distance to the cloud.

   The ionized hydrogen cloud is a thermal distribution of electrons, and you should use the non-relativistic version of the Compton y parameter.

   Note: In the real cosmological case, with the cloud being a cluster of galaxies at some redshift z receding from us due to the expansion of the universe, the X-ray surface brightness is reduced below the true one by a factor $(1+z)^4$, and the X-ray spectrum gets redshifted leading to an apparent temperature lower by a factor $1+z$. The numbers given in this problem are typical values for a rich cluster at $z \simeq 0.3$ after correction for the redshift effects.

10 pts

3. We receive X-ray radiation from a source a distance D away with a flux F with a spectrum corresponding to the figure below. We assume the spectrum is a result of bremsstrahlung for an optically thin, hot plasma cloud. The cloud has a thickness a radius R $\Delta R \sim R$ and is in hydrostatic equilibrium around a central mass M.

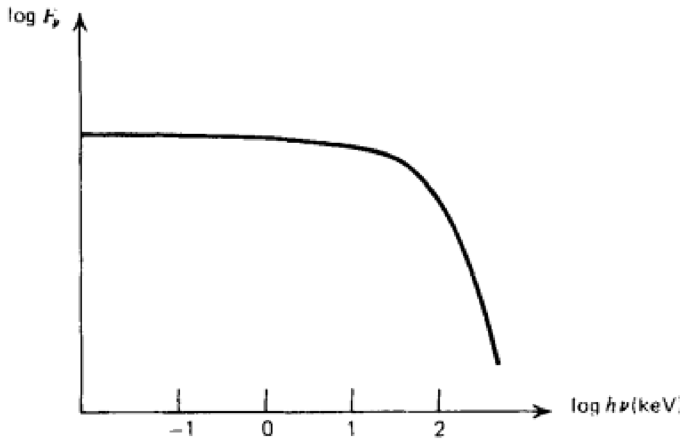
   (a) Find R in terms of the observables D and F, and the unknown mass M.

   (b) Find the cloud density ρ in terms of the same variables.

   (c) If $F = 10^{-8}$ erg cm^{-2} s^{-1} and $D = 10$ kpc, what are the constraints on M such that the source would actually be optically thin, like we assumed?

   (d) Calculate the Compton y parameter in terms of the unknown mass M. You'll need κ_{es} ("es" for electron scattering, aka Thomson scattering), revisit Chapter 1.

   (e) How large or small mass would be required to result in Compton scattering being insignificant? This source is inside our Galaxy (since $D = 10kpc$) and one well-known



source of Bremsstrahlung is HII regions. What parameters are consistent or inconsistent with the source being an HII region?

4. In this problem we will take another look at the Compton cooling catastrophe. The textbook arrives at the ratio which I'll call η:

$$\eta = \frac{P_{\rm IC}}{P_{\rm synch}} = \frac{U_{ph}}{U_B}$$

10 pts where $U_B = B^2/8\pi$ is the magnetic energy density and U_{ph} is the photon energy density.

The photon energy density comes from synchrotron emission. U_{ph} is defined in Eq 7.15, though you don't need the numerical definition as written here. Instead, look back to Chapter 1 for the definition of the specific energy density (which is per unit frequency), and assume an isotropic distribution (which means $I_\nu = J_\nu$ – why?). For quasars, synchrotron emission is in the radio regime and we'll consider a self-absorbed synchrotron source. You can approximate the total flux F as $F_\nu \nu$ (why does this get you energy?), evaluated at a characteristic frequency. For the characteristic frequency, use the synchrotron critical frequency, ν_c and assume isotropic scattering.

(a) What happens when $\eta > 1$?

(b) Write a function for η of the form $\eta \propto T_b^5 \nu_c$. To get here, you'll be doing some substitutions, including to remove some factors of ν_c (this will also conveniently get rid of your B); but, you'll want to leave one ν_c unsubstituted to get this form of the equation.

(c) Solve numerically for the critical brightness temperature, T_c, as a function of ν, putting ν in units of GHz and T_c in K. This should show that the critical brightness temperature that gives $\eta = 1$ is $\sim 10^{12}$ K, the value presented in the lecture notes.