A74, Dartmouth College
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A74 EXERCISES: Thompson Scattering (3.4)

1. The solar corona is sometimes decomposed into different parts, which reflect the origins of how it shines. The "K-corona" is that one that shines via Thomson scattering.

   Scattering can be taken into account in the radiative transfer equation like so:

   $$dI_\nu = \alpha_{\nu s} I_\nu(s) ds$$

   where $\alpha_{\nu s}$ is the scattering coefficient, and the amount of scattering depends on the amount of incident radiation. $\alpha_\nu s$ can be used equivalently to the absorption coefficient, even though here scattering is adding light rather than absorbing it. When we consider the solar corona, we can make the assumption that the corona is optically thin, and therefore that the incident radiation is constant and equal to the photospheric intensity, $I_\odot$. The Thompson scattering cross section is $\sigma_T \approx 7 \times 10^{-25}$ cm^2.

   (a) Estimate the electron density of the corona by considering that Thomson scattering over a characteristic length equal to $R_\odot$ produces an intensity of about $I_K = 10^{-6} I_\odot$. I_K here refers to the K-corona. Assume the electron density is constant.

   (b) Verify that the corona is optically thin to Thompson scattering.

2. For both parts, you do not need to do any calculations, just think about geometry.

   From the lecture notes, recall that we can decompose the unpolarized incoming wave into two component waves with polarization that differs by $\pi/2$. In this example, the incoming wave is from the Solar photosphere, and the electrons being hit are the ones in the K corona. The wave polarized in the plane of scattering contributes to the outgoing wave with magnitude proportional to $\cos^2\theta$, while the wave polarized perpendicular to the plane of scattering contributes the outgoing wave with constant magnitude 1. This combines to give the $(1+\cos^2\theta)$ term in Eq. 3.40. The outgoing wave components have different polarizations, which match the polarization of the incoming wave component (pg 91 & Eq. 3.41).

   (a) We resolve the solar corona. Consider the corona as just a single scattering surface, a sphere, and assume you are looking at the limb. As a distant observer, θ is going to be pretty close to $\pi/2$ when you are looking at the limb: the outgoing lightwave travels radially outwards from the Sun, then must be redirected $90\circ$ to be sent in your direction. Choose to decompose the incident waves into components that are parallel to the direction of travel (radial) and perpendicular (tangent to the limb of the Sun).

   Sketch the polarization of the coronal limb, considering several key directions. I found this easiest to do by placing the observer out of the page, such that the outgoing wave is also directed out of the page. Consider how each of the two incident wave components contributes to the polarization of the final scattered wave.

   (b) If we were to observe a corona around another star, the corona would be unresolved. What would this mean for the polarization we would observe? Assume the stellar corona is unresolved.