Category: 242 Homework 2
Name: Emily Wu
SID: 3034504344

Problem 0

P1 a) 3 b) 3 c) 3 d) 3 e) 3

P2: 10

Problem 1

a)

Pr(Y=1|X=30, Z=1) = $\frac{1}{1+exp(-(-3.5+0.18*30+1.24))} = 0.9588$

So the probability of the team getting good grades is 0.9588.

b)

(a)

If the team uses Python, which means Z=1 and (1-Z=0), then the predicted probability of Y=1 would be $\frac{1}{1+exp(-(-\alpha_0+\alpha_1 X))}$

If the team uses R, which means Z=0, the predicted probability of Y=1 would be $\frac{1}{1+exp(-(-\beta_0+\beta_1 X))}$

(b)

I would select the teams that use Python(Z=1), and fit these teams with the standard logistic regression model: $\frac{1}{1+exp(-(-\alpha_0+\alpha_1 X))}$

Then the rest teams that use R(Z=0) will be fit to the standard logistic regression model: $\frac{1}{1+exp(-(-\beta_0+\beta_1 X))}$

(c)

Since the team uses Python, then the Z should be one, which means the model would be $\frac{1}{1+exp(-(-\alpha_0+\alpha_1 X))}$.

Since α_0 is -4.02 and α_1 is 0.24 and X is 30, then the result should be 0.96. The probability of a team getting good grades under the conditions above is 0.96.

(d)

I think the model in Part(b) would be more accurate. Let's first analyze the information given by the training set data. The training data shows that if a team is using Python, the probability of getting good grades is around 60%. If a team is using R, the probability of getting good grades should be around 52%. As is revealed by the training data, whether a team uses Python or R would make a significant impact on the team's probability of getting good grades, so in this case, the model in part(b) should be more accurate since it uses differents models for team that uses Python and the team that uses R.

(e)

I think the new training set data shows that there is no significant difference between the team that uses Python and the team that uses R in the probability of getting good grades. So there is no need to fit them with different models. In this case, I think the model in Part(a) would be more accurate.

Problem 2

The model should always predict Y to be the value that would generate the least expected loss. We should calculate the expected loss of both predicting Y to be 0 and predicting Y to be 1.
$ExpectedLoss(Y=0) = L_{FN} * f(x)$
$ExpectedLoss(Y=1) = L_{FP} * (1-f(x))$
$ExpectedLoss(Y=1) = ExpectedLoss(Y=0)$

We can get the break-even f(x) to be $\frac{L_{FP}}{L_{FN}+L_{FP}}$, so $\hat{P}$ is $\frac{L_{FP}}{L_{FN}+L_{FP}}$

If we try to the probability of Y=1 is below $\hat{P}$, then we should predit it to be Y=0, otherwise we should predict it to be Y=1

Problem 3

In [1]:

```
# Train Logistic Regression model using training set data.
import numpy as np
import pandas as pd
from plotnine import ggplot


fram_train = pd.read_csv('framingham_train.csv')
fram_test = pd.read_csv('framingham_test.csv')
fram_train.head()
print(fram_train.info)
fram_no = fram_train[fram_train['TenYearCHD'] == 0]
fram_yes = fram_train[fram_train['TenYearCHD'] == 1]
print("No Disease: " + str(len(fram_no)))
print("Yes Disease: " + str(len(fram_yes)))

#Baseline training set prediction: Always No Disease
#Baseline accuracy:
base_accur = len(fram_no)/len(fram_train)
print(base_accur)
```

```
<bound method DataFrame.info of       male  age        education  currentSmoker  cigsPerDay  BPMeds  \
0        0   41  Some high school              1          43       0
1        0   38   High school/GED              1          15       0
2        1   42   High school/GED              1           9       0
3        0   42   High school/GED              0           0       0
4        0   53   High school/GED              0           0       0
...    ...  ...               ...            ...         ...     ...
2555     1   39  Some high school              1          40       0
2556     0   52  Some high school              0           0       0
2557     1   39  Some high school              0           0       0
2558     1   49   High school/GED              1          50       0
2559     0   64  Some high school              0           0       0

      prevalentStroke  prevalentHyp  diabetes  totChol  sysBP  diaBP    BMI  \
0                   0             1         0      306  199.0  106.0  38.75
1                   0             0         0      176  110.0   80.0  24.03
```

```
2                    0              0          0       205  110.0   73.0  22.40
3                    0              1          0       263  150.0   88.0  23.68
4                    0              1          0       272  146.0   89.0  25.50
...                ...            ...        ...       ...    ...    ...    ...
2555                 0              0          0       209  134.0   82.0  28.34
2556                 0              0          0       292  125.0   87.0  31.92
2557                 0              0          0       188  105.0   65.0  22.85
2558                 0              0          0       252  156.0   91.0  25.35
2559                 0              0          0       280  127.0   77.0  30.39

      heartRate  glucose  TenYearCHD
0           100       75           0
1           100      113           0
2            61       66           0
3            96       78           0
4            73       67           0
...         ...      ...         ...
2555         70       75           0
2556         75       67           0
2557         63       76           0
2558         70      114           1
2559         56       78           1

[2560 rows x 16 columns]>
No Disease: 2178
Yes Disease: 382
0.85078125
```

i) The Logistic Regression Model on Training Set

The fitted logistic regression is

$$y = \frac{1}{1+exp(-(-9.274-0.1053*eduHigh/GED-0.1025*eduCol+0.061*eduHigh+0.56*male+0.07*age+0.15*currSmoker+0.0 \\ *prevStroke+0.2075*prevHyp-0.2975*diabetes+0.002*totChol+0.0181*sysBP-0.0045*diaBP+0.0136*BMI-0.0046*he}$$

In [2]:

```python
import statsmodels.formula.api as smf

logreg = smf.logit(formula = 'TenYearCHD ~ male + age + education + currentSmoker + cigsPerDay + BPMeds + prevalentStro
                   data = fram_train).fit()
print(logreg.summary())
```

```
corr = fram_train.corr()
print(corr)
import seaborn as sns
sns.heatmap(corr,xticklabels = corr.columns.values, yticklabels=corr.columns.values)
```

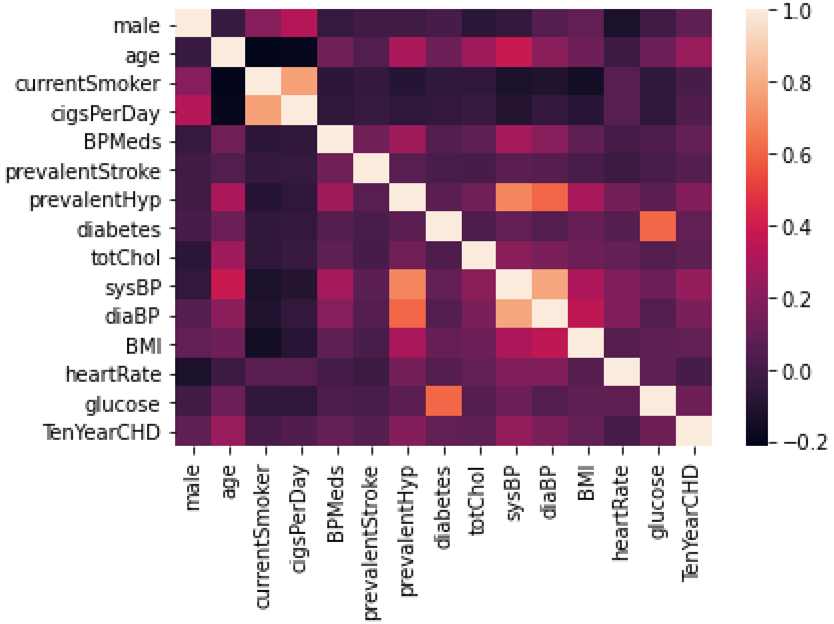
```
Optimization terminated successfully.
         Current function value: 0.365281
         Iterations 7
                           Logit Regression Results
==============================================================================
Dep. Variable:              TenYearCHD   No. Observations:                 2560
Model:                           Logit   Df Residuals:                     2542
Method:                            MLE   Df Model:                           17
Date:                 Tue, 05 Oct 2021   Pseudo R-squ.:                  0.1331
Time:                         17:29:41   Log-Likelihood:                -935.12
converged:                        True   LL-Null:                       -1078.7
Covariance Type:             nonrobust   LLR p-value:                 5.181e-51
==================================================================================================================
                                                  coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------------------------------------------
Intercept                                      -9.2740      0.882    -10.516      0.000     -11.002      -7.546
education[T.High school/GED]                   -0.1053      0.217     -0.485      0.628      -0.531       0.321
education[T.Some college/vocational school]    -0.1025      0.241     -0.425      0.671      -0.575       0.370
education[T.Some high school]                   0.0610      0.202      0.302      0.762      -0.334       0.456
male                                            0.5621      0.134      4.189      0.000       0.299       0.825
age                                             0.0689      0.008      8.303      0.000       0.053       0.085
currentSmoker                                   0.1539      0.189      0.816      0.415      -0.216       0.524
cigsPerDay                                      0.0155      0.007      2.077      0.038       0.001       0.030
BPMeds                                          0.1528      0.281      0.544      0.587      -0.398       0.704
prevalentStroke                                 0.8209      0.570      1.441      0.150      -0.296       1.938
prevalentHyp                                    0.2075      0.167      1.246      0.213      -0.119       0.534
diabetes                                       -0.2975      0.395     -0.753      0.452      -1.072       0.477
totChol                                         0.0020      0.001      1.445      0.148      -0.001       0.005
sysBP                                           0.0181      0.005      3.900      0.000       0.009       0.027
diaBP                                          -0.0045      0.008     -0.584      0.560      -0.020       0.011
BMI                                             0.0136      0.016      0.867      0.386      -0.017       0.044
heartRate                                      -0.0046      0.005     -0.888      0.374      -0.015       0.006
glucose                                         0.0096      0.003      3.439      0.001       0.004       0.015
==================================================================================================================
                     male       age  currentSmoker  cigsPerDay    BPMeds  \
male             1.000000 -0.029909       0.204207    0.327397 -0.041037
age             -0.029909  1.000000      -0.212590   -0.189122  0.136687
currentSmoker    0.204207 -0.212590       1.000000    0.769880 -0.069135
cigsPerDay       0.327397 -0.189122       0.769880    1.000000 -0.057161
BPMeds          -0.041037  0.136687      -0.069135   -0.057161  1.000000
prevalentStroke -0.006701  0.046245      -0.043744   -0.040217  0.134277
prevalentHyp    -0.005309  0.302459      -0.093215   -0.062643  0.264434
```

```
diabetes          0.018548  0.123349       -0.054224   -0.049984  0.056487
totChol          -0.076278  0.265056       -0.056851   -0.028286  0.081220
sysBP            -0.046821  0.388899       -0.123977   -0.097569  0.280192
diaBP             0.056319  0.208740       -0.105782   -0.051435  0.196694
BMI               0.094351  0.130222       -0.154004   -0.079980  0.086536
heartRate        -0.127251 -0.014015        0.063123    0.070557  0.008832
glucose          -0.005773  0.122493       -0.056205   -0.065092  0.037385
TenYearCHD        0.087473  0.249548        0.014960    0.042388  0.096434

                 prevalentStroke  prevalentHyp  diabetes   totChol     sysBP  \
male                   -0.006701     -0.005309  0.018548 -0.076278 -0.046821
age                     0.046245      0.302459  0.123349  0.265056  0.388899
currentSmoker          -0.043744     -0.093215 -0.054224 -0.056851 -0.123977
cigsPerDay             -0.040217     -0.062643 -0.049984 -0.028286 -0.097569
BPMeds                  0.134277      0.264434  0.056487  0.081220  0.280192
prevalentStroke         1.000000      0.069614  0.019806  0.016979  0.071750
prevalentHyp            0.069614      1.000000  0.076278  0.140809  0.695793
diabetes                0.019806      0.076278  1.000000  0.037066  0.101584
totChol                 0.016979      0.140809  0.037066  1.000000  0.206660
sysBP                   0.071750      0.695793  0.101584  0.206660  1.000000
diaBP                   0.058465      0.607500  0.048151  0.169206  0.783140
BMI                     0.022100      0.292661  0.106759  0.119180  0.307599
heartRate              -0.014379      0.148294  0.054499  0.104704  0.177102
glucose                 0.020678      0.073367  0.614267  0.049336  0.119062
TenYearCHD              0.054035      0.190281  0.090978  0.085381  0.239592

                    diaBP       BMI  heartRate   glucose  TenYearCHD
male             0.056319  0.094351  -0.127251 -0.005773    0.087473
age              0.208740  0.130222  -0.014015  0.122493    0.249548
currentSmoker   -0.105782 -0.154004   0.063123 -0.056205    0.014960
cigsPerDay      -0.051435 -0.079980   0.070557 -0.065092    0.042388
BPMeds           0.196694  0.086536   0.008832  0.037385    0.096434
prevalentStroke  0.058465  0.022100  -0.014379  0.020678    0.054035
prevalentHyp     0.607500  0.292661   0.148294  0.073367    0.190281
diabetes         0.048151  0.106759   0.054499  0.614267    0.090978
totChol          0.169206  0.119180   0.104704  0.049336    0.085381
sysBP            0.783140  0.307599   0.177102  0.119062    0.239592
diaBP            1.000000  0.359543   0.183339  0.052196    0.162136
BMI              0.359543  1.000000   0.058991  0.083668    0.090559
heartRate        0.183339  0.058991   1.000000  0.088158    0.013079
glucose          0.052196  0.083668   0.088158  1.000000    0.126177
TenYearCHD       0.162136  0.090559   0.013079  0.126177    1.000000
```

Out[2]: <AxesSubplot:>



ii)

The variables that will have significant impact on the probability of getting CHD are 'male', 'age', 'cigsPerDay', 'sysBP' and 'glucose'.

The coefficient of 'male' is 0.5621, which means that holding other variables constant, if a person is male, his odds of getting CHD within 10 years would multiply by $e^{0.5621}$, which means the odds would increase by 75.44%.

iii)

The expected cost of prescribing medication is $775000 * 0.15p + 75000 * (1 - 0.15p)$

The expected cost of not prescribing medication is $700000 * p$

$775000 * 0.15p + 75000 * (1 - 0.15p) = 700000 * p$

We can achieve the break-even P should be 0.1261

So if the probability of a patient getting CHD is larger than 0.1261, we should give the medicine to the patient. If the probability is lower than 0.1261, then we shouldn't give the medication to the patient.

iv)

The accuracy of logistic regression model using threshold value of 0.126 is 0.627. This suggests that the possibility of our model correctly predicting whether the patient would get CHD within 10 years is 62.7%.

The TPR is 0.68, suggesting that given that a patient would get CHD in reality, there is a probability of 68% that our model would predict it correctly.

The FPR is 0.38, suggesting that given that a patient wouldn't get CHD in reality, there is a probability of 38% that our model would wrongfully classify him to be in the group that would get CHD.

In [3]:

```
print(fram_test)
print(fram_test['education'].dtypes)
fram_test.dtypes
y_predProb = logreg.predict(fram_test)


print(y_predProb)
y_predResult = pd.Series([1 if i > 0.1261 else 0 for i in y_predProb], index = y_predProb.index)
y_realResult = fram_test['TenYearCHD']

from sklearn.metrics import confusion_matrix
cm = confusion_matrix(y_realResult, y_predResult)
print(cm)
print(type(cm))
cm_result = cm.ravel()
print(cm_result)

#Accuracy
acc = (cm_result[0]+cm_result[3])/(sum(cm_result))
print(acc)

#TPR
tpr = cm_result[3]/(cm_result[2]+cm_result[3])
print(tpr)

#FPR
```

```
fpr = cm_result[1]/(cm_result[1]+cm_result[0])
print(fpr)
```

```
      male  age                        education  currentSmoker  cigsPerDay  \
0        1   46                 Some high school              1          20
1        0   65  Some college/vocational school              0           0
2        0   41                          College              0           0
3        0   39                 Some high school              0           0
4        0   51                 Some high school              0           0
...    ...  ...                              ...            ...         ...
1093     0   64                 Some high school              0           0
1094     1   54                 Some high school              0           0
1095     0   55  Some college/vocational school              0           0
1096     1   64                          College              0           0
1097     0   46                 Some high school              1          40

      BPMeds  prevalentStroke  prevalentHyp  diabetes  totChol  sysBP  diaBP  \
0          0                0             0         0      219  118.0   79.0
1          0                0             1         0      216  163.0  102.0
2          0                0             0         0      159  119.0   76.0
3          0                0             0         0      229  125.0   80.0
4          0                0             1         0      225  155.0   92.5
...      ...              ...           ...       ...      ...    ...    ...
1093       0                0             1         0      273  155.0   86.0
1094       0                0             0         0      250  123.0   75.0
1095       0                0             0         0      260  136.5   87.5
1096       0                0             0         0      210  123.0   81.0
1097       0                0             0         0      253  118.0   74.0

        BMI  heartRate  glucose  TenYearCHD
0     24.17         70       90           0
1     30.12         91       73           0
2     27.49         55       70           0
3     24.10         75       58           0
4     23.84         72       63           0
...     ...        ...      ...         ...
1093  27.53        100       91           0
1094  25.91         65       71           0
1095  25.41         75       60           0
1096  26.49         60       75           1
1097  26.42         75       64           1

[1098 rows x 16 columns]
object
0        0.126993
1        0.254170
2        0.028517
```

```
3       0.025976
4       0.113625
          ...
1093    0.301143
1094    0.141394
1095    0.082371
1096    0.228751
1097    0.088837
Length: 1098, dtype: float64
[[569 354]
 [ 56 119]]
<class 'numpy.ndarray'>
[569 354  56 119]
0.6265938069216758
0.68
0.38353196099674974
```

v)

If the outcomes in the test set aren't affected by the treatment decision, then the expected cost per patient would be:

$$\frac{775000*119+75000*354+700000*56+0*569}{569+354+56+119} = 143875.23$$

This assumption is not reasonable because the prescribtion of medicine is supposed to prevent the CHD and therefore less people would catch CHD after the prescribtion, which is expected to decrease the cost per person. But now that this is not taken into consideration, the expected cost per person would be estimated to be higher than it should be.

If the effect of treatment decision is taken into consideration, only 15% of those prescribed would be expected to catch CHD. The expected cost per patient would be calculated as below:

$$\frac{775000*119*0.15+75000*(119+354-119*0.15)+700000*56+0*569}{569+354+56+119} = 79389.80$$

vi)

The baseline model uses the principle of majority to do the prediction. Since most people in the training set doesn't have CHD, the baseline model predicts everyone in the testing set doesn't have CHD.
This model has high accuracy of 84%, which is higher than the logistic regression model produced.But its True Positive Rate is 0 and its False Positive Rate is also 0.
Although this model has higher accuracy, suggesting this model may be more accurate in predicting whether a person would have CHD or not

than the logistic regression. But the cost per person in the baseline model would be:

$$\frac{775000*0+75000*0+700000*175+0*569}{569+354+56+119} = 111566.48$$

This is significantly higher than cost per person in logistic regression, which means if our overall goal is to minimize cost per person, then we should choose the logistic regression although its prediction may be less accurate.

In [4]:

```
fram_test2 = fram_test['TenYearCHD']


fram_basePredict = pd.Series([0 for i in fram_test2], index = fram_test2.index)
cm_base = confusion_matrix(fram_test2, fram_basePredict)
print(cm_base)
cm_resultB = cm_base.ravel()
#Acc
acc_b = (cm_resultB[0]+cm_resultB[3])/(sum(cm_resultB))
print(acc_b)
tpr_b = cm_resultB[3]/(cm_resultB[2]+cm_resultB[3])
print(tpr_b)
fpr_b = cm_resultB[1]/(cm_resultB[1]+cm_resultB[0])
print(fpr_b)
```

```
[[923   0]
 [175   0]]
0.8406193078324226
0.0
0.0
```

vii)

In [5]:

```
print(fram_test.columns)
newPerson = {'male': [0], 'age': [45], 'education':['College'], 'currentSmoker':[1],'cigsPerDay':[9],'BPMeds':[1],
             'prevalentStroke':[1], 'prevalentHyp':[0], 'diabetes':[1], 'totChol':[220], 'sysBP':[140.0],
             'diaBP':[100.0], 'BMI':[33.00], 'heartRate':[69], 'glucose':[74], 'TenYearCHD':[0]}

print(newPerson)
newData = pd.DataFrame(newPerson, index = [0])
print(type(newData.iloc[0]))
print(newData.iloc[0])
```

```
print(newData['male'])
print(type(newData['male']))
newPred = logreg.predict(newData)
print(newPred)

#newPredict = logreg.predict(df_new)
```

```
Index(['male', 'age', 'education', 'currentSmoker', 'cigsPerDay', 'BPMeds',
       'prevalentStroke', 'prevalentHyp', 'diabetes', 'totChol', 'sysBP',
       'diaBP', 'BMI', 'heartRate', 'glucose', 'TenYearCHD'],
      dtype='object')
{'male': [0], 'age': [45], 'education': ['College'], 'currentSmoker': [1], 'cigsPerDay': [9], 'BPMeds': [1], 'prevalentS
troke': [1], 'prevalentHyp': [0], 'diabetes': [1], 'totChol': [220], 'sysBP': [140.0], 'diaBP': [100.0], 'BMI': [33.0],
'heartRate': [69], 'glucose': [74], 'TenYearCHD': [0]}
<class 'pandas.core.series.Series'>
male                     0
age                     45
education          College
currentSmoker            1
cigsPerDay               9
BPMeds                   1
prevalentStroke          1
prevalentHyp             0
diabetes                 1
totChol                220
sysBP                140.0
diaBP                100.0
BMI                   33.0
heartRate               69
glucose                 74
TenYearCHD               0
Name: 0, dtype: object
0    0
Name: male, dtype: int64
<class 'pandas.core.series.Series'>
0    0.137213
dtype: float64
```

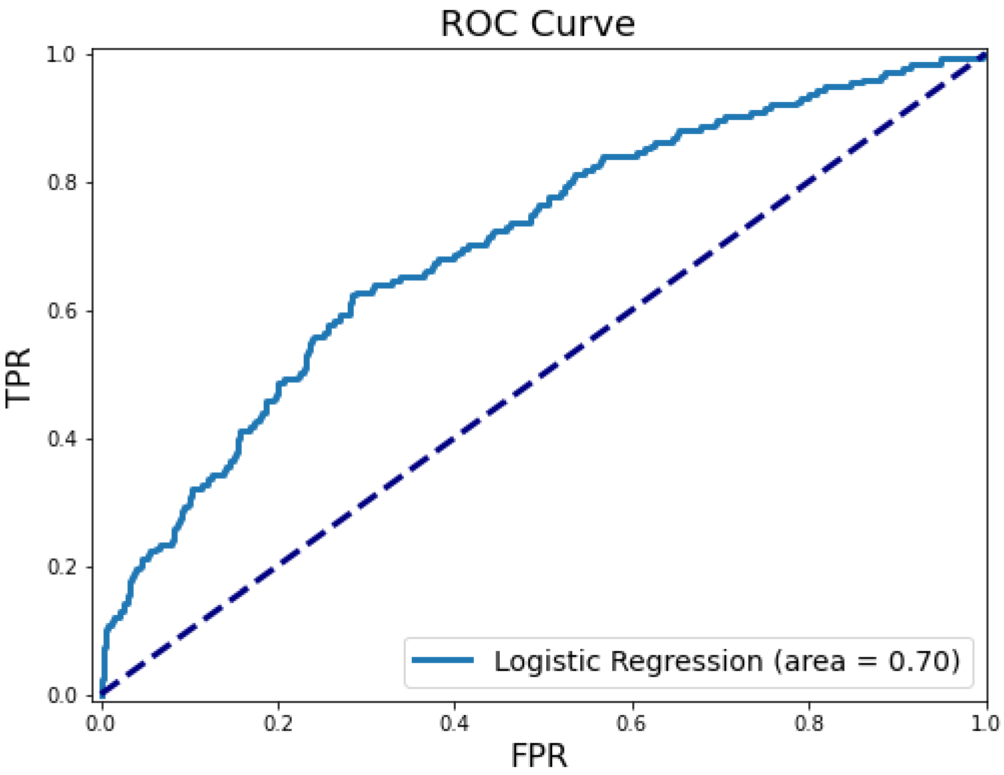
b)

The AUC is 0.7 in the test set, which suggests that there is 70% probability that the model can successfully distinguish a person that will get CHD in 10 years from a person that won't catch CHD in 10 years.
The AUC in our model is larger than the naive bayes model and therefore our model is better than the Naive Bayes Model in the discriminative ability.

In [6]:

```
import matplotlib.pyplot as plt
from sklearn.metrics import roc_curve, auc

fpr, tpr, _ = roc_curve(y_realResult, y_predProb)
roc_auc = auc(fpr,tpr)
plt.figure(figsize = (8,6))
plt.title('ROC Curve', fontsize=18)
plt.xlabel('FPR', fontsize=16)
plt.ylabel('TPR', fontsize=16)
plt.xlim([-0.01, 1.00])
plt.ylim([-0.01, 1.01])
plt.plot(fpr, tpr, lw=3, label='Logistic Regression (area = {:0.2f})'.format(roc_auc))
plt.plot([0, 1], [0, 1], color='navy', lw=3, linestyle='--')
plt.legend(loc='lower right', fontsize=14)
plt.show()
```



c)

For the scenario discussed in part a, if the probability of a patient getting CHD is larger than 0.1261, then we should prescribe the patient with the medication.
Suppose the co-payment amount is C.
If this threshold is still true, the probability of people who prescribe the medicine but still catch CHD would be 0.15 $*$ 0.1261. The cost incurred for the patient would be 500000+C.
The probability of person who prescribes the medicine but doesn't catch CHD would be 1-0.15$*$0.1261. The cost incurred for the patient would be C.
The probability of person who doesn't prescribes the medicine but catched CHD would be 0.1261. The cost incurred for the patient would be 500000 since the treatment cost is covered by the insurance company.
The probability of person who doesn't prescribes the medicine and doesn't catched CHD would be 1-0.1261. The cost incurred for the patient would be 0.
Set the expected cost incurred to person prescribed with medicine equal to the expected cost incurred to person that's not prescribed with medicine, we can solve the following equation:

$$(500000 + C) * 0.15 * 0.1261 + (1 - 0.15 * 0.1261) * C = 500000 * p$$

And we get that

$$C = 53592.5$$

When the co-payment cost is set to be 53592.5, then if the co-payment is larger than 53592.5, there would be a group of people not prescribing the medicine and the number of people in this group is the same as the number of people predicted by our logistic regression model to have lower than 0.1261 probability of catching CHD within future 10 years.

In this way, the co-payment cost of 53592.5 could match with the optimal strategy we have raised in part (a).

d)

I think one key issue here is that if we predict the patient to have the probability of less than 0.126 to catch CHD in 10 years, then we would not let the patient prescribe medication even though there is still a chance for them to catch the disease, which is unfair because everyone deserves the right to receive medication not to mention they have a chance to get infected.

So I think one way to solve this question is the approach discussed in part c, which is to give everyone the right to prescribe the medical prescription but setting a co-payment amount so that this cost would do self selection.