MTH 351: Homework 1

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1. Write a Matlab function that outputs the last d digits of the n-th Fibonacci number.
   (a) The first method is a modified version of the naive method. The 0-th and first Fibonacci number are defined as 0 and 1 respectively. All other Fibonacci numbers are computed by adding the current Fibonacci number to the previous Fibonacci number in a for loop until the desired Fibonacci number is reached. The sum produced by each addition is divided by 10^{17} and the remainder is taken to be the next Fibonacci. This ensures that precision to the ones place is maintained. 17 digits is the maximum digits saved since the datatype uint64 can only store numbers up to $1.84 * 10^{19}$ $(2^{16} - 1)$ and the two largest possible 18 digit numbers add to $2 * 10^{19}$. Since only 17 digits are kept of each addition d will only be accurate to 17 digits. Since the addition will never overflow the uint64 datatype the output will be accurate for any n. Time could be a concern as each increase in n will increase the number of for loops executed by 1.
   (b) The second method follows the matrix exponential method. This simply computes the desired Fibonacci number using the formula $\begin{bmatrix} a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{n-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The last d digits are extracted from the Fibonacci number a_n by using the modulo operator such that a_n is divided by 10^d. For this method the result is only accurate up to the 78th Fibonacci number. This is because the 79th Fibonacci number is the first to exceed $2^{53} - 1$ which is the largest number that can be represented by the 53 bit mantissa of the double datatype in Matlab. Once this number is exceeded precision would be lost making the last digits inaccurate. In this method d can become arbitrarily large, but any larger d than total digits in the Fibonacci number will just output the Fibonacci number as the additional digits would be leading zeroes.