Think Beyond 0 And 1

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Area of Research Interests

Mathematics

- Complex Analysis
  One complex variable and Several complex variables
- Complex Geometry And Algebraic Geometry

$\because \quad \therefore \quad \because$

Data Science

- Quantum Computing
- Monte-Carlo Tree Search
  Board game playing algorithms
- New Type of Search Algorithms
  Probabilistic search algorithms

D-Wave + AlphaGo

Have you ever heard of the company D-Wave?

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First Example

Question (Deutsch's Problem)

Alice, in Amsterdam, selects a number x from 0 to $2^n - 1$, and mails it in a letter to Bob, in Boston. Bob calculates some function $f(x)$ and replies with the result, which is either 0 or 1. Bob has promised to use f as one of the follwoing:

1. *$f(x) = c$, where c is a constant, or*
2. *$f(x)$ is balanced, that is, equal to 1 for exactly half of all the possible x, and 0 for the other half.*

Alice's goal is to determine whether Bob has chosen (1) or (2), corresponding with him as little as possible. How fast can she succeed?

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- Classically, at least $2^n/2 + 1$ times.
- In quantum computing, you only need **1** time.

Go Down to The Very Earth

Classically,

- Pythom, R
- C/C++, Java

- (X86) Assembly Language
- Switching Circuit
- bit

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Now, let us go to

- q-bit (reads "qubit")
- Quantum Circuit

How to Get from 0 and 1 to $\alpha_0 0 + \alpha_1 1$?

- Quantum Mechanics

Schrödinger equation (Wave Equation):

$$i\hbar \frac{\mathrm{d}\psi}{\mathrm{d}t} = H\psi,$$

where H is called the **Hamiltonian** of the quantum system. The equation roughly says (if H does not depend on t),

$$\psi = e^{\frac{-it}{\hbar}H}\psi_0,$$

where ψ_0 is the initial state.

How to Get from 0 and 1 to $\alpha_0 0 + \alpha_1 1$?

- Linear Algebra (Hilbert Space)

Consider the **solution space** of Schrödinger equation

$$V = \{\psi \mid i\hbar \frac{\mathrm{d}\psi}{\mathrm{d}t} = H\psi\}.$$

- V is a **vector space**
- $\dim_{\mathbb{C}} V = \infty$

In this setting, H can be treated as a **Linear transformation** of V, namely

$$H : V \longrightarrow V.$$

Quantum State

A q-bit is a quantum system consisting two states $|0\rangle$ and $|1\rangle$ which are also denoted as ψ_0 and ψ_1. They satisfy

$$i\hbar\frac{\mathrm{d}\psi_i}{\mathrm{d}t} = H\psi_i,\ i = 0, 1.$$

• **Superposition Principle**:

For any $\alpha_0, \alpha_1 \in \mathbb{C}$

$$i\hbar\frac{\mathrm{d}(\alpha_0\psi_0 + \alpha_1\psi_1)}{\mathrm{d}t} = H(\alpha_0\psi_0 + \alpha_1\psi_1).$$

That is why a quit has multiple states in the same time!

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- **Quantum Measurement**:

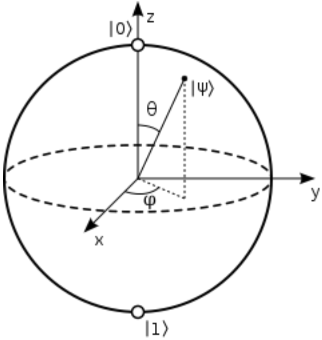
The probability to measure $|0\rangle$ is $|\alpha_0|^2$, i.e. $P(M = |0\rangle) = |\alpha_0|^2 \leq 1$.

Bloch Sphere

Let ψ be a quantum state, i.e. $\psi = \alpha_0|0\rangle + \alpha_1|1\rangle$ and $|\alpha_0|^2 + |\alpha_1|^2 = 1$. Since only the relative phase between the coefficients of $|0\rangle$ and $|1\rangle$ has any physical meaning, we reassign the parameters and write

$$\psi = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\varphi}|1\rangle.$$

Then, a state ψ can be marked on a sphere:



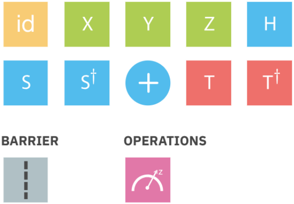
Common Q & A

- If quantum computer is commercialized, do we still need **traditional computers**?
  I think so. In most cases, quantum computers do not perform better than traditional computers.
- Compared current **super computer** with **quantum computer**, which one is faster?
  It really depends on the problem.
- What feature makes quantum computer significantly different from traditional computer?
  Quantum superposition and quantum entanglement
- University of Waterloo, Quantum computing 101
- Can I program a quantum computer using C/C++?
  Yes, you can. Thera are some simulators.

Quantum Gates And Quantum Circuit

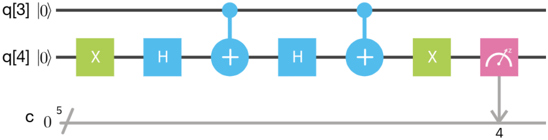
- **Basic quantum gates:**

Each gate represent a **elementary linear transformation** of V.



- **Quantum circuit:**

A circuit is a sequence of linear transformations.



Basic Gates And Basic Superposition States

- **Superposition states:**
  - $|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
  - $|-\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

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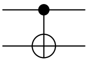
$$\because \qquad \therefore \qquad \because$$

- $-\boxed{H}-$ H gate: $|0\rangle \xrightarrow{H} |+\rangle$ and $|1\rangle \xrightarrow{H} |-\rangle$
- $-\boxed{X}-$ X gate: $|0\rangle \xrightarrow{X} |1\rangle \xrightarrow{X} |0\rangle$
-  $CNOT$ gate: $|y, x\rangle \xrightarrow{\oplus} |y \oplus x, x\rangle$, where $\oplus$ is $+ \mod 2$
- $-\boxed{Y}-$, $-\boxed{Z}-$, $-\boxed{S}-$, $-\boxed{T}-$ Other gates: Change phase

Oracle Function

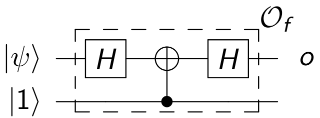
Let $f(x)$ be balanced such that $f(|0\rangle) = |0\rangle$ and $f(|1\rangle) = |1\rangle$.

Definition (Oracle Function)

Oracle function $\mathcal{O}_f$ is defined as

$$|x\rangle \xrightarrow{\mathcal{O}_f} (-1)^{f(x)}|x\rangle.$$

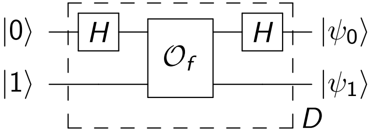
Hence, in our case, $\mathcal{O}_f(|0\rangle) = |0\rangle$ and $\mathcal{O}_f(|1\rangle) = -|1\rangle$.



If $|\psi\rangle = |0\rangle$, $|0\rangle \xrightarrow{H} |+\rangle \xrightarrow{\oplus} |+\rangle \xrightarrow{H} |0\rangle$.
If $|\psi\rangle = |1\rangle$, $|1\rangle \xrightarrow{H} |-\rangle \xrightarrow{\oplus} -|-\rangle \xrightarrow{H} -|1\rangle$.

Deutsch's Algorithm I

Oracle Function solves Deutsch's problem.



If $f(x) = 0 \Longrightarrow \mathcal{O}_f(|x\rangle) = (-1)^0|x\rangle = |x\rangle$.

- $|\psi_0\rangle$:

$$|0\rangle \xrightarrow{H} |+\rangle \xrightarrow{\mathcal{O}_{f,0}} |+\rangle \xrightarrow{H} |0\rangle$$

- $|\psi_1\rangle$:

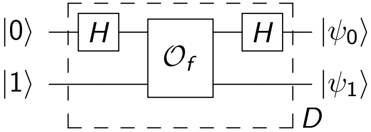
$$|1\rangle \xrightarrow{\mathcal{O}_{f,1}} |1\rangle$$

In summary, we have

$$|10\rangle \xrightarrow{D} |10\rangle \Longrightarrow P(\psi_0 = |0\rangle) = 1.$$

Deutsch's Algorithm II

Oracle Function solves Deutsch's problem.



If $f(x)$ is balance, $\mathcal{O}_f(|0\rangle) = |0\rangle$ and $\mathcal{O}_f(|1\rangle) = -|1\rangle$.

- $|\psi_0\rangle$:

$$|0\rangle \xrightarrow{H} |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \xrightarrow{\mathcal{O}_{f,0}} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle \xrightarrow{H} |1\rangle$$

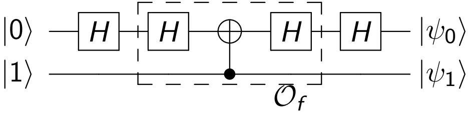
- $|\psi_1\rangle$:

$$|1\rangle \xrightarrow{\mathcal{O}_{f,1}} |1\rangle$$

In summary, we have

$$|10\rangle \xrightarrow{D} |11\rangle \implies P(\psi_0 = |0\rangle) = 0.$$

Equivalent Circuits



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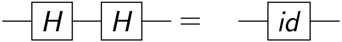

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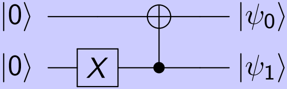

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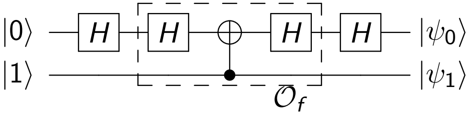

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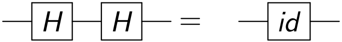

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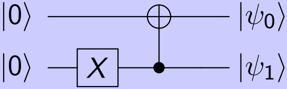

- **In summary:**

We only need to measure $|\psi_1\rangle$. If $P(\psi_0 = |0\rangle) = 0$ f is balanced, and if $P(\psi_0 = |0\rangle) = 1$, f is a constant.

Quantum Search

Question

Given 4 cards, there is only one Q in them. Shuffle the cards and line them on the table. How fast can you find the Q?

- Classically, the expectation is $(1+2+3+4)\cdot\frac{1}{4}=2.5$ times.

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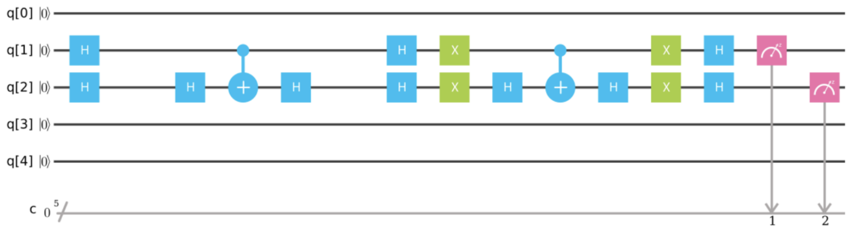
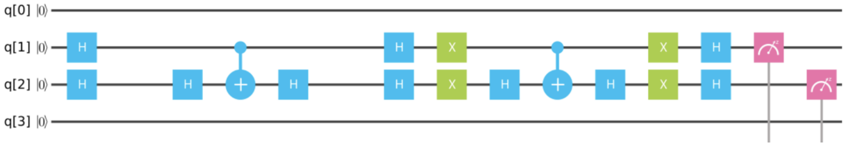
The quantum circuit:

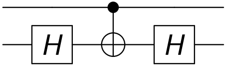


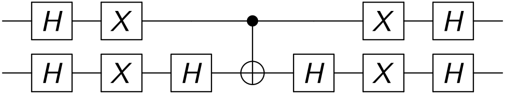
Grover's Algorithm

Quantum search algorithm consists of three parts.



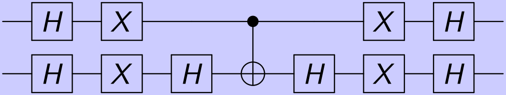
- **Superposition** 

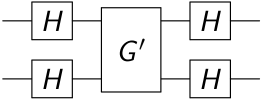
- **Oracle reflection** 

$: \ |x\rangle \xrightarrow{\mathcal{O}_f} (-1)^{f(x)}|x\rangle$
- **Grover reflection** 

Grover Reflection (Grover Diffusion Operator)

It requires **linear algebra** to understand Grover reflection.



- $-\boxed{H}-$ Since $H^\dagger = H$,  means we take $|\psi\rangle = |+,+\rangle$ as the new basis and carry out $-\boxed{G'}-$.

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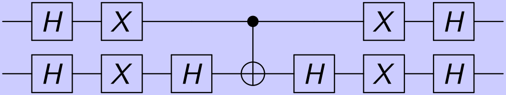
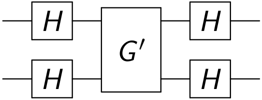


- $-\boxed{H}-$ Since $H^\dagger = H$, 

means we take $|\psi\rangle = |+,+\rangle$ as the new basis and carry out $-\boxed{G'}-$.
- $-\boxed{G'}-$ What G' does is $|00\rangle \to |00\rangle$ and $|x\rangle \to -|x\rangle$ for $x \neq 00$, which is equivalent to $|00\rangle \to -|00\rangle$ and $|x\rangle \to |x\rangle$ for $x \neq 00$, e.g.

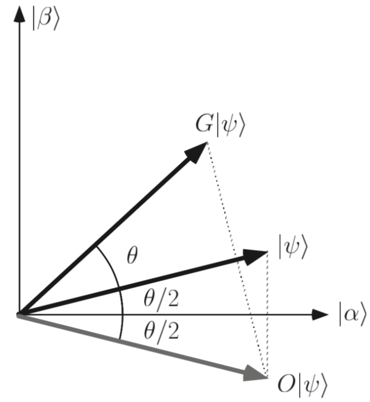
$$|0\rangle \xrightarrow{X} |1\rangle \xrightarrow{X} |0\rangle$$

$$|0\rangle \xrightarrow{X} |1\rangle \xrightarrow{H} |-\rangle \xrightarrow{\oplus} -|-\rangle \xrightarrow{H} -|1\rangle \xrightarrow{X} -|0\rangle.$$

Geometry

Let $|\alpha\rangle = \sum_{x \text{ is not a sol}} |x\rangle$, $|\beta\rangle = \sum_{x \text{ is a sol}} |x\rangle$, and $|\psi\rangle = \frac{1}{\sqrt{4}}|\alpha\rangle + \frac{1}{\sqrt{4}}|\beta\rangle = |+,+\rangle$. Then,

- **Oracle reflection** reflects with respect to $|\alpha\rangle$.
- **Grover reflection** reflects with respect to $|\psi\rangle$.



Common Q & A II And Research Topics

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  Unfortunately, **no**.
- Is there a auto machine which transfers a regular algorithm into its quantum version?
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- Is it possible to combine neuron network, MCTS, and quantum computing?
  Quantized MCTS algorithm, i.e. D-Wave + AlphaGo

Reference

Cloud Quantum Computers

- IBM Q.
  Developed by IBM, quantum circuit
- D-Wave Leap
  Developed by D-Wave, use Phython

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Textbook

- Nielsen, Michael A., Chuang, Isaac L., *Quantum computation and quantum information, Cambridge University Press, Cambridge*
  Detailed introduction
- *Gershenfeld, Neil, The physics of information technology, Cambridge University Press, Cambridge*
  More theoretical discussion