Sheath equations

1 Equations describing plasma-wall interaction

Boltzmann-Poisson system The `sheath` executable solves the following Boltzmann-Poisson system

$$\partial_t f_a + v\partial_x f_a + \frac{q_a E}{m_a}\partial_v f_a = \mathcal{C}(f_a) + \mathcal{S}(f_a), \tag{1}$$

$$-\partial_x^2 \phi = \rho/\varepsilon_0 \quad \text{with} \quad \rho = \sum_{\text{species}} q_a n_a. \tag{2}$$

Where we use the x variable to specify the position. The v variable expresses the velocity of the considered particle. $f_a(v, x, t)$ is the distribution function of species a, which expresses the density of particles at time t and at point (x, v) of phase space. The subscript a denotes the species the quantity refers to:—e for electrons, i for ions. The mass of any particle of species a is written m_a, and its charge is q_a. ε_0 is the dielectric permittivity of vacuum. The electric field is written E, and we have $E = -\partial_x \phi$. We also introduced the local density n_a of species a, which is a function of time and space. It is defined as the integral over the velocity space $n_a = \int dv\, f_a$. The $\mathcal{C}(f_a)$ term accounts for collisions, and the $\mathcal{S}(f_a)$ operator represents sources and sinks. Depending on the simulation parameters, these operators can be a combination of the terms described in the following. For more detail about each operator, see the corresponding documentation.

The Boltzmann-Poisson system is normalized using a reference density n_0 and temperature T_0. Time is normalized to the inverse of the electron plasma frequency $\omega_{\text{pe}_0} = \sqrt{n_0 e^2/m_\text{e}\varepsilon_0}$. The space variable x is normalized to the length scale relevant for plasma-wall interaction studies, i.e. the Debye length $\lambda_{\text{D}_0} = \sqrt{\varepsilon_0 T_0/n_0 e^2}$. The electrostatic potential is normalized to T_0/e. The phase space velocity variable v is normalized to the thermal velocity of each species $v_{T_0 a}$ as $v_a = v/v_{T_0 a}$, with the thermal velocity of species a written as $v_{T_0 a} = \sqrt{T_0/m_a}$. Particles distribution functions are normalized to the reference particle density in phase space $n_0/v_{T_0 a}$. We use the notation $A_a = m_\text{e}/m_a$ to express the mass ratio between electrons and species a (in particular $A_\text{e} = 1$). The normalized charge of species a is written as $Z_a = q_a/e$. The normalized Boltzmann-Poisson system then reads as follows

$$\partial_t f_a + \sqrt{A_a}\,(v_a \partial_x f_a - Z_a \partial_x \phi\, \partial_{v_a} f_a) = \mathcal{C}(f_a) + \mathcal{S}(f_a), \tag{3}$$

$$-\partial_x^2 \phi = \rho \quad \text{with} \quad \rho = \sum_{\text{species}} Z_a n_a. \tag{4}$$

In all the following each quantity of interest is normalized. If a reference to some dimensional quantities is needed, it will be explicitly pointed out.

Sources and sinks The source term $\mathcal{S}(f_a)$ can be composed of the following operators.

1. A Bhatnagar-Gross-Krook operator [1] of the form

$$\mathcal{S}(f_a) = -\nu_a \mathcal{M}(x)\,(f_a - g)\,, \tag{5}$$

that relaxes the distribution function f_a towards a target function g. Depending on the characteristics of the target function this operator can inject or absorb particles. The $\mathcal{M}(x)$ term is a function of space that defines the simulation region where the operator becomes active. Typically this function has a hyperbolic tangent shape and transitions smoothly between a region where it is equal to zero—therefore where the BGK operator is not active—and another region where it is equal to one. ν_a is a coefficient that sets the operator magnitude. This coefficient can be either a constant, or can be adapted at each spatial position and each timestep to ensure that this BGK term conserves the charge locally.

2. A source term [3, Appendix A] defined by

$$S(f_a) = s_{\mathrm{k}} \frac{\mathcal{M}(x)}{\int_0^{L_x} \mathcal{M}(x)\,dx} S_{\mathrm{v}}(v), \tag{6}$$

where the mask function $\mathcal{M}$ defines the spatial extent of the source, similarly to its counterpart of Eq. 5. L_x stands for the simulation box length. The S_{v} term is written as

$$S_{\mathrm{v}}(v) = \left\{ s_0 \left(\frac{3}{2} - \frac{v^2}{2T_{\mathrm{s}}} \right) + s_2 \left(-\frac{1}{2} + \frac{v^2}{2T_{\mathrm{s}}} \right) \right\} \frac{1}{\sqrt{2\pi T_{\mathrm{s}}}}\, e^{-\dfrac{v^2}{2T_{\mathrm{s}}}} \tag{7}$$

The T_{s} is a constant parameter that defines the source temperature. The s_0, s_2 and s_{k} parameters are numerical inputs of the code that define the properties of the source. In particular when using $s_0 = s_2 = 1$, the source adopts a Maxwellian shape whose magnitude is defined by the s_{k} parameter:

$$S(f_a) = s_{\mathrm{k}} \frac{\mathcal{M}(x)}{\int_0^{L_x} \mathcal{M}(x)\,dx} \frac{1}{\sqrt{2\pi T_{\mathrm{s}}}}\, e^{-\dfrac{v^2}{2T_{\mathrm{s}}}}. \tag{8}$$

Collisions The collisions term $\mathcal{C}(f_a)$ can be composed of the following operators.

1. An intra species collision operator [2] that accounts for ion-ion and electron-electron collisions which is written as

$$\mathcal{C}_{aa}(f_a) = \partial_v \left[D_v f_a^M\, \partial_v \left(\frac{f_a}{f_a^M} \right) \right]. \tag{9}$$

In this expression D_v is a function of space and velocity, and depends on the collisionality $\nu_0^\star$. This latter parameter is an input of the code that sets the magnitude of the collision operator. The f_a^M function is a Maxwellian that depends on space x, velocity v and time. This Maxwellian is constructed such that the collision operator conserves locally the density, momentum and energy of the distribution function f_a.

References

[1] P L Bhatnagar, E P Gross, and M Krook. "A Model for Collision Processes in Gases. I. Small Amplitude Processes in Charged and Neutral One-Component Systems". In: *Physical Review* 94.3 (May 1954). Publisher: American Physical Society, pp. 511–525. DOI: 10.1103/PhysRev.94.511. URL: https://link.aps.org/doi/10.1103/PhysRev.94.511.

[2] G Dif-Pradalier et al. "Neoclassical physics in full distribution function gyrokinetics". In: *Physics of Plasmas* 18.6 (June 2011). Publisher: American Institute of Physics, p. 62309. ISSN: 1070-664X. DOI: 10.1063/1.3592652. URL: https://doi.org/10.1063/1.3592652.

[3] Y Sarazin et al. "Predictions on heat transport and plasma rotation from global gyrokinetic simulations". In: *Nuclear Fusion* 51.10 (2011). Publisher: IOP Publishing, p. 103023. ISSN: 0029-5515. DOI: 10.1088/0029-5515/51/10/103023. URL: http://dx.doi.org/10.1088/0029-5515/51/10/103023.