Calculus 3, Fall 2024
Lia Vas

Matlab Project 2

Due on Monday Nov 25, 2024

Instructions. Use Matlab to solve the problems below. In the problems below, a is the last nonzero digit, b is the second to last nonzero digit, and c is second to second to last nonzero digit of the phone number you listed on your student record. For example, if your phone number is 256-0307 then $a = 7$, $b = 3$, and $c = 6$.

You should copy all the Matlab commands as well as the answers and figures which you obtained when executing the commands, into a text file (you can use Microsoft Word or any other text editor). You can add your comments, answers and any sketches you may want to enclose (by hand or in an electronic format). In particular, for problems 1 and 3 and, possibly, parts of 4 **you should turn in all your (non-Matlab) work** which illustrates how you obtained the bounds of integration and the function you integrated. Presenting only the final Matlab command that evaluates certain integral is not full credit even if the command is correct.

When your file contains all the answers and figures, you can print the file and bring the printout and the by-hand work to our class on the due date listed above.

1. Find the surface area of the part of the elliptical paraboloid
$$z = ax^2 + (a+b)y^2$$
which is between the cylinders $x^2 + y^2 = c^2$ and $x^2 + y^2 = (c+2)^2$. Find the bounds and set up the integrals by hand and then evaluate them using Matlab. Show all your work.

2. Evaluate the triple integrals.

   (a) $\displaystyle\int_0^a \int_0^{\sqrt{bp}} \int_0^{cp+q^2} (p^2+q)r \; dp\, dq\, dr$

   (b) $\displaystyle\int_a^{a+1} \int_0^{\sqrt{y}} \int_0^{by+cz} (xz+y^2)\; dx\, dy\, dz$

3. Let R be the solid region between the spheres
$$x^2 + y^2 + z^2 = \tfrac{a^2}{(b+1)^2} \text{ and } x^2 + y^2 + z^2 = a^2$$
above the xy-plane and *to the right of the xz-plane.*

   (a) Find the triple integral of function
   $$f(x, y, z) = (x^2 z)^c$$
   over R.

   (b) Find the average value of $f(x, y, z)$ over R.

4. Recall that the parametrization of the sphere $x^2 + y^2 + z^2 = a^2$ in spherical coordinates is $x = a\cos\theta\sin\phi$, $y = a\sin\theta\sin\phi$, $z = a\cos\phi$.

(a) Graph the sphere $x^2 + y^2 + z^2 = a^2$ using the spherical coordinates.

(b) If the radius of the sphere $r = a$ is changed to

$$r = a\left(1 + \frac{1}{p}\sin(m\theta)\sin(n\phi)\right),$$

the resulting surface looks like a sphere with bumps and it is called the **bumpy sphere** (recall also your third assignment). The constants m and n determine the number and the position of the bumps and p is inversely proportional to the height of the bump. Graph the bumpy sphere with $p = a + 9$, $m = b + 2$ and $n = c + 2$.

(c) Find the volume of the bumpy sphere with $p = a + 9$, $m = b + 2$ and $n = c + 2$.

5. Determine whether the following series are convergent or divergent. For convergent series, find the sum.

$$(a) \quad \sum_{n=0}^{\infty} \frac{(n+1)^{2b}}{a^{n+1}} \qquad\qquad (b) \quad \sum_{n=1}^{\infty} \frac{n^c + a(n+1)}{n^{c+1} + b(n+2)}$$