Propensity Score Diagnostics

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Background

- Commonly used in observational data to deal with confounding bias.

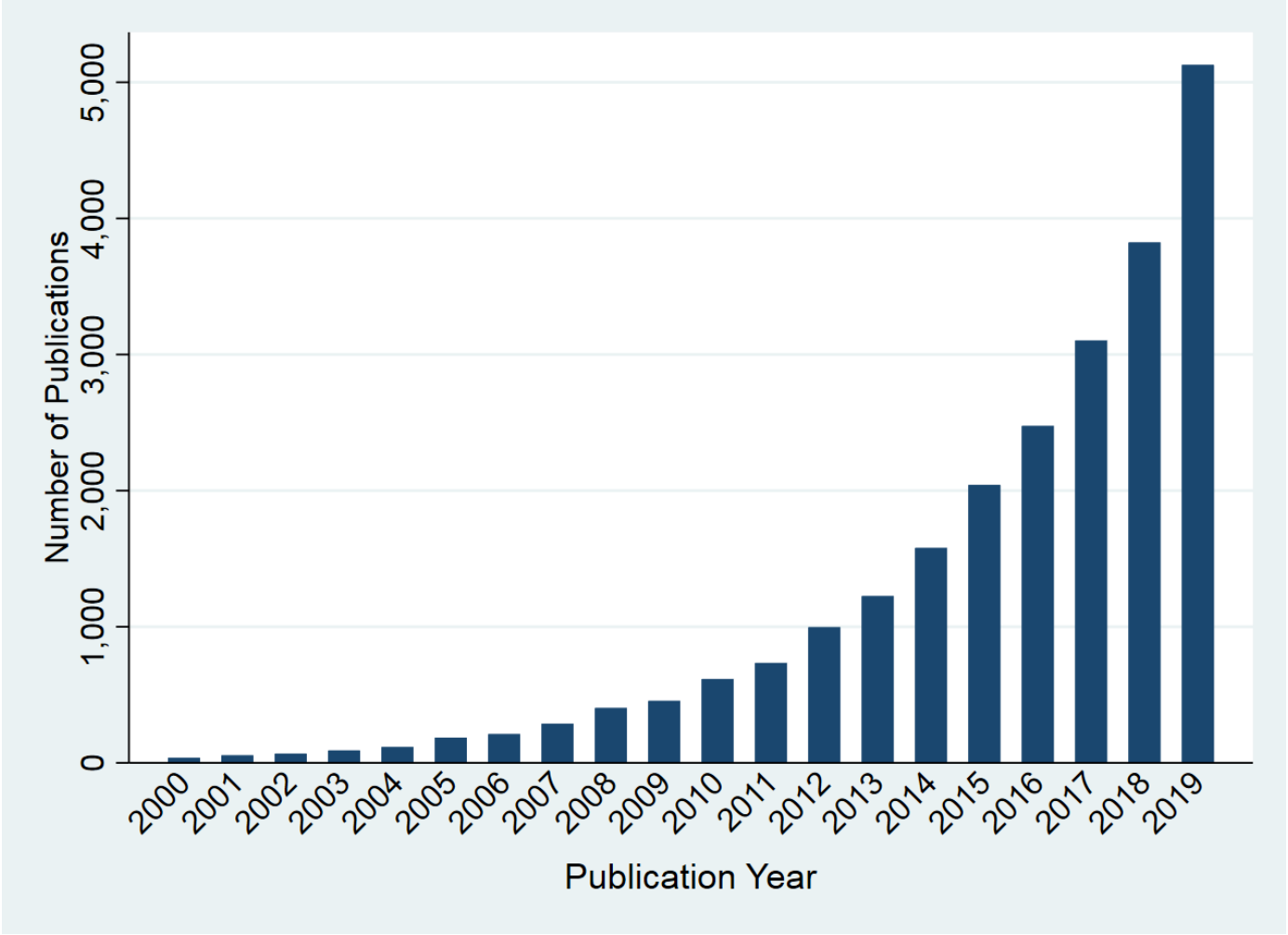


Figure 1: Number of propensity score publications in medical research by year

Background

- Commonly used in observational data to deal with confounding bias.
- Poorly estimated propensity scores may lead to biased estimates.
- Use of diagnostics to assess propensity scores is important.
- Currently unknown how best to assess propensity scores.

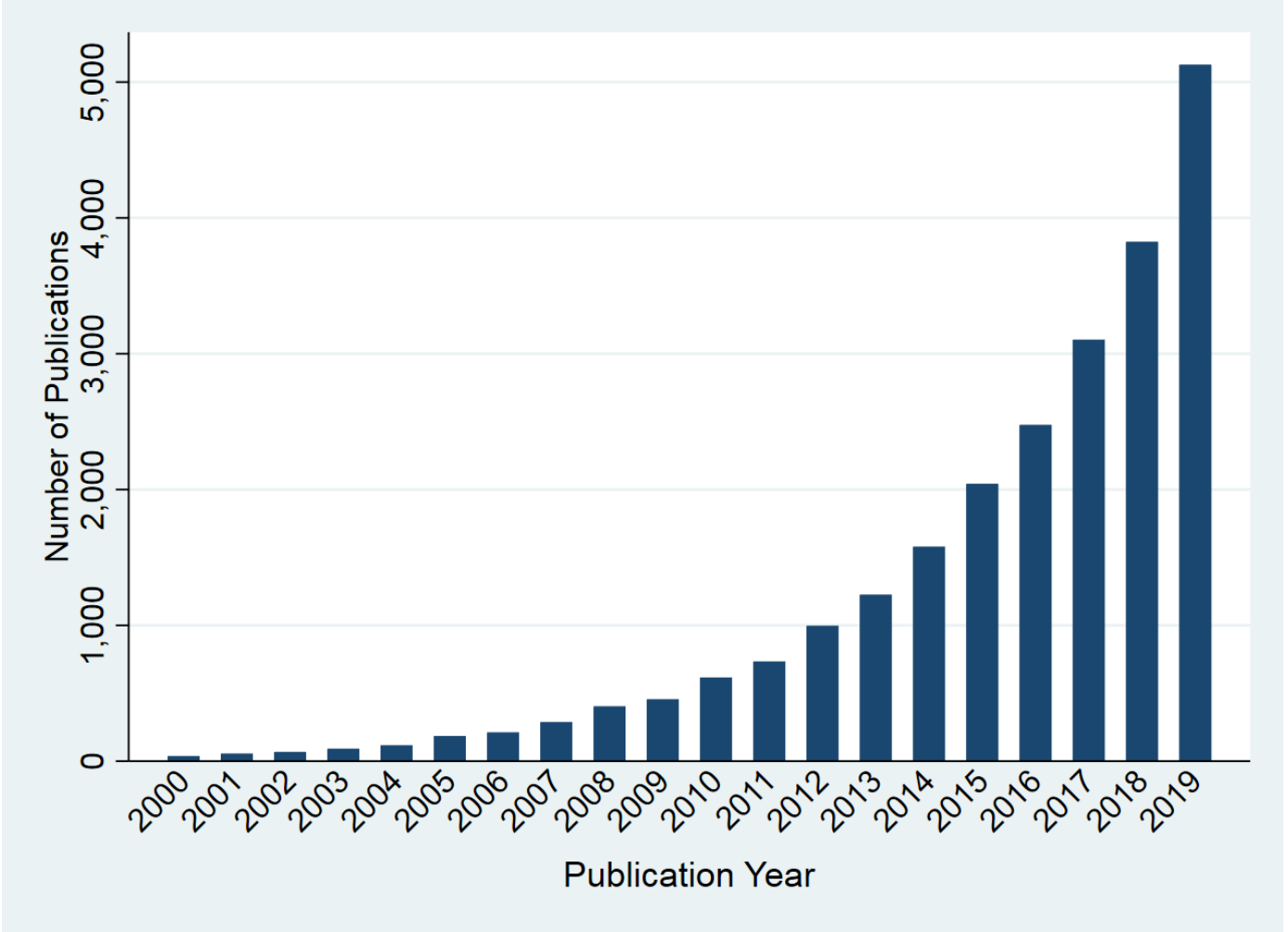


Figure 1: Number of propensity score publications in medical research by year

Aims of research

Aim 1:

Review and compare the existing propensity score diagnostics.

Aim 2:

Develop guidelines for how to build and assess propensity score models.

Aims of research

Individual Diagnostics

Overall Diagnostics

Aim 2:

Develop guidelines for how to build and assess propensity score models.

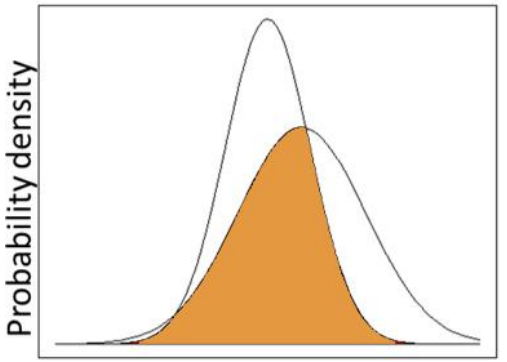
Individual diagnostics

Mean-based

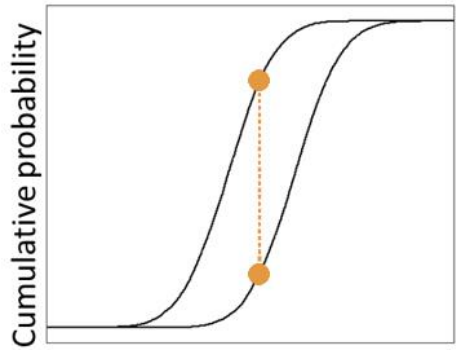
- Standardised difference (SD)
- t-test statistic (t)
- Percent reduction in mean difference (PR)

Distribution-based

- Overlapping coefficient (OVL)



- Kolmogorov-Smirnov Statistic (KS)

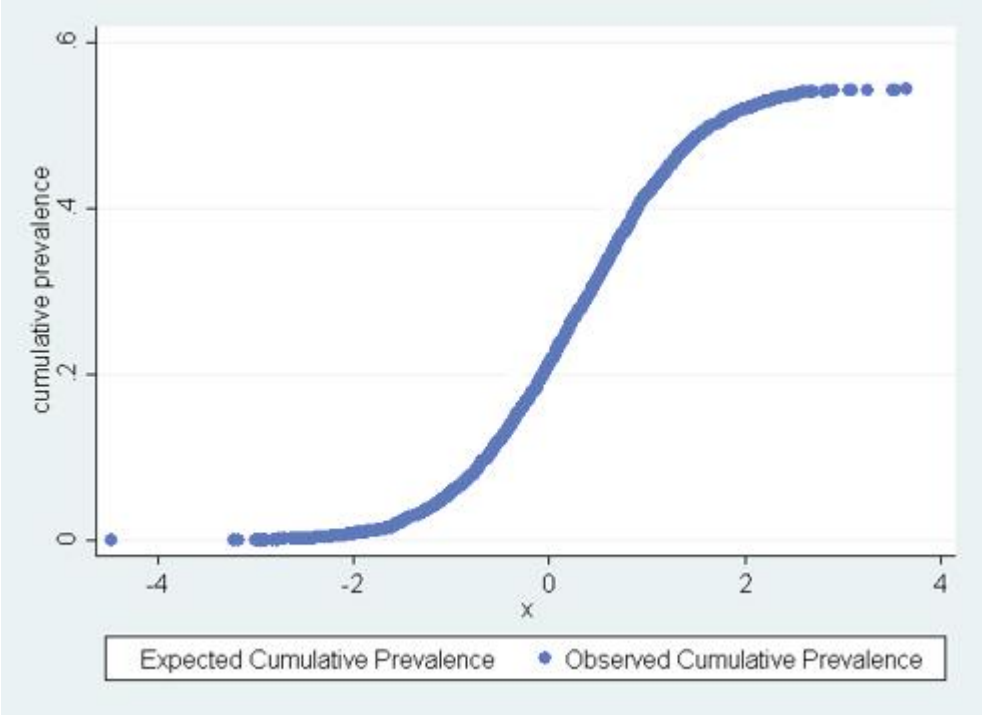


Cumulative prevalence of exposure

Notation: exposure indicator for subject i: E_i, propensity score for subject i: PS_i, sample size: n.

For continuous variable X:

- $OCP_X(X_0) = \frac{1}{n}\sum_{i:X_i \leq X_0} E_i$



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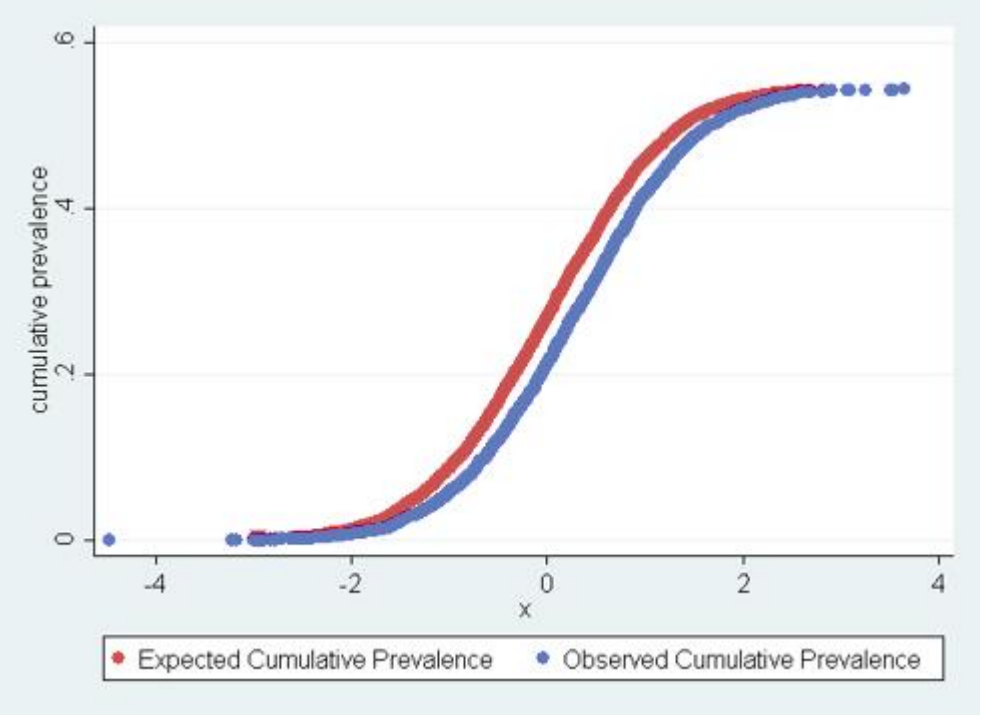

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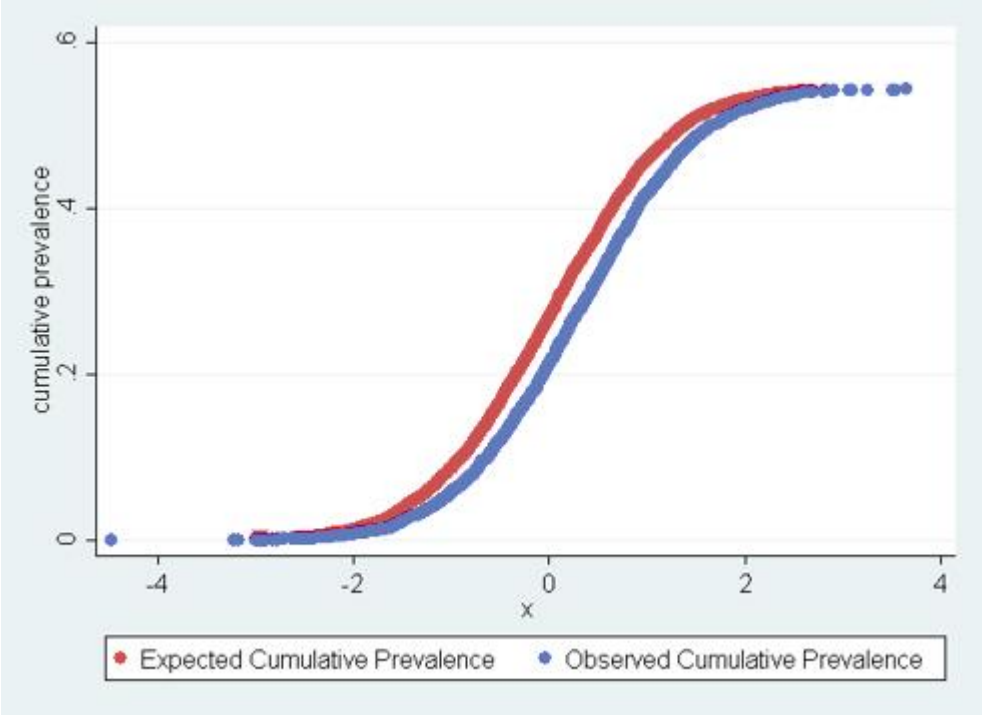
- $D_X = |\, OCP_X - ECP_X \,|$



Simulated data

Propensity score model:

- logit(PS)=$\alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_7 X_7 + \alpha_8 X_8$

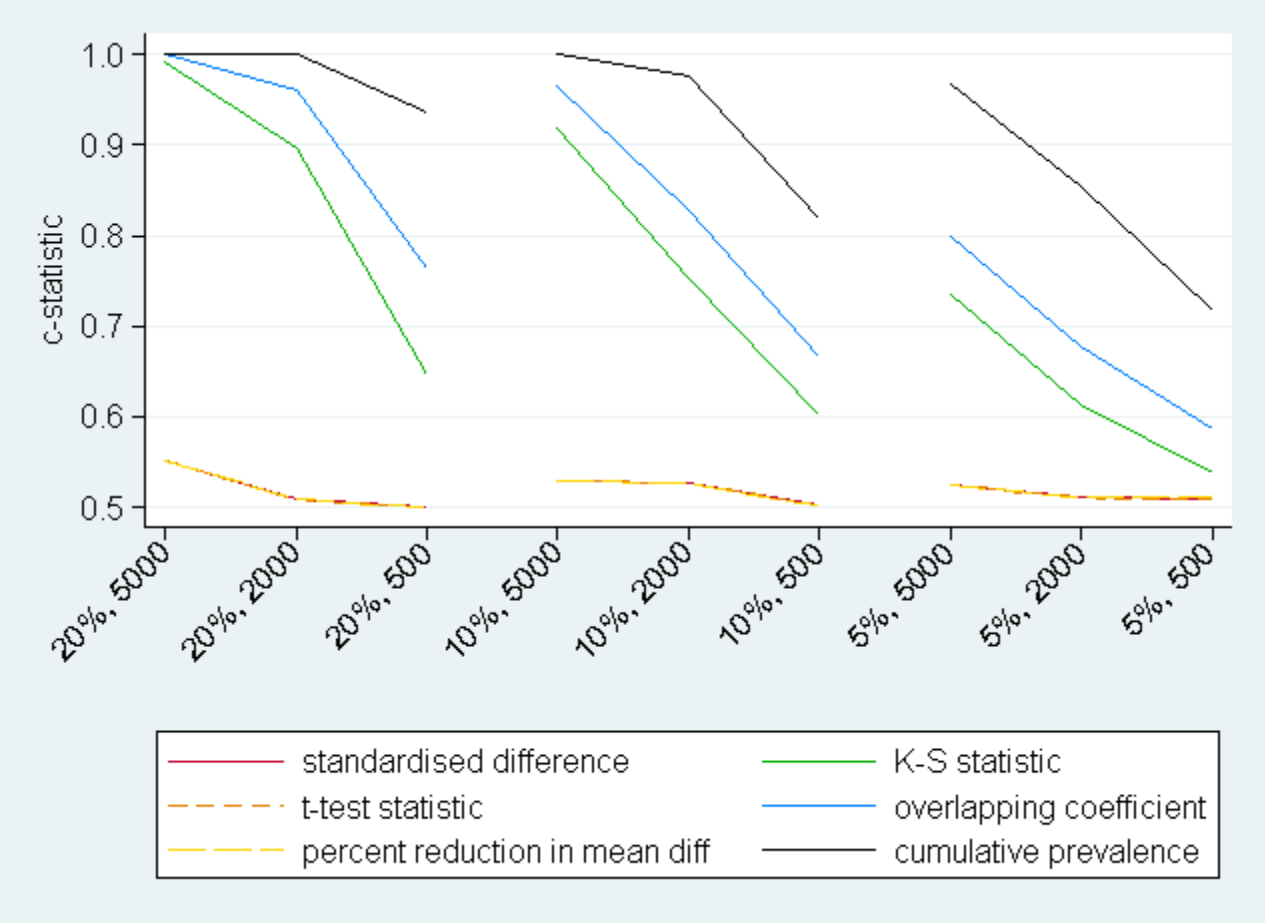
Variation between scenarios:

		Correct PS: / **Incorrect PS:**
S1: $X_8 = 0.4(3.5^{X_1} - 1)$	Nonlinearity added (monotonic)	$X_8 = 0$
S2: $X_8 = X_4 X_5$	Binary-binary interaction	$X_8 = 0$
S3: $X_8 = X_4 X_1$	Binary-continuous interaction	$X_8 = 0$
S4: $X_8 = X_1 X_2$	Continuous-continuous interaction	$X_8 = 0$

Data Analysis:

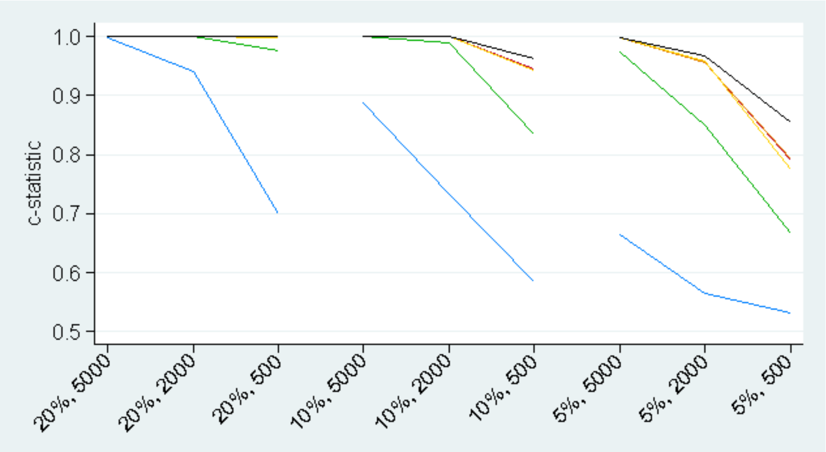
- Logistic Regression: PS indicator ~ PS diagnostic

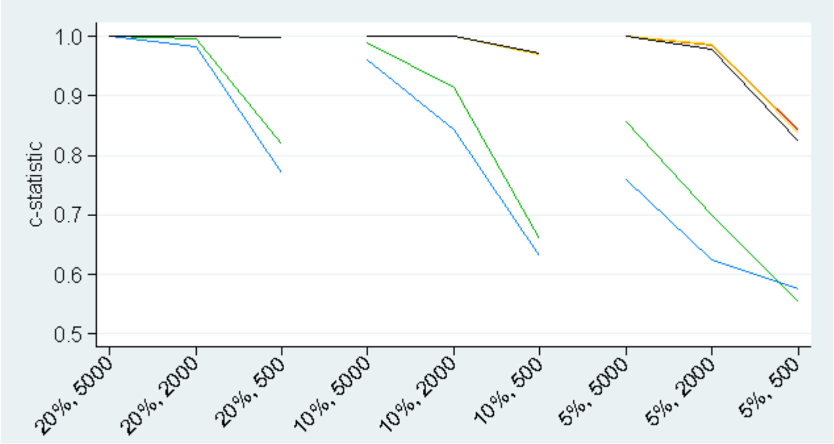
Scenario 1: Misspecification of a non-linear term

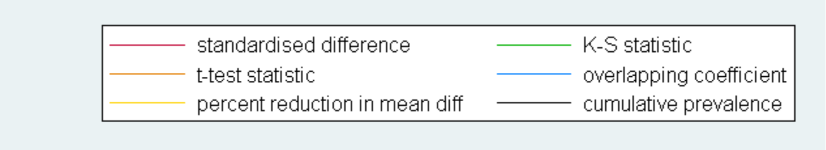


*diagnostics used to assess balance/specification the non-linear covariate X_1; balance assessed in PS-matched samples.

Scenarios 2-4: Omission of an interaction term







Figures:

- Scenario 2 (top left) binary-binary
- Scenario 3 (top right) binary-continuous
- Scenario 4 (bottom left) continuous-continuous

*diagnostics used to assess balance/specification the interaction term;
balance assessed in PS-matched samples.

Overall diagnostics

Disease risk score

- Disease risk scores (DRS) defined as predicted outcome under the control condition
- Standardised mean difference in DRS as a propensity score diagnostic [Stuart et al. 2013]

Weighted average of balance

Let w_j denote the weight for covariate X_j.
Then:

- $w_j = \gamma_j Std.Dev(X_j)$ [Caruana et al. 2015]
  - γ_j is the coefficient for X_j obtained after regressing outcome on X_j.

Balance measured using either:

- Standardised difference (SD)
- Overlapping coefficient (OVL)
- Kolmogorov-Smirnov Statistic (KS)

Simulated data

Propensity score model:

- logit(PS)=$\alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_7 X_7$

Outcome model:

- Y=$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_7 X_7 + \beta_8 X_8$

Variation between scenarios:

S1: $X_8 = 0$	Independent baseline covariates
S2: $X_8 = 0$	Correlated baseline covariates
S3: $X_8 = 0.2(6.0^{X_1} - 1)$	Monotonic non-linearity

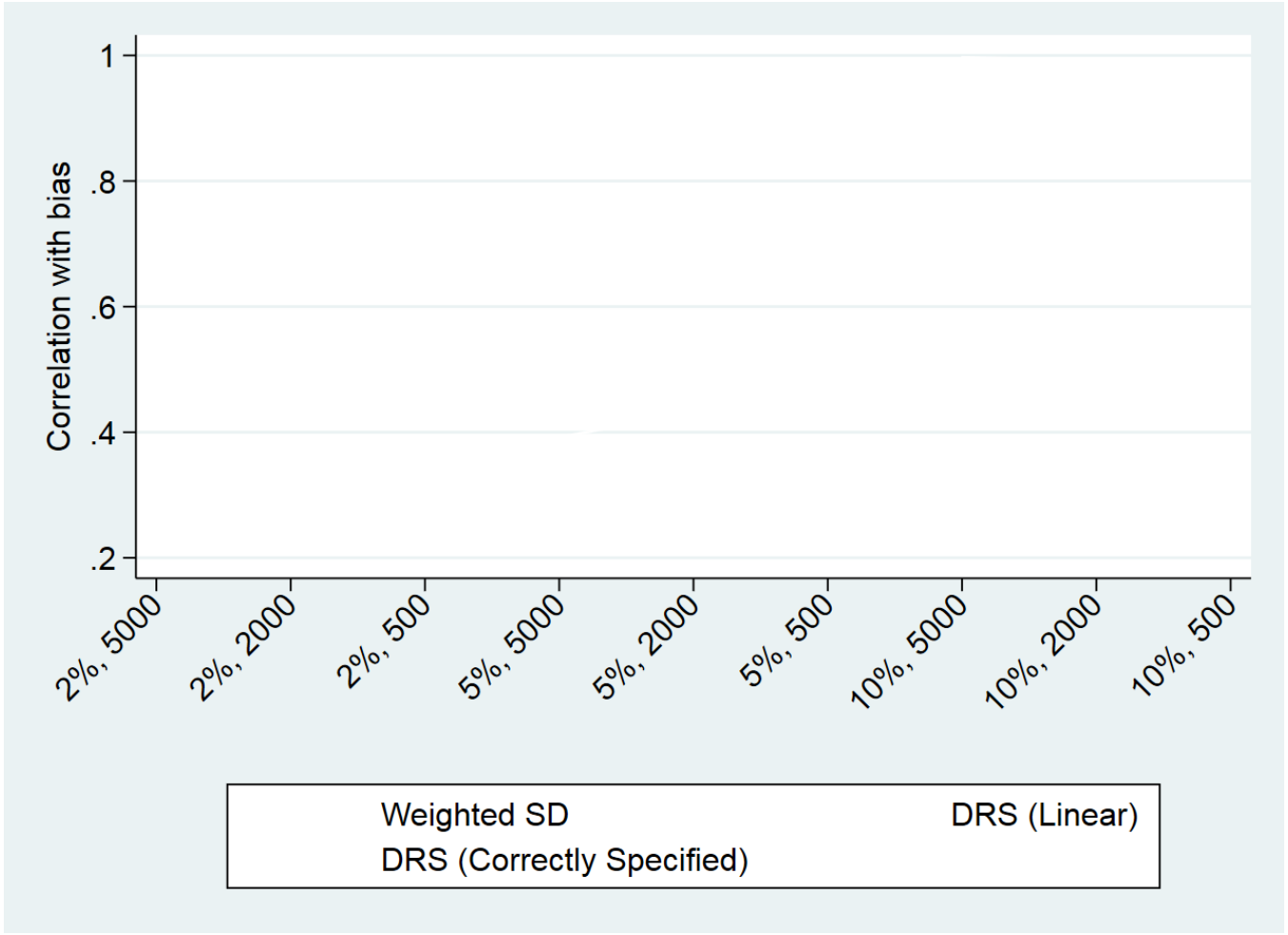
Scenarios 1 and 2: Linear outcomes

Table 1: Spearman rank correlation between overall diagnostics and bias (sample sizes 5000)

Scenario	Weighted SD	Weighted KS	Weighted OVL	SD(DRS)
Scenario 1	0.992	0.137	0.012	1.000
Scenario 2	0.129	0.102	0.031	1.000

*SD: Standardised difference; KS: Kolmogorov-Smirnov statistic; OVL: Overlapping coefficient; DRS: Disease risk score

Scenario 3: Non-linear term in outcome model



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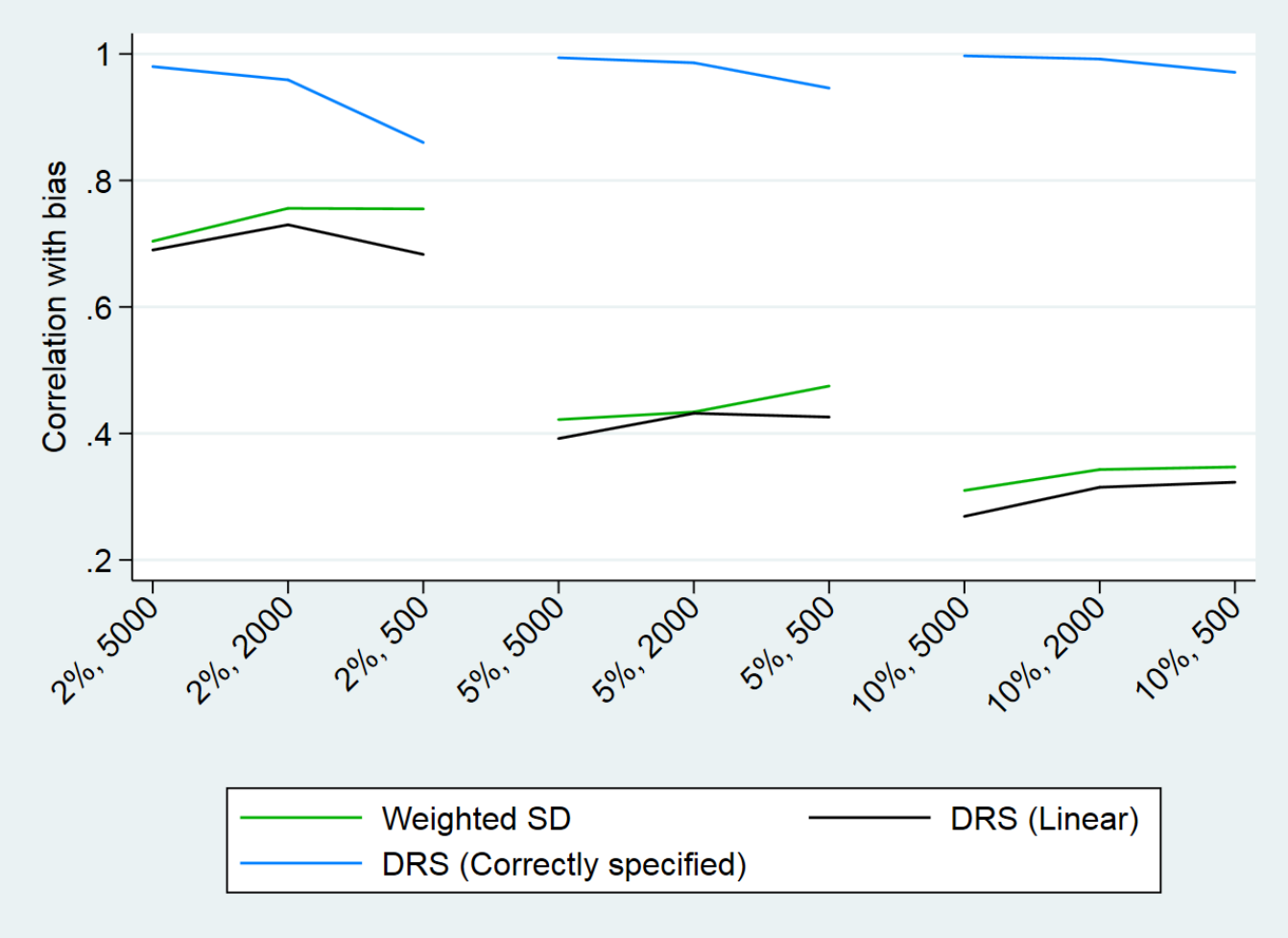


Conclusions

- Cumulative prevalence (CP) diagnostic most useful for identifying all types of propensity score misspecification.
- Standardised mean difference in the disease risk score (DRS) is a promising overall diagnostic.
- Main limitation:
  - DRS not robust to misspecifications in the outcome model
  - Could use CP diagnostics to check specification
  - Future research into different estimation methods for the DRS.

Aims of research

Aim 1:

Review and compare the existing propensity score diagnostics.

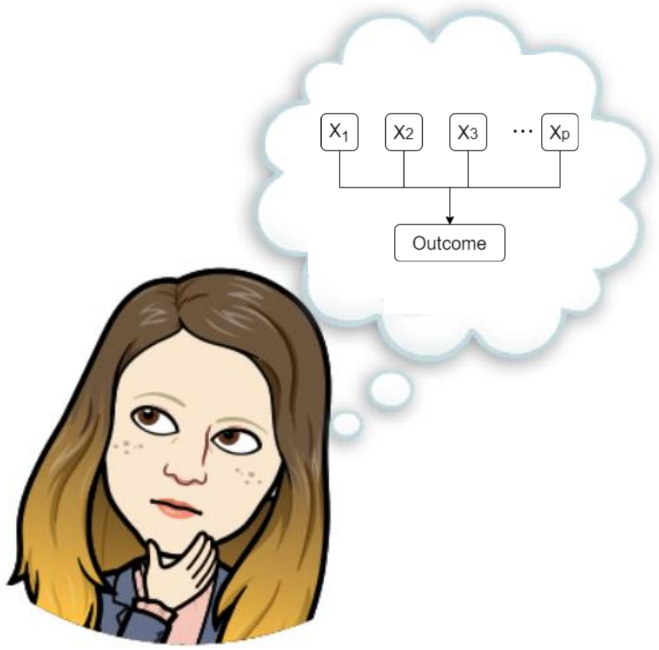
Aim 2:

Develop guidelines for how to build and assess propensity score models.

Proposed guidelines for propensity score assessment

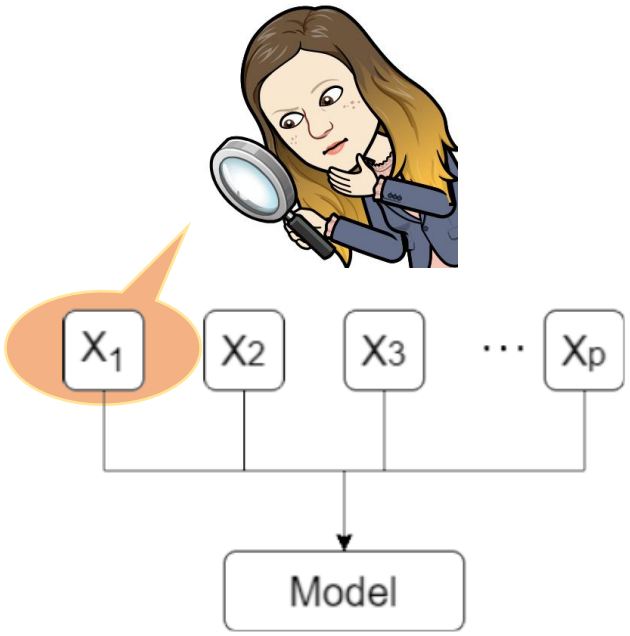
STEP 1:

Choose variables



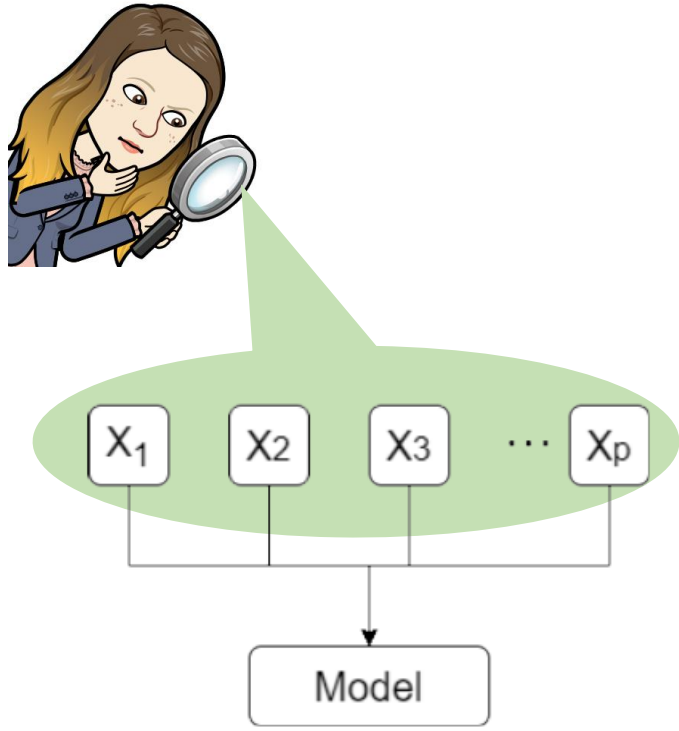
STEP 2:

Check individual covariates using CP diagnostics



STEP 3:

Check overall balance using DRS



References

[1] Stuart, EA et al. Prognostic score-based balance measures for propensity score methods in comparative effectiveness research. *Journal of Clinical Epidemiology.* 2013

[2] Caruana, E et al. A new weighted balance measure helped to select the variables to be included in a propensity score model. *Journal of Clinical Epidemiology.* 2015.

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