

# Productivity Index - Output - By Quantity

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## 1 Math Theory: General Total Factor Productivity (*TFP*) Equation

The general form of the *TFP* can be measured as aggregate output ( $Y$ ) divided by real total inputs ( $X$ ). Rates of TFP growth are constructed using the Törnqvist index approach. The TFP growth over two time periods is defined as:

$$\ln(TFP_t/TFP_{t-1}) = \sum_{i=1}^n \left( \left( \frac{R_{it} + R_{it-1}}{2} \right) * \ln \left( \frac{Y_{it}}{Y_{it-1}} \right) \right) - \sum_{j=1}^m \left( \left( \frac{W_{jt} + W_{jt-1}}{2} \right) * \ln \left( \frac{X_{jt}}{X_{jt-1}} \right) \right)$$

Such that:

- Output =  $\sum_{i=1}^n \left( \left( \frac{R_{it} + R_{it-1}}{2} \right) * \ln \left( \frac{Y_{it}}{Y_{it-1}} \right) \right)$
- Input =  $\sum_{j=1}^m \left( \left( \frac{W_{jt} + W_{jt-1}}{2} \right) * \ln \left( \frac{X_{jt}}{X_{jt-1}} \right) \right)$

where:

- $Y_i$  are individual outputs. This will later be referred to as  $Q_i$  in the following equations.
- $X_j$  are individual inputs
- $R_i$  are output revenue shares
- $W_j$  are input cost shares
- $t$  and  $t - 1$  are time subscripts, where 1 is the minimum year in the dataset
- $i$  is category, e.g., Finfish (=1), Shellfish (=2)
- $s$  is species, e.g., Salmon, Alewife, Surf Clams

## 2 Output Method: From Quantity to Quantity Measures

### 2.0.1 Variable Summary

Variables

- $Q$  are individual quantity outputs in pounds (lbs).
- $V$  are individual value outputs in dollars (\$)
- $R$  are output revenue shares
- $baseyr$  is the year to base all indices from

Indices

- $t$  and  $t - 1$  are time subscripts, where 1 is the minimum year in the dataset
- $i$  is category, e.g., Finfish (=1), Shellfish (=2)
- $s$  is species, e.g., Salmon, Alewife, Surf Clams

### 2.0.2 Data requirements

We need time series data for the value of all species ( $V_t$ ; e.g., Total), value of all species in a category ( $i$ ) ( $V_{i=1}$ ; e.g., Finfish), value of each species in a category ( $i$ ) ( $V_{i=1,s=n}$ ; e.g., Salmon and Summer Flounder), quantity of all species in a category ( $i$ ) (in lbs,  $Q_{i=1}$ ; e.g., Finfish and others), and the quantity of each species in a category ( $i$ ) ( $Q_{i=1,s=n}$ ; e.g., Salmon and Flounder):

#### 2.0.2.1 Edit Data

Here we summate the category and total  $V$  because there may be instances where these values may not be the sum of their parts (though they are here). The calculation Price Index aims to deal with this potential issue.

	Q1_1Salmon	Q1_2Cod	Q2_1Shrimp	Q2_2Clam	Q1_3Flounder	Q1_4SeaBass	QE0_0Total	QE1_0Finfish
2007	NA	2000	100	150	NA	1000	3250	
2008	NA	1900	120	160	NA	1200	3380	
2009	NA	2000	110	140	NA	900	3150	
2010	20	2500	90	NA	NA	NA	2610	
2011	10	2400	80	NA	NA	NA	2490	
2012	12	2300	100	NA	NA	NA	2412	
2013	11	2000	100	140	NA	1000	3251	
2014	11	2300	110	110	NA	900	3431	
2015	10	2400	90	130	NA	1000	3630	
2016	15	2200	100	160	NA	1100	3575	

#### 2.0.2.2 The naming conventions of the column names.

For example, in “V1\_0Finfish”:

- “V”... refers to the variable represented in the column (here  $V$  = “Value”)
- ...“1”... refers to the category index (here, = Finfish)
- ...“\_”... is simply a separator in the title
- Since this is the total, ...“0”.. refers to the index of the species, which is not relevant since this is the

sum of the category, hence = 0

- ... “Finfish” is purely descriptive (here the name of the category), so you can follow along with what is happening!

Similarly for “Q2\_2Clam”:

- “Q”... refers to the variable represented in the column (here Q = “Quantity”)
- ... “2”... refers to the category index (here, = Shellfish)
- ... “\_”... is simply a separator in the title
- ... “2”.. refers to the index of the species, such that this organism happens to be the second species of this category.
- ... “Clams” is purely descriptive (here the name of the species), so you can follow along with what is happening!

### 2.0.3 Lets get started

In most of the following examples, we will just focus on the finfish ( $i=1$ ) side of the equation. Here *baseyr* is set to 2010 and the *pctmiss* (The percent of data in a column that we will allow to be missing for analysis; more on that later) is set to 0.5%.

### 2.0.4 Remove any V and Q data where V column has less data than the specified *pctmiss*

*No warning.*

	Q1_1Salmon	Q1_2Cod	Q2_1Shrimp	Q2_2Clam	REMOVED_Q1_3Flounder	Q1_4SeaBass
2007	NA	2000	100	150	NA	1000
2008	NA	1900	120	160	NA	1200
2009	NA	2000	110	140	NA	900
2010	20	2500	90	NA	NA	NA
2011	10	2400	80	NA	NA	NA
2012	12	2300	100	NA	NA	NA
2013	11	2000	100	140	NA	1000
2014	11	2300	110	110	NA	900
2015	10	2400	90	130	NA	1000
2016	15	2200	100	160	NA	1100

### 2.0.5 Caluclate Category Sums of V and Q

Because we removed some columns for not meeting a perecent missing threshold of 0.5% and those columns will not be used at all in any part of the further analysis, we need to re-calculate the totals of *V* and *Q* for the catagories and the fishery as a whole.

	QE1_0Finfish	VE1_0Finfish	Q1_0Finfish	V1_0Finfish
2007	3000	3800	3000	2800
2008	3100	4020	3100	2820
2009	2900	3910	2900	3010
2010	2520	3190	2520	3190
2011	2410	3280	2410	3280
2012	2312	3150	2312	3150

	QE1_0Finfish	VE1_0Finfish	Q1_0Finfish	V1_0Finfish
2013	3011	4080	3011	3080
2014	3211	4270	3211	3370
2015	3410	4700	3410	3700
2016	3315	4480	3315	3380

There may be instances where there are no or too few  $Q$  data for that species in a year or ever. The next goal will be to calculate the quantity change, so we need to have a value in there that won't show change. If we left a 0 in the spot, then the price change from 0 to the next year would be huge and misrepresented on the index. To avoid this, we have to deal with four scenarios:

**2.0.5.1 1. If there are instances for a species where there are too few pairs of  $Q$  are completely missing from the timeseries or where a percent of  $Q$  is missing from the timeseries, we will remove the offending price columns entirely, so they don't influence the downstream price change or price index calculations.**

Let's say here that if 50% of the data is missing in a given  $Q_{t,i,s}$ , don't use that to calculate that species  $Q_{t,i}$

	Q1_1Salmon	Q1_2Cod	Q1_4SeaBass
2007	NA	2000	1000
2008	NA	1900	1200
2009	NA	2000	900
2010	20	2500	NA
2011	10	2400	NA
2012	12	2300	NA
2013	11	2000	1000
2014	11	2300	900
2015	10	2400	1000
2016	15	2200	1100

*No warning.*

**2.0.5.2 2. If the first value of  $Q_{t,i,s}$  is 0/NA in a timeseries, we (impute) let the next available non-zero/non-NA value of  $Q$  in the timeseries inform the past.**

$$where \begin{cases} if : Q_{t,i=1} = 0, then : Q_{t,i=1} = Q_{t,i=1+1...} \\ if : Q_{t,i \neq 1} = 0, then : Q_{t,i} = Q_{t-1,i} \end{cases}$$

	Q1_1Salmon	Q1_2Cod	Q1_4SeaBass
2007	20	2000	1000
2008	NA	1900	1200
2009	NA	2000	900
2010	20	2500	NA
2011	10	2400	NA
2012	12	2300	NA
2013	11	2000	1000
2014	11	2300	900
2015	10	2400	1000

	Q1_1Salmon	Q1_2Cod	Q1_4SeaBass
2016	15	2200	1100

**2.0.5.3 3.** If there is a value in the middle of  $P_{t,i,s}$ 's timeseries that is 0/NA, we (impute) let the most recent past available non-zero/non-NA of  $P_{t,i,s}$  in the timeseries inform the future.

	Q1_1Salmon	Q1_2Cod	Q1_4SeaBass
2007	20	2000	1000
2008	20	1900	1200
2009	20	2000	900
2010	20	2500	900
2011	10	2400	900
2012	12	2300	900
2013	11	2000	1000
2014	11	2300	900
2015	10	2400	1000
2016	15	2200	1100

## 2.0.6 Impute values of $V_{t,i,s}$ where P was able to be calculated

To ensure that the price index does not rise or fall too quickly with changes (that are really because of NA values) we fill in the missing instances of  $V_{t,i,s}$ .

$$where \begin{cases} if : V_{t,i=1} = 0, then : V_{t,i=1} = V_{t,i=1+1...} \\ if : V_{t,i \neq 1} = 0, then : V_{t,i} = V_{t-1,i} \end{cases}$$

**2.0.6.1 1.** If the first value of  $V_{t,i,s}$  is 0/NA in a timeseries, we let the next available non-zero value of  $V_{t,i,s}$  in the timeseries inform the past.

	V1_1Salmon	V1_2Cod	V1_4SeaBass
2007	100	2800	120
2008	NA	2700	120
2009	NA	2900	110
2010	100	3000	90
2011	100	3100	80
2012	150	2900	100
2013	180	2800	100
2014	170	3200	NA
2015	200	3500	NA
2016	180	3200	NA

**2.0.6.2 2.** If there is a value in the middle of  $V_{t,i,s}$ 's timeseries that is 0/NA, we let the most recent past available non-zero of  $V_{t,i,s}$  in the timeseries inform the future.

	V1_1Salmon	V1_2Cod	V1_4SeaBass
2007	100	2800	120
2008	100	2700	120

	V1_1Salmon	V1_2Cod	V1_4SeaBass
2009	100	2900	110
2010	100	3000	90
2011	100	3100	80
2012	150	2900	100
2013	180	2800	100
2014	170	3200	100
2015	200	3500	100
2016	180	3200	100

### 2.0.7 Value of species $VV_{t,i}$ where Q available

$R_{t,i}$ , as defined and discussed in the subsequent step, will need to sum to 1 across all species in a category. Therefore, you will need to sum a new total of  $V_{t,i}$  available (called  $VV_{t,i}$ ) for the category using only values for species that were used to calculate  $Q_{t,i}$  (called  $V_{t,i,s,available}$ ).

$$VV_{t,i} = \sum_{s=1}^n (V_{t,i,s,available})$$

where:

- $VV_{t,i}$  is the new total of  $V_{t,i}$  (called  $VV_{t,i}$ ) for the category using only values for species that were used to calculate  $Q_{t,i}$
- $V_{t,i,s,available}$  are the  $V_{t,i,s}$  where  $Q_{t,i,s}$  were able to be calculated

V1__	1Salmon	V1__	2Cod	V1__	4SeaBass	VV1	_0Finfish
2007	100		2800		120		3020
2008	100		2700		120		2920
2009	100		2900		110		3110
2010	100		3000		90		3190
2011	100		3100		80		3280
2012	150		2900		100		3150
2013	180		2800		100		3080
2014	170		3200		100		3470
2015	200		3500		100		3800
2016	180		3200		100		3480

### 2.0.8 Revenue Share for each species ( $R_{t,i,s}$ ; e.g., Salmon and Flounder)

$$R_{t,i,s} = V_{t,i,s}/VV_{t,i}$$

where:

- $R_{t,i,s}$  is the revenue share per individual species (s), category (i), for each year (t)
- $V_{t,i,s}$  is the value (\$) per individual species (s), category (i), for each year (t)

Here we divide  $V_{t,i,s}$  by  $VV_{t,i}$  because  $VV_{t,i}$  only includes species used to calculate  $V_{t,i,s}$  as per the above price calculations.

	R1_1Salmon	R1_2Cod	R1_4SeaBass
1	0.0331126	0.9271523	0.0397351
2	0.0342466	0.9246575	0.0410959
3	0.0321543	0.9324759	0.0353698
4	0.0313480	0.9404389	0.0282132
5	0.0304878	0.9451220	0.0243902
6	0.0476190	0.9206349	0.0317460
7	0.0584416	0.9090909	0.0324675
8	0.0489914	0.9221902	0.0288184
9	0.0526316	0.9210526	0.0263158
10	0.0517241	0.9195402	0.0287356

### 2.0.8.1 Analysis Warnings Checks

As an additional check, let's make sure that each row sums to 1.

	x
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

Is there a warning?

*No warning.*

### 2.0.9 Revenue Share-Weighted Qunatity Changes for each species ( $QCW_{t,i,s}$ ; e.g., Salmon and Flounder)

$$QCW_{t,i,s} = \frac{R_{t,i,s} + R_{s,t-1,i}}{2} * \ln\left(\frac{Q_{t,i,s}}{Q_{s,t-1,i}}\right) = \frac{R_{t,i,s} + R_{s,t-1,i}}{2} * [\ln(Q_{t,i,s}) - \ln(Q_{s,t-1,i})]$$

Where:

- $QCW_{t,i,s}$  = Revenue share-weighted quantity change for a species (s)

Such that:

- category's (i) Quantity Change for each species (s) =  $\frac{R_{t,i,s} + R_{s,t-1,i}}{2}$
- category's (i) Revenue Share for each species (s) =  $\ln\left(\frac{Q_{t,i,s}}{Q_{s,t-1,i}}\right) = [\ln(Q_{t,i,s}) - \ln(Q_{s,t-1,i})]$

### 2.0.10 Quantity Changes for the category ( $QC_{t,i}$ ; e.g., Finfish)

$$QC_{t,i} = \ln\left(\frac{Q_{t,i}}{Q_{t-1,i}}\right) = \sum_{s=1}^n (QCW_{t,i,s})$$



Where:

- $QC_{t,i}$  = Quantity change for a category (i)

QCW	1_1Salmon QC	W1_2Cod QCW	1_4SeaBass QC1	_0Finfish
1	0.0000000	0.0000000	0.0000000	0
2	0.0000000	-0.0474927	0.0073686	0
3	0.0000000	0.0476292	-0.0109989	0
4	0.0000000	0.2089644	0.0000000	0
5	-0.0214306	-0.0384862	0.0000000	0
6	0.0071203	-0.0397029	0.0000000	0
7	-0.0046142	-0.1278630	0.0033828	0
8	0.0000000	0.1279717	-0.0032286	0
9	-0.0048429	0.0392239	0.0029045	0
10	0.0211563	-0.0800763	0.0026235	0

### 2.0.11 Quantity Index for the each category ( $QI_{t,i}$ )

We calculate the quantity index first by comparing by multiplying the previous years  $QI_{t-1}$  by that year's quantity change  $QC_t$ , where the  $QI$  of the first year  $QI_{t=firstyear,i} = 1$

$$QI_{t,i} = QI_{t-1,i} * \exp(\ln(\frac{Q_{t,i,s}}{Q_{t-1,i,s}})) = QI_{t-1,i} * \exp(QC_{t,i})$$

Where

$$QI_{i,t=firstyear} = 1$$

Note that the first row of this column is = 1

Then, to change the price index into base year dollars, we use the following equation:

$$QI_t = QI_t / QI_{t=baseyear}$$

	QI1_0Finfish
2007	0.8142640
2008	0.7822391
2009	0.8114241
2010	1.0000000
2011	0.9418429
2012	0.9116497
2013	0.8012406
2014	0.9076914
2015	0.9421740
2016	0.8905983

## 2.1 Redo Analysis for Shellfish

Now lets redo that whole analysis up to this point (via function) for the two species of the shellfish group, as we will need them for the next steps of this analysis.

What does the Shellfish data look like?

	R2_1Shrimp	R2_2Clam	QCW2_1Shrimp	QCW2_2Clam	QC2_0Shellfish	QI2_0Shellfish
2007	0.4444444	0.5555556	0.0000000	0.0000000	0.0000000	1.0918916
2008	0.4545455	0.5454545	0.0819526	0.0355288	0.1174814	1.2280076
2009	0.5000000	0.5000000	-0.0415282	-0.0698005	-0.1113287	1.0986305
2010	0.4375000	0.5625000	-0.0940644	0.0000000	-0.0940644	1.0000000
2011	0.5000000	0.5000000	-0.0552108	0.0000000	-0.0552108	0.9462857
2012	0.5263158	0.4736842	0.1145079	0.0000000	0.1145079	1.0610904
2013	0.5454545	0.4545455	0.0000000	0.0000000	0.0000000	1.0610904
2014	0.5500000	0.4500000	0.0522040	-0.1090710	-0.0568670	1.0024330
2015	0.5000000	0.5000000	-0.1053521	0.0793507	-0.0260014	0.9767043
2016	0.5217391	0.4782609	0.0538255	0.1015627	0.1553882	1.1408993

### 2.1.1 Value for all fisheries for species where Q was able to be calculated

$R_{t,i}$ , defined and discussed in the subsequent step, will need to sum to 1 across all species in a category. Therefore, you will need to sum a new total of  $V_{t,i}$  (called  $VV_t$ ) for the category using only values for species that were used to calculate  $QI_{t,i}$ .

$$VV_t = \sum_{s=1}^n (VV_{t,i})$$

where:

- $VV_t$  is the new total of  $V_{t,i}$  for the entire fishery using only values for species that were used to calculate  $P_{t,i}$

VV1	_0Finfish	VV2	_0Shellfish	VV0	_0Total
2007	3020	1800		4820	
2008	2920	2200		5120	
2009	3110	1800		4910	
2010	3190	1600		4790	
2011	3280	1800		5080	
2012	3150	1900		5050	
2013	3080	2200		5280	
2014	3470	2000		5470	
2015	3800	2000		5800	
2016	3480	2300		5780	

V1__	0Finfish	V2__	0Shellfish
2007	2800	1800	
2008	2820	2200	
2009	3010	1800	
2010	3190	700	
2011	3280	900	
2012	3150	1000	
2013	3080	2200	
2014	3370	2000	
2015	3700	2000	

V1_	0Finfish	V2_	0Shellfish
2016	3380		2300

### 2.1.2 Revenue Share for the each category ( $R_{t,i}$ )

$$R_{t,i} = V_{t,i}/V_t$$

where:

- $R_{t,i}$  is the revenue share per individual species (s), category (i), for each year (t)
- $V_{t,i}$  is the value (\$) per individual species (s), category (i), for each year (t)

Here, we don't use  $VV_t$  because we want to expand the proportion to include all of the species caught, regardless if they were used in the quantity calculations.

	R1_0Finfish	R2_0Shellfish	V1_0Finfish	V2_0Shellfish	V0_0Total
2007	0.6086957	0.3913043	2800	1800	4600
2008	0.5617530	0.4382470	2820	2200	5020
2009	0.6257796	0.3742204	3010	1800	4810
2010	0.8200514	0.1799486	3190	700	3890
2011	0.7846890	0.2153110	3280	900	4180
2012	0.7590361	0.2409639	3150	1000	4150
2013	0.5833333	0.4166667	3080	2200	5280
2014	0.6275605	0.3724395	3370	2000	5370
2015	0.6491228	0.3508772	3700	2000	5700
2016	0.5950704	0.4049296	3380	2300	5680

#### 2.1.2.1 Analysis Warnings Checks

As an additional check, let's make sure that each row sums to 1.

	x
2007	1
2008	1
2009	1
2010	1
2011	1
2012	1
2013	1
2014	1
2015	1
2016	1

Is there a warning?

*No warning.*

### 2.1.3 Revenue Share-Weighted Qunatity Changes for each category ( $QCW_{t,i}$ ; e.g., Finfish and Shellfish)

$$QCW_{t,i} = \frac{R_{t,i,s} + R_{s,t-1,i}}{2} * \ln\left(\frac{QI_{t,i,s}}{QI_{s,t-1,i}}\right) = \frac{R_{t,i,s} + R_{s,t-1,i}}{2} * [\ln(QI_{t,i,s}) - \ln(QI_{s,t-1,i})]$$

Where:

- $QCW_{t,i}$  = Revenue share-weighted quantity change for each category (i)

Such that:

- category's (i) Quantity Change for each category (i) =  $\frac{R_{t,i} + R_{t-1,i}}{2}$
- category's (i) Revenue Share for each category (i) =  $\ln\left(\frac{QI_{t,i}}{QI_{t-1,i}}\right) = [\ln(QI_{t,i}) - \ln(QI_{t-1,i})]$

### 2.1.4 Quantity Changes for the entire fishery ( $QC_t$ )

$$QC_t = \ln\left(\frac{QI_{t,i}}{QI_{t-1,i}}\right) = \sum_{s=1}^n (QCW_{t,i})$$

Where:

- $QC_t$  = Quantity change for the entire fishery

QCW	1_0Finfish	QCW	2_0Shellfish	QC0	_0Total
1	0.0000000		0.0000000		0
2	-0.0234816		0.0487284		0
3	0.0217499		-0.0452255		0
4	0.1510636		-0.0260638		0
5	-0.0480755		-0.0109113		0
6	-0.0251493		0.0261235		0
7	-0.0866462		0.0000000		0
8	0.0755254		-0.0224371		0
9	0.0238009		-0.0094036		0
10	-0.0350218		0.0587217		0

### 2.1.5 Quantity Index for the entier fishery ( $QI_t$ )

We calculate the quantity index first by comparing by multiplying the previous years  $QI_{t-1}$  by that year's quantity change  $QC_t$ , where the  $QI$  of the first year  $QI_{t=firstyear,i} = 1$

$$QI_t = QI_{t-1} * \exp\left(\ln\left(\frac{Q_{t,i}}{Q_{t-1,i}}\right)\right) = QI_{t-1} * \exp(QC_t)$$

Where

$$QI_{t_{firstyear}} = 1$$

Note that the first row of this column is = 1

Then, to change the price index into base year dollars, we use the following equation:

$$QI_t = QI_t / QI_{t=baseyear}$$

	QI0_0Total
2007	0.8809353
2008	0.9034593
2009	0.8824970
2010	1.0000000
2011	0.9427192
2012	0.9436381
2013	0.8653175
2014	0.9124970
2015	0.9257294
2016	0.9479312

### 2.1.6 Sum Total Simple Sum Quantity Output Index

$$QEI_t = QE_t / QE_{t=baseyr}$$

Where:

- $QE_t$  is the sum of Q before these calculations; the simple sum
- $QE_t$  is the index of the sum of Q before these equations

QE0	_0Total	QEI	0_0Total
1	3250		1.2452107
2	3380		1.2950192
3	3150		1.2068966
4	2610		1.0000000
5	2490		0.9540230
6	2412		0.9241379
7	3251		1.2455939
8	3431		1.3145594
9	3630		1.3908046
10	3575		1.3697318

## 2.2 Other Analysis Warnings Checks

To make sure our analyses worked as inteded, let's see if we can back calculate our numbers.

We want the calcuated V to equal this check:

### 2.2.0.1 When back calculated, growth rate?

$$\ln(Q_t/Q_{t-1}) = \sum((\frac{R_{i,t} + R_{i,t-1}}{2}) * \ln(\frac{Q_{t,i}}{Q_{t-1,i}}))$$

	part1	part2
2007	NA	NA

	part1	part2
2008	-0.0252468	0.0252468
2009	0.0234756	-0.0234756
2010	-0.1249998	0.1249998
2011	0.0589868	-0.0589868
2012	-0.0009742	0.0009742
2013	0.0866462	-0.0866462
2014	-0.0530883	0.0530883
2015	-0.0143972	0.0143972
2016	-0.0236999	0.0236999

Is there a warning?

*Warning: When back calculated,  $\ln(Q_t/Q_{t-1}) = \text{did not equal } \text{sum}((R_{i,t} - R_{i,t-1})(2))$*   
 $\ln((Q_{t,i})(Q_{t-1,i}))^*$

### 2.2.1 View Total Outputs

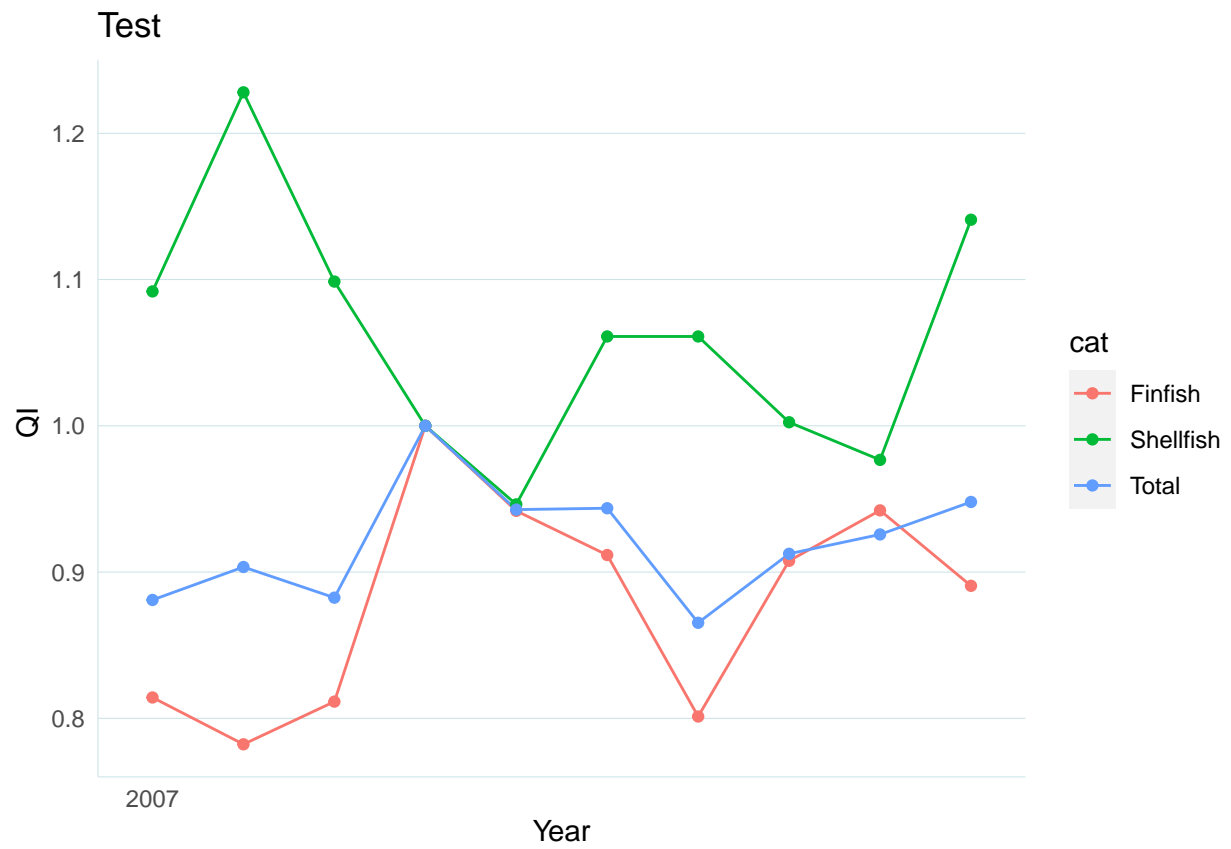
	QE0_0Total	VE0_0Total	VV0_0Total	V0_0Total	QC0_0Total	QI0_0Total	QEI0_0Total
2007	3250	5600	4820	4600	0.0000000	0.8809353	1.2452107
2008	3380	6220	5120	5020	0.0252468	0.9034593	1.2950192
2009	3150	5710	4910	4810	-0.0234756	0.8824970	1.2068966
2010	2610	3890	4790	3890	0.1249998	1.0000000	1.0000000
2011	2490	4180	5080	4180	-0.0589868	0.9427192	0.9540230
2012	2412	4150	5050	4150	0.0009742	0.9436381	0.9241379
2013	3251	6280	5280	5280	-0.0866462	0.8653175	1.2455939
2014	3431	6270	5470	5370	0.0530883	0.9124970	1.3145594
2015	3630	6700	5800	5700	0.0143972	0.9257294	1.3908046
2016	3575	6780	5780	5680	0.0236999	0.9479312	1.3697318

### 2.2.2 Missing Data

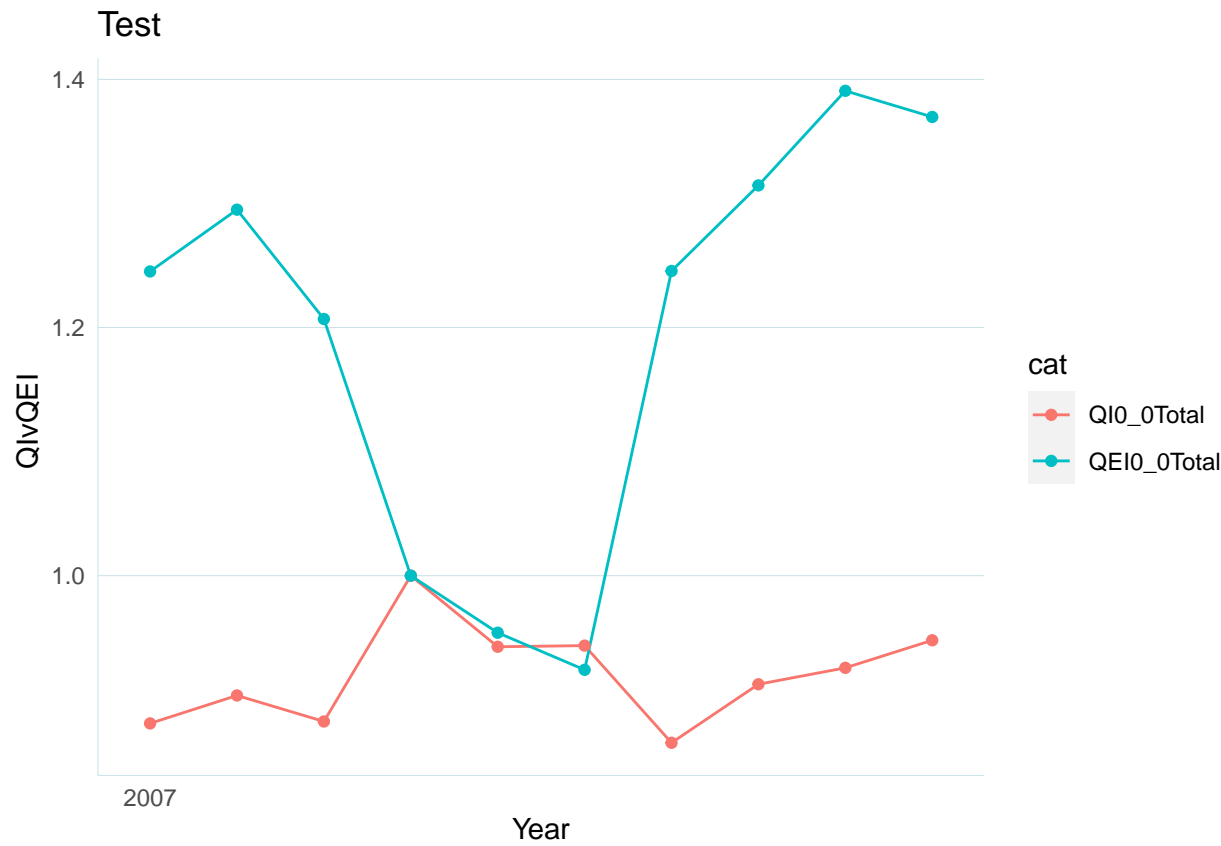
*FYI: 0 of species V columns are completely empty, 1 of species Q columns are completely empty.*

### 2.2.3 Graph 1: Price Index

In theory, *PI* should be negative slope after the baseyear and positive after the base year, but because this data was fabricated without thinking of this, we don't see that here. The index value for the base year is = 1, however.

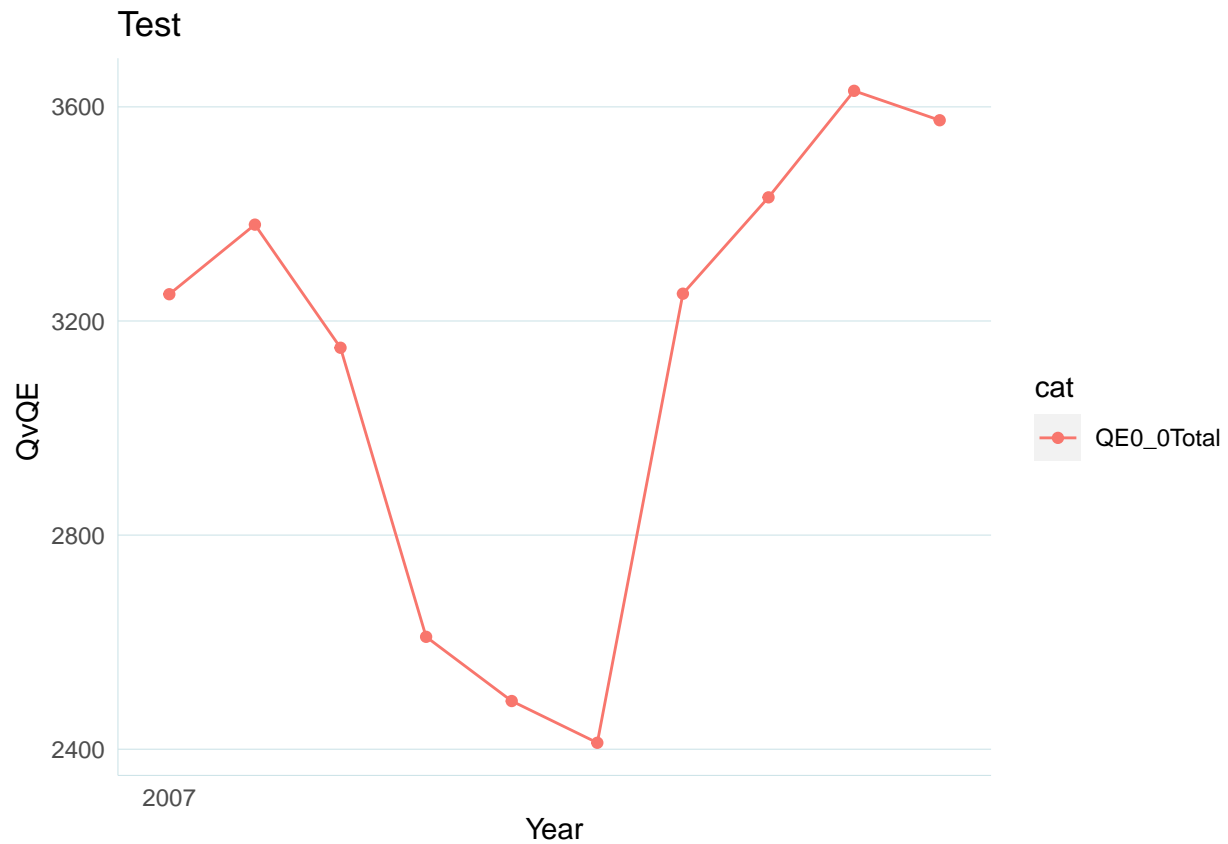


2.2.4 Graph 2: Quantity Index Compare





### 2.2.5 Graph 3: Quantity Compare



## 2.3 Do same analysis via a function!

Now that we know the method, we can simplify most of it into a function and do this whole analysis in 4 easy steps:

- A. Import and Edit data
- B. Enter base year
- C. Run the function
- D. Obtain the implicit quantity estimates

### 2.3.1 Function

#### 2.3.2 A. Import and Edit data

#### 2.3.3 B. Enter base year

#### 2.3.4 C. Run the function

#### 2.3.5 D. Obtain the implicit quantity estimates

	QE0_0Total	VE0_0Total	VV0_0Total	V0_0Total	QC0_0Total	QI0_0Total	QEI0_0Total
2007	3250	5600	4820	4600	0.0000000	0.8809353	1.2452107
2008	3380	6220	5120	5020	0.0252468	0.9034593	1.2950192
2009	3150	5710	4910	4810	-0.0234756	0.8824970	1.2068966
2010	2610	3890	4790	3890	0.1249998	1.0000000	1.0000000
2011	2490	4180	5080	4180	-0.0589868	0.9427192	0.9540230
2012	2412	4150	5050	4150	0.0009742	0.9436381	0.9241379
2013	3251	6280	5280	5280	-0.0866462	0.8653175	1.2455939
2014	3431	6270	5470	5370	0.0530883	0.9124970	1.3145594
2015	3630	6700	5800	5700	0.0143972	0.9257294	1.3908046
2016	3575	6780	5780	5680	0.0236999	0.9479312	1.3697318

Did all of the analyses work as intended?

*Warning: When back calculated,  $\ln(Q_t/Q_{t-1}) = \text{did not equal } \sum( (R_{i,t} - R_{i,t-1}) / 2 ) \times \ln( (Q_{i,t}) / (Q_{i,t-1}) )$ , FYI: 0 of species V columns are completely empty, 2 of species Q columns are completely empty.*

## 2.3.6 E. Graph

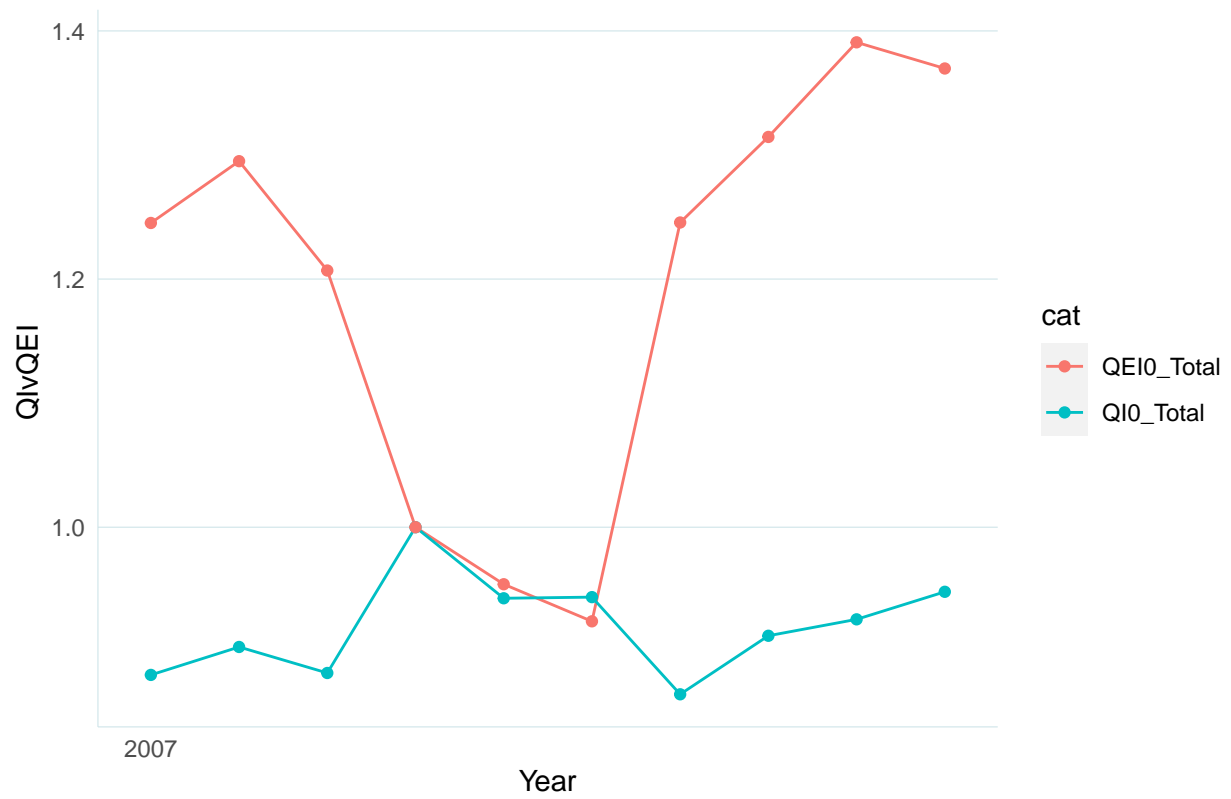
### 2.3.6.1 Graph 1: Price Index

For comparison, let's recreate those graphs to make sure we are getting the same output:

```
## NULL
```

### 2.3.6.2 Graph 2: Quantity Index Compare

For comparison, let's recreate those graphs to make sure we are getting the same output:



### 2.3.6.3 Graph 3: Quantity Compare

## NULL