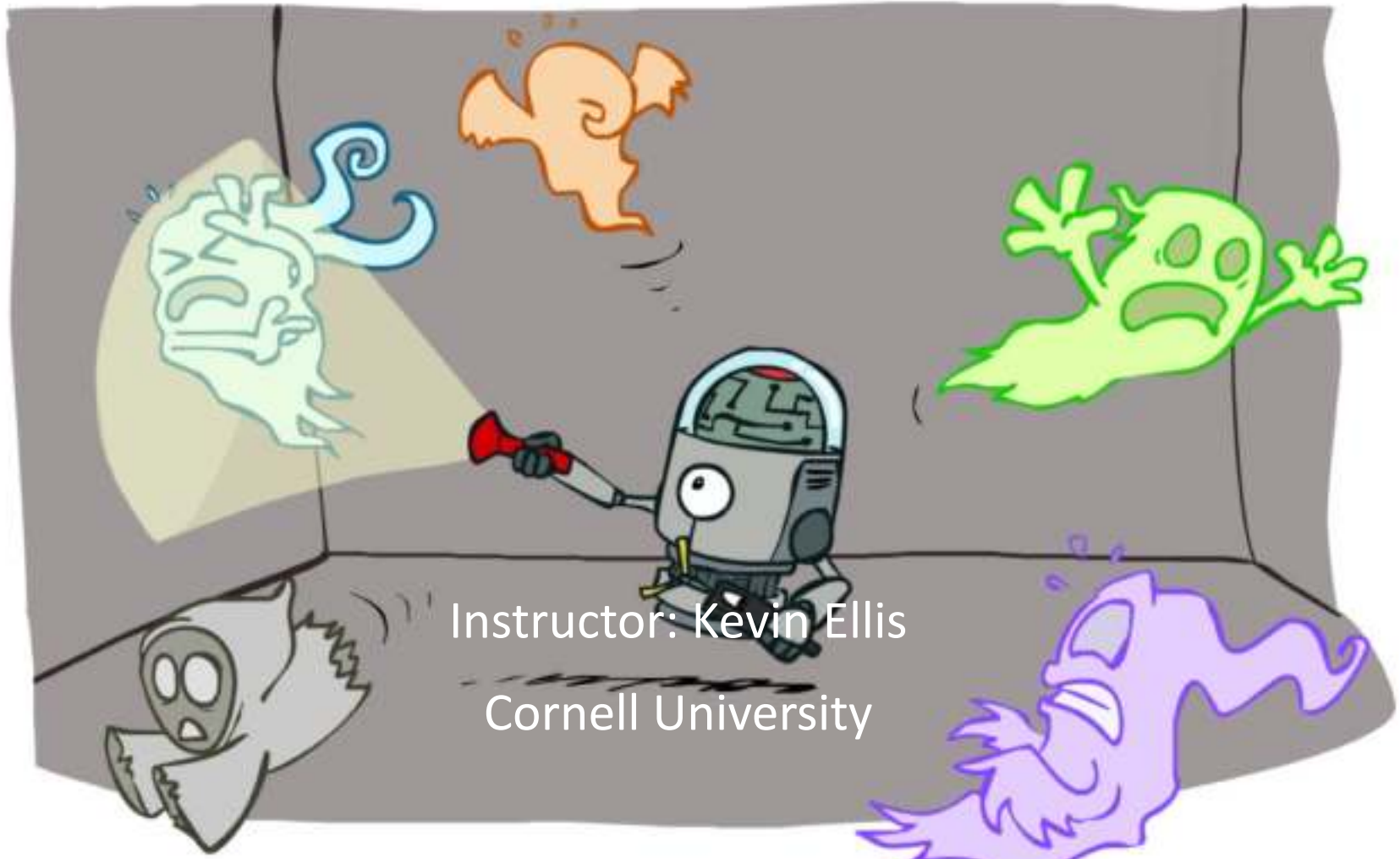
CS 4700: Foundations of Artificial Intelligence

HMMs, Particle Filters, and Applications



Instructor: Kevin Ellis

Cornell University

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

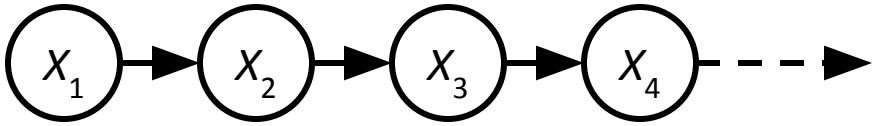
Today

- HMMs
  - Particle filters
  - Demos!
  - Most-likely-explanation queries
- Applications:
  - Robot localization / mapping
  - Speech recognition
  - Espionage

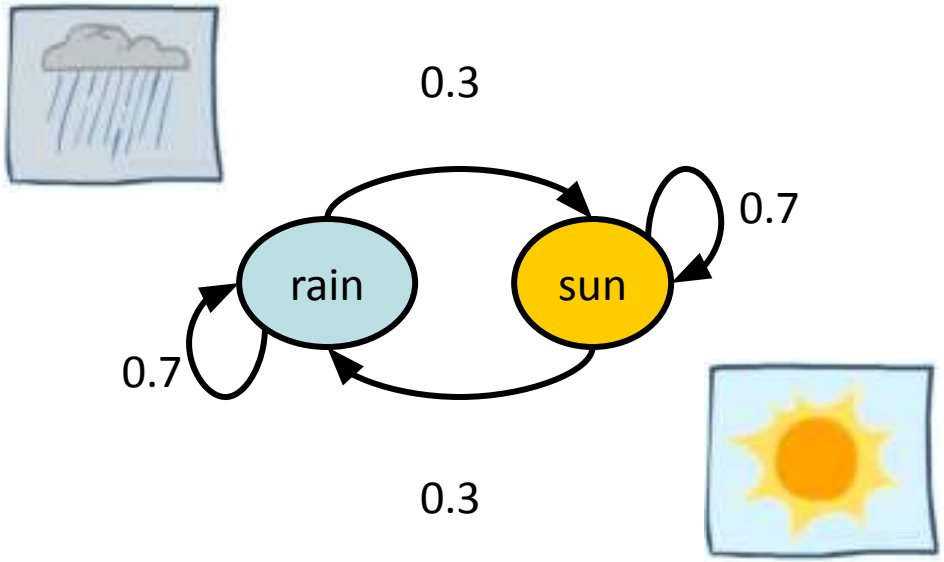
[Demo: Ghostbusters Markov Model (L15D1)]

Recap: Reasoning Over Time

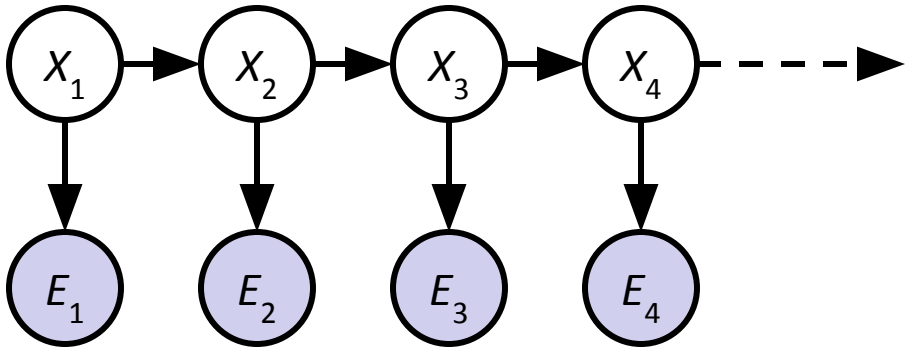
- Markov models



$$P(X_1) \qquad P(X|X_{-1})$$



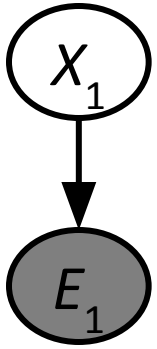
- Hidden Markov models



$$P(E|X)$$

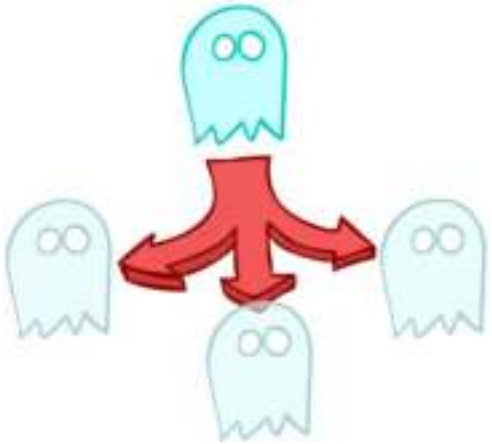
X	E	P
rain	umbrella	0.9
rain	no umbrella	0.1
sun	umbrella	0.2
sun	no umbrella	0.8

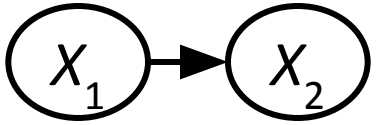
Inference: Base Cases



$$P(X_1|e_1)$$

$$\begin{aligned} P(x_1|e_1) &= P(x_1, e_1)/P(e_1) \\ &\propto_{X_1} P(x_1, e_1) \\ &= P(x_1)P(e_1|x_1) \end{aligned}$$

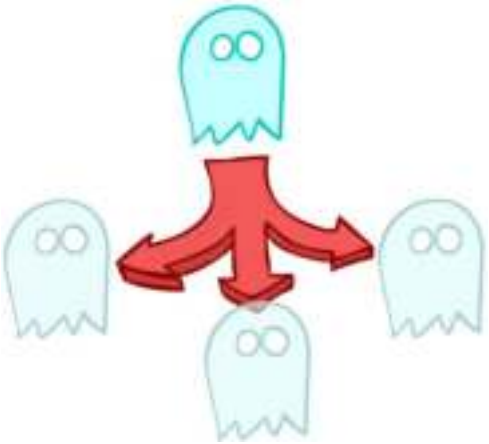


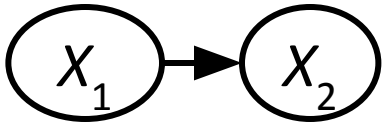


$$P(X_2)$$

$$\begin{aligned} P(x_2) &= \sum_{x_1} P(x_1, x_2) \\ &= \sum_{x_1} P(x_1)P(x_2|x_1) \end{aligned}$$

Inference: Base Cases





$$P(X_2)$$

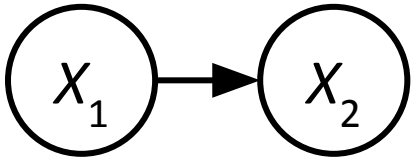
$$P(x_2) = \sum_{x_1} P(x_1, x_2)$$

$$= \sum_{x_1} P(x_1) P(x_2|x_1)$$

Passage of Time

- Assume we have current belief P(X | evidence to date)

$$B(X_t) = P(X_t|e_{1:t})$$



- Then, after one time step passes:

$$\begin{aligned} P(X_{t+1}|e_{1:t}) &= \sum_{x_t} P(X_{t+1}, x_t|e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1}|x_t, e_{1:t}) P(x_t|e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1}|x_t) P(x_t|e_{1:t}) \end{aligned}$$

- Or compactly:

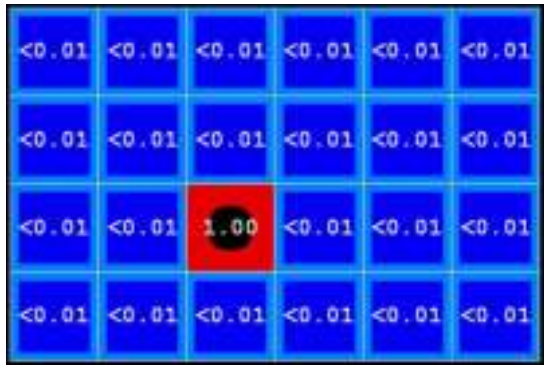
$$B'(X_{t+1}) = \sum_{x_t} P(X'|x_t) B(x_t)$$

- Basic idea: beliefs get "pushed" through the transitions
  - With the "B" notation, we have to be careful about what time step t the belief is about, and what evidence it includes

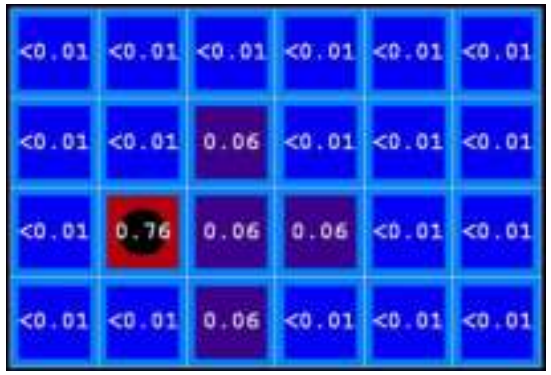
Example: Passage of Time

- As time passes, uncertainty “accumulates”

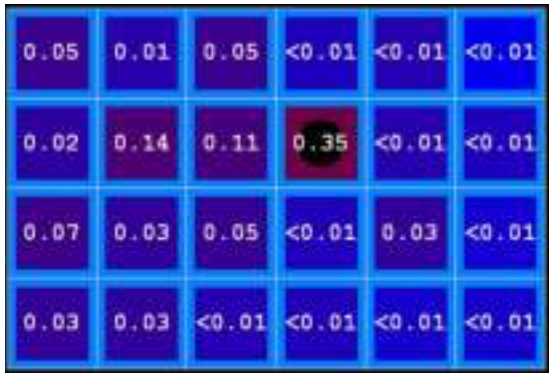
(Transition model: ghosts usually go clockwise)



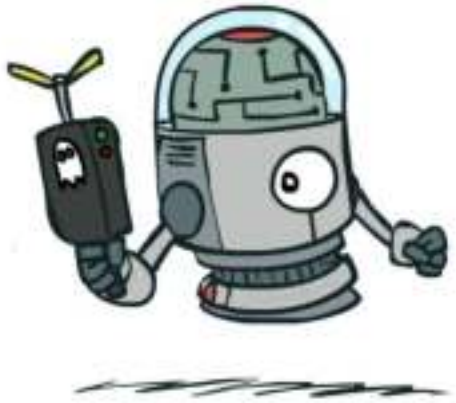
T = 1



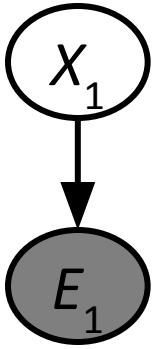
T = 2



T = 5



Inference: Base Cases



$$P(X_1|e_1)$$

$$P(x_1|e_1) = P(x_1, e_1)/P(e_1)$$
$$\propto_{X_1} P(x_1, e_1)$$
$$= P(x_1)P(e_1|x_1)$$

Observation

- Assume we have current belief P(X | previous evidence):

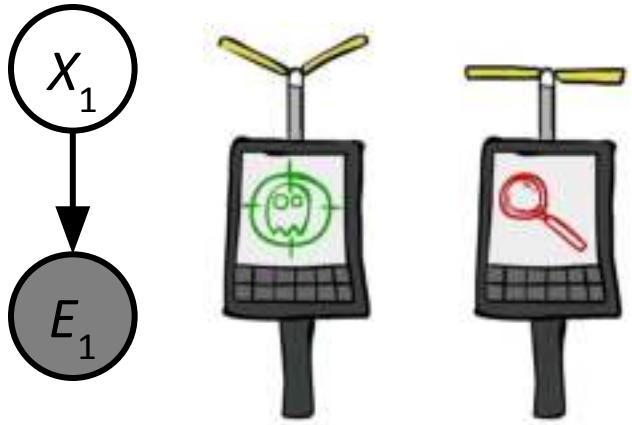
$$B'(X_{t+1}) = P(X_{t+1}|e_{1:t})$$

- Then, after evidence comes in:

$$\begin{aligned} P(X_{t+1}|e_{1:t+1}) &= P(X_{t+1}, e_{t+1}|e_{1:t})/P(e_{t+1}|e_{1:t}) \\ &\propto_{X_{t+1}} P(X_{t+1}, e_{t+1}|e_{1:t}) \\ &= P(e_{t+1}|e_{1:t}, X_{t+1})P(X_{t+1}|e_{1:t}) \\ &= P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t}) \end{aligned}$$

- Or, compactly:

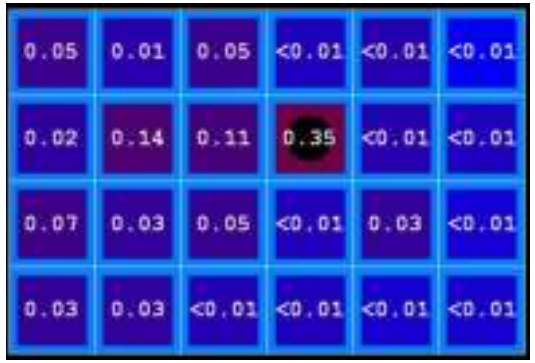
$$B(X_{t+1}) \propto_{X_{t+1}} P(e_{t+1}|X_{t+1})B'(X_{t+1})$$



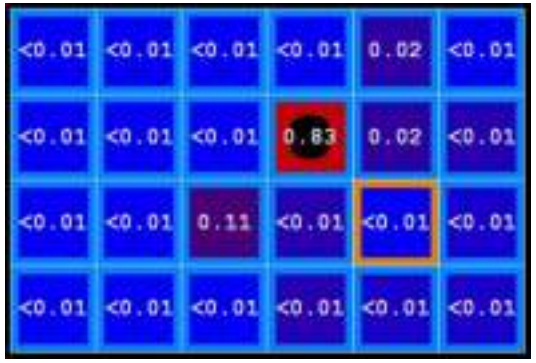
- Basic idea: beliefs "reweighted" by likelihood of evidence
- Unlike passage of time, we have to renormalize

Example: Observation

- As we get observations, beliefs get reweighted, uncertainty “decreases”



Before observation



After observation

$$B(X) \propto P(e|X)B'(X)$$

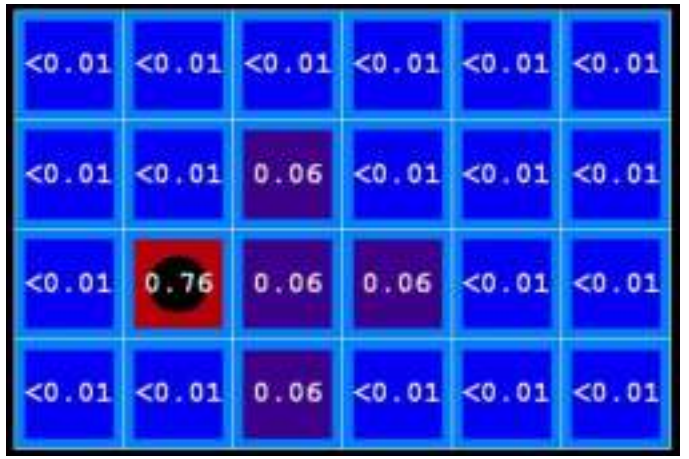
Filtering

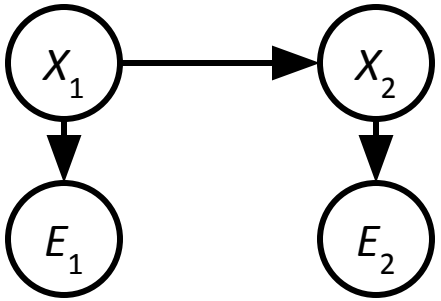
Elapse time: compute P(X_t | $e_{1:t-1}$)

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

Observe: compute P(X_t | $e_{1:t}$)

$$P(x_t|e_{1:t}) \propto P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$





Belief: <P(rain), P(sun)>

$P(X_1)$	<0.5, 0.5>	*Prior on X_1*
$P(X_1 \mid E_1 = umbrella)$	<0.82, 0.18>	*Observe*
$P(X_2 \mid E_1 = umbrella)$	<0.63, 0.37>	*Elapse time*
$P(X_2 \mid E_1 = umb, E_2 = umb)$	<0.88, 0.12>	*Observe*

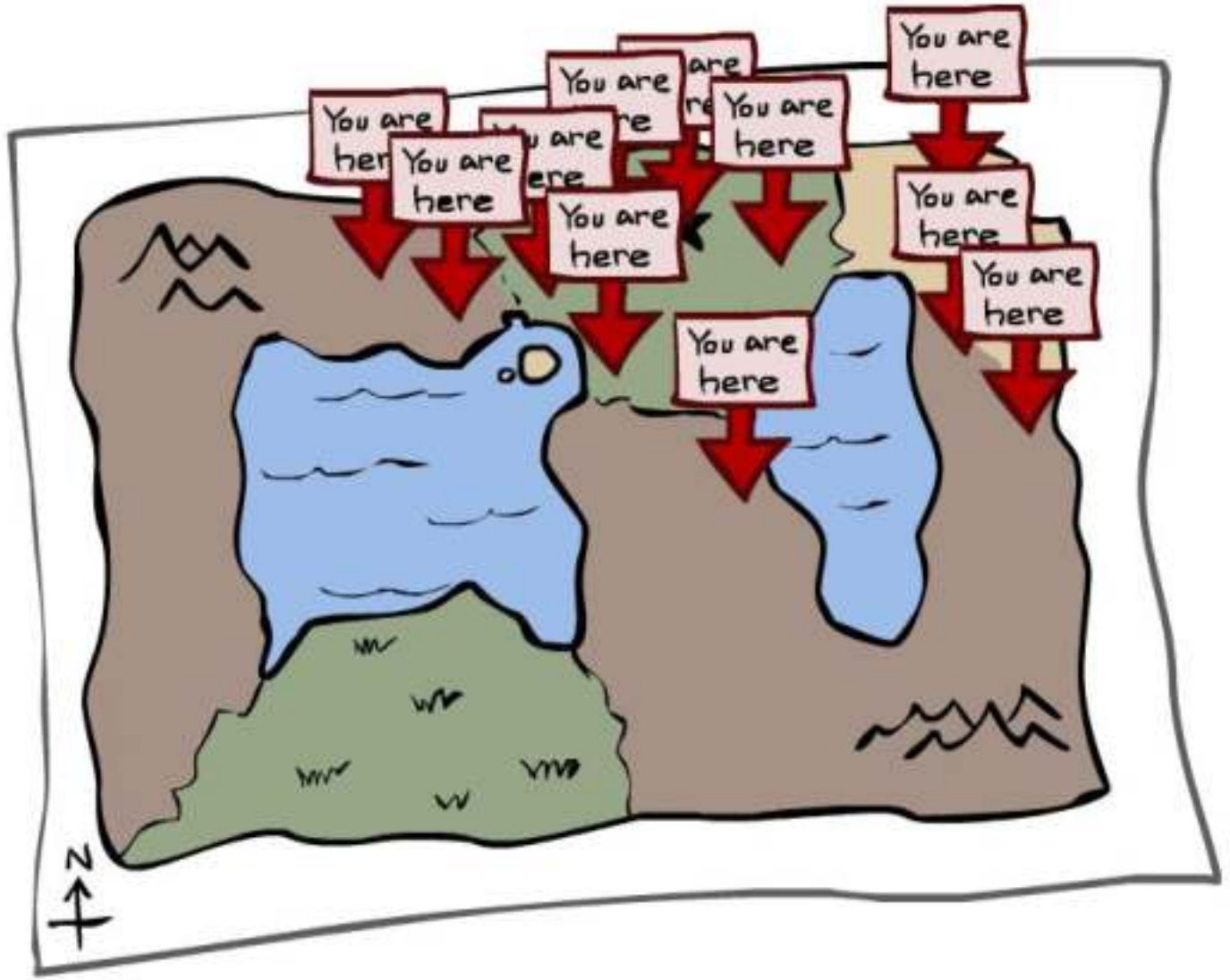
[Demo: Ghostbusters Exact Filtering (L15D2)]

Demo

python autograder.py -q q2

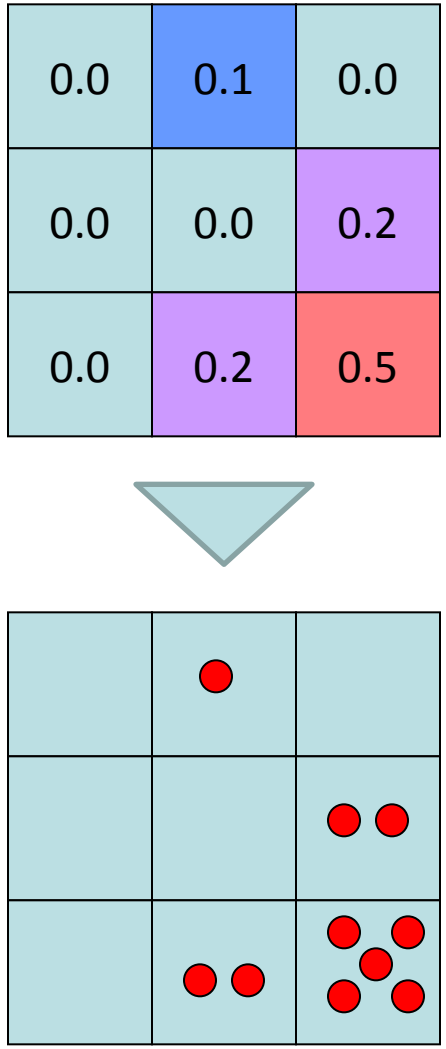
python autograder.py -q q4

Particle Filtering



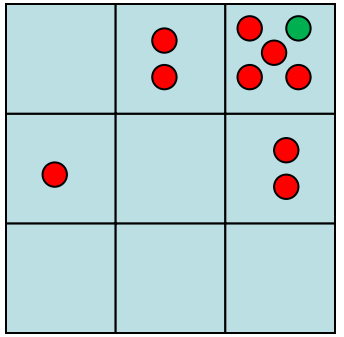
Particle Filtering

- Filtering: approximate solution
- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. X is continuous
- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states
- This is how robot localization works in practice
- Particle is just new name for sample



Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - Generally, N << |X|
  - Storing map from X to counts would defeat the point
- P(x) approximated by number of particles with value x
  - So, many x may have P(x) = 0!
  - More particles, more accuracy
- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

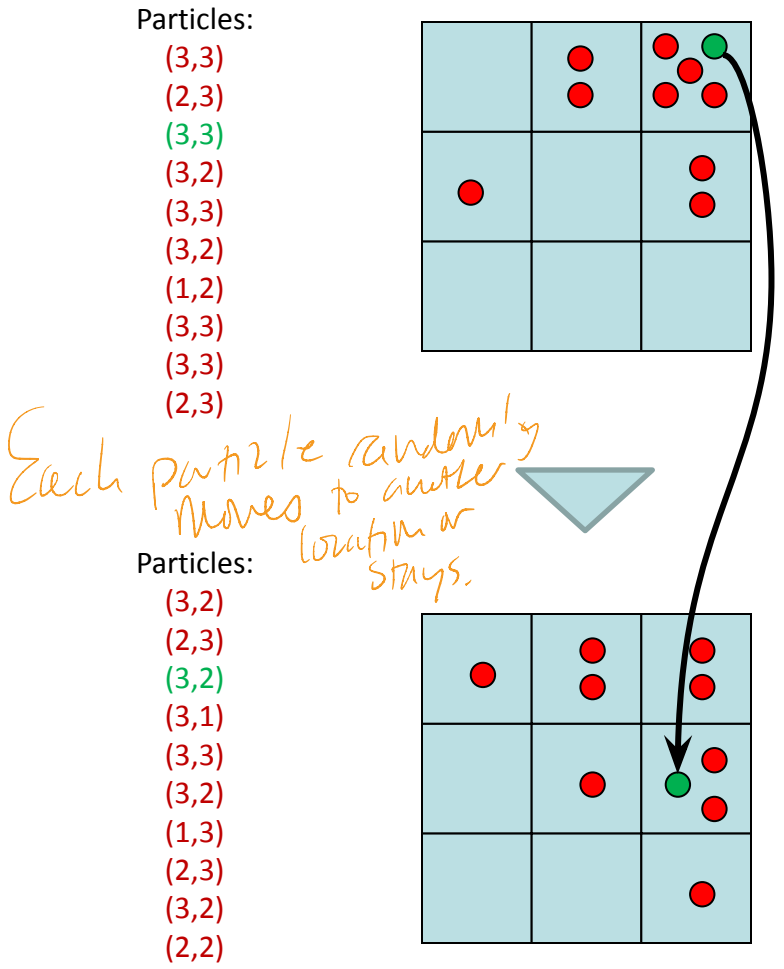
Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probabilities
  - Here, most samples move clockwise, but some move in another direction or stay in place

- This captures the passage of time
  - If enough samples, close to exact values before and after (consistent)



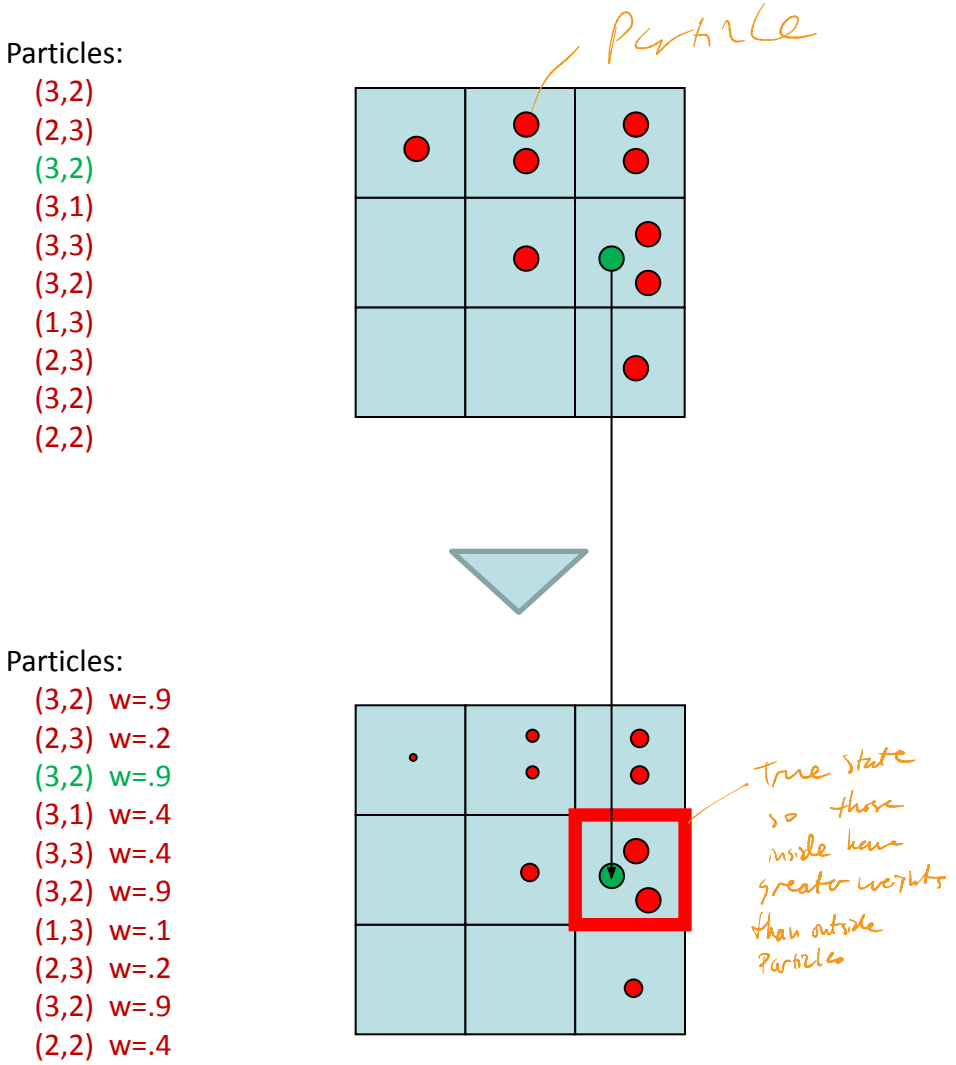
Particle Filtering: Observe

- Slightly trickier:
  - Don't sample observation, fix it
  - Similar to likelihood weighting, downweight samples based on the evidence

$$w(x) = P(e|x)$$

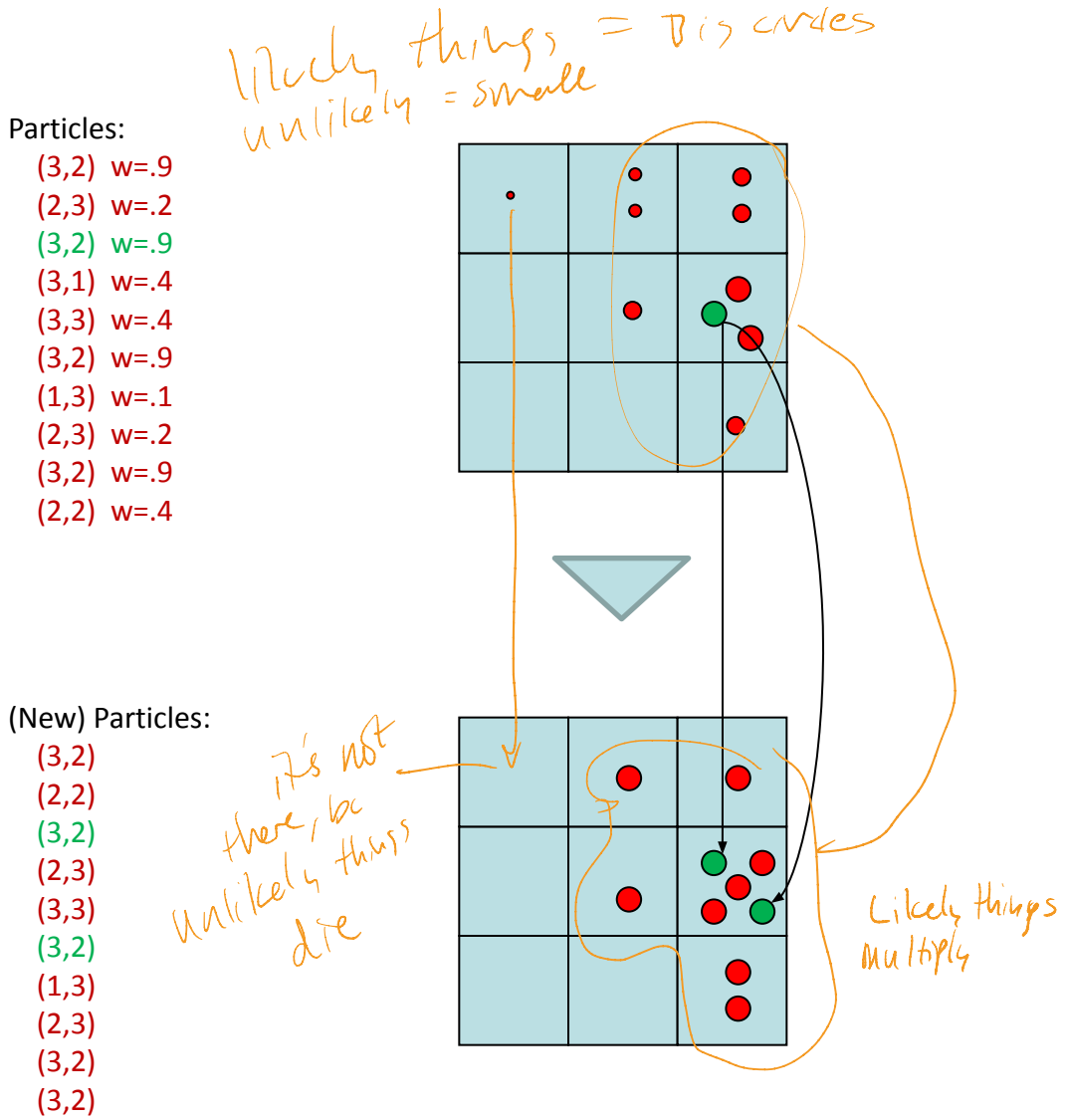
$$B(X) \propto P(e|X)B'(X)$$

  - As before, the probabilities don't sum to one, since all have been downweighted (in fact they now sum to (N times) an approximation of P(e))



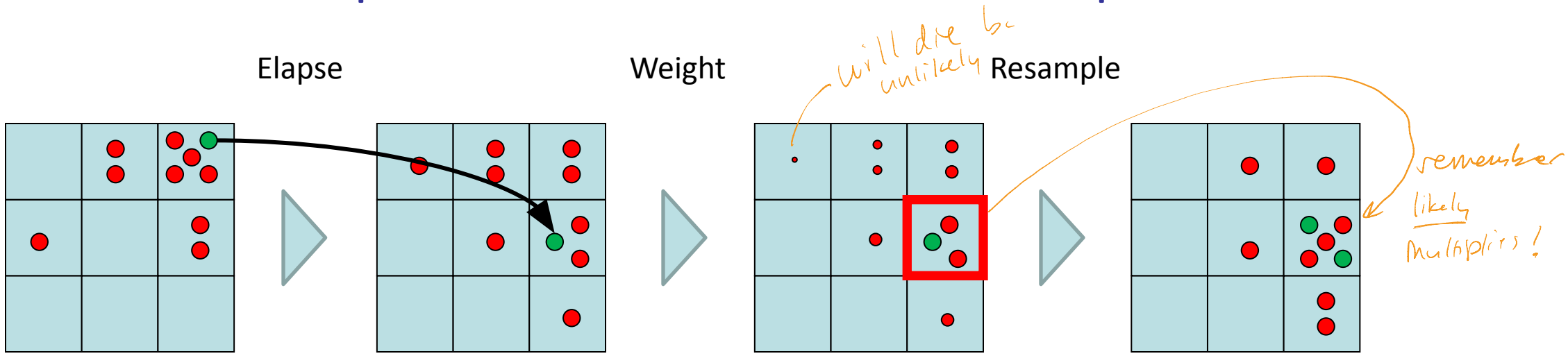
Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one



Recap: Particle Filtering

- Particles: track samples of states rather than an explicit distribution



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Particles:
(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)

Particles:
(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

(New) Particles:
(3,2)
(2,2)
(3,2)
(2,3)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(3,2)

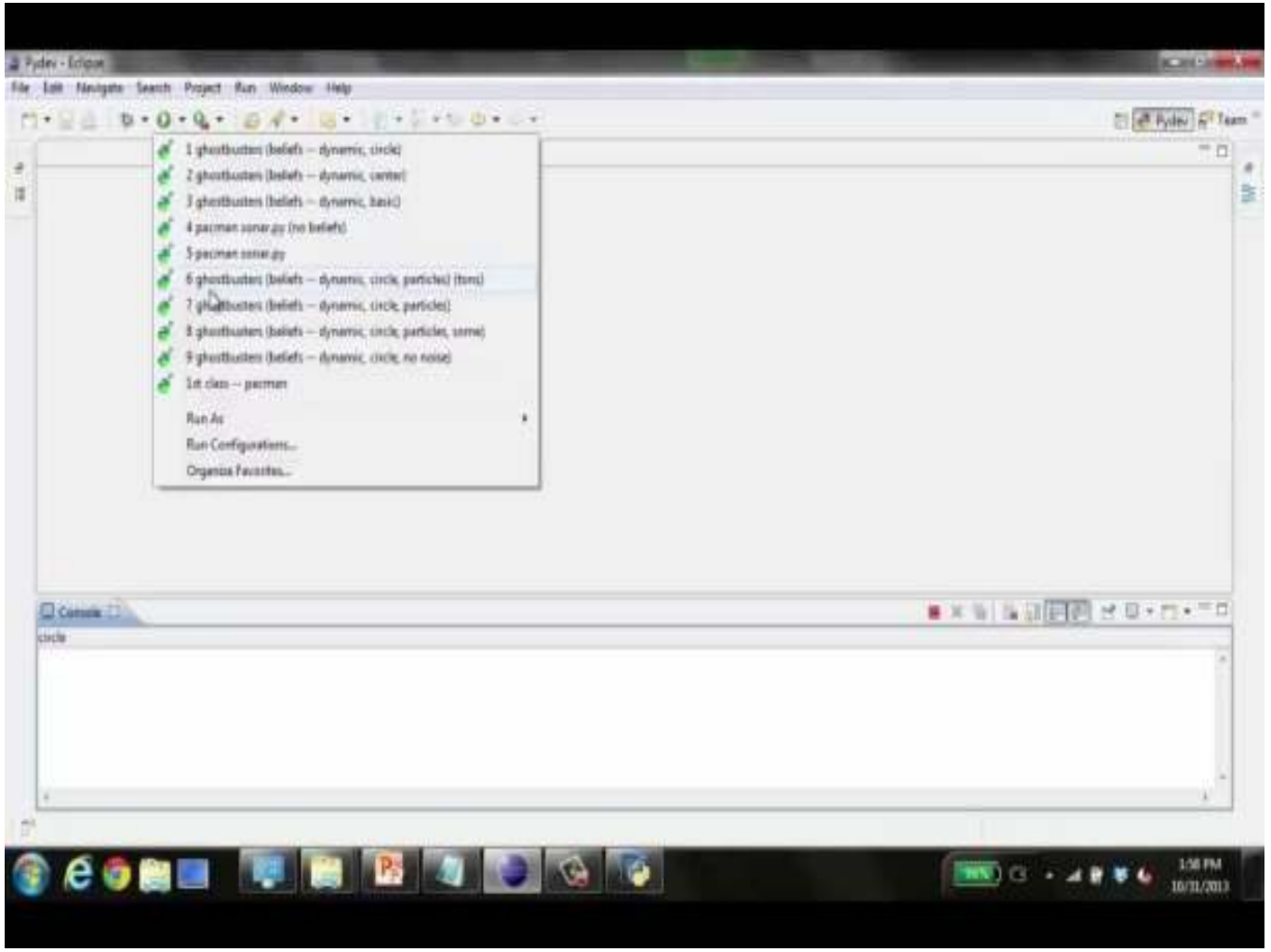
[Demos: ghostbusters particle filtering (L15D3,4,5)]

Demo

python autograder.py -q q6

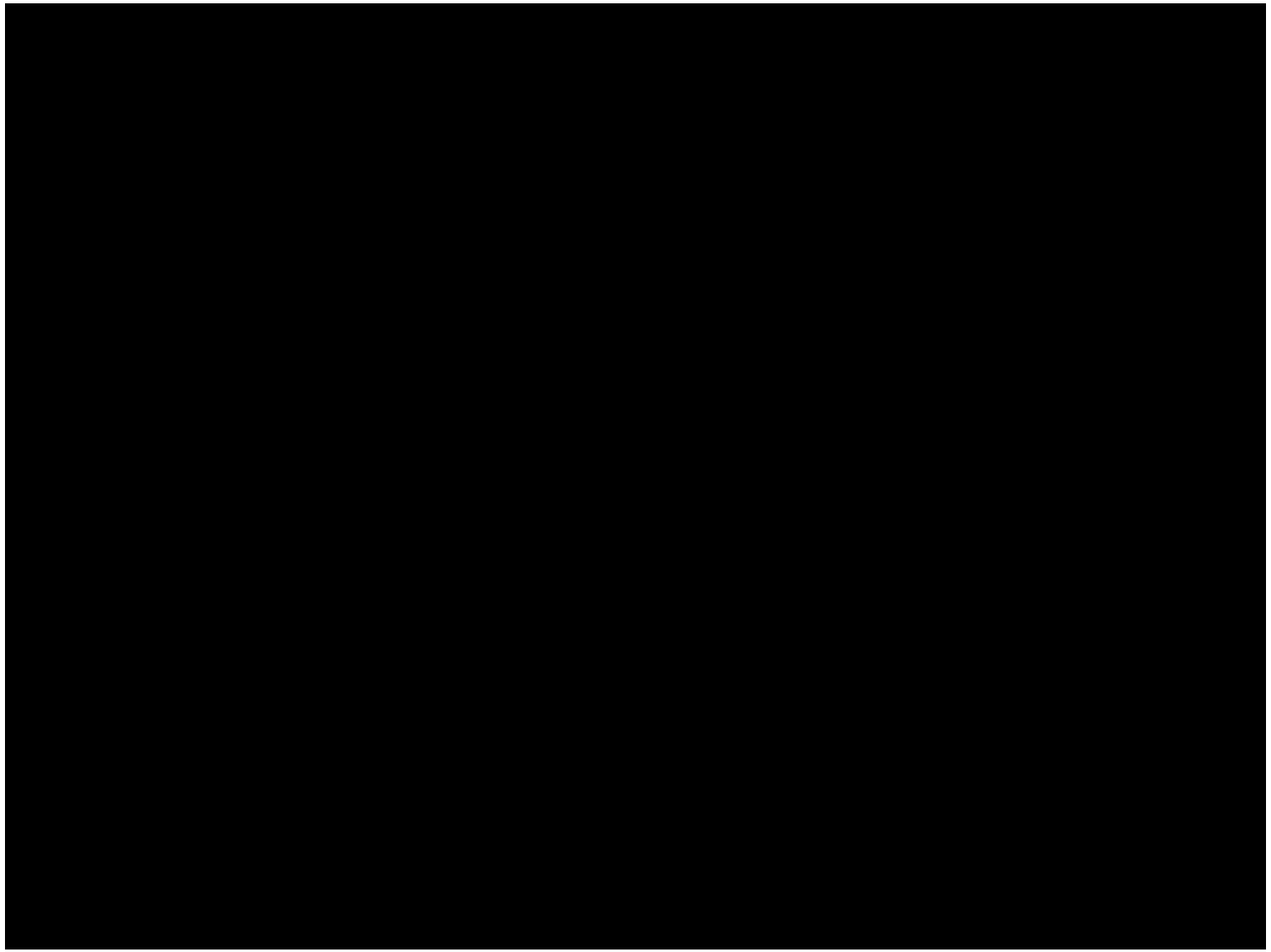
* All your particles can die, so many that are generate new random particles (Resample)

Video of Demo – Moderate Number of Particles

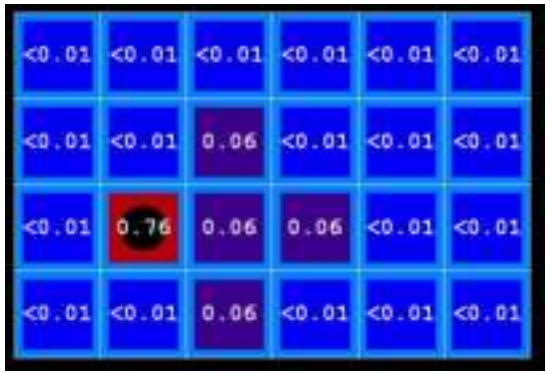


Video of Demo – One Particle

Video of Demo – Huge Number of Particles



More Demos!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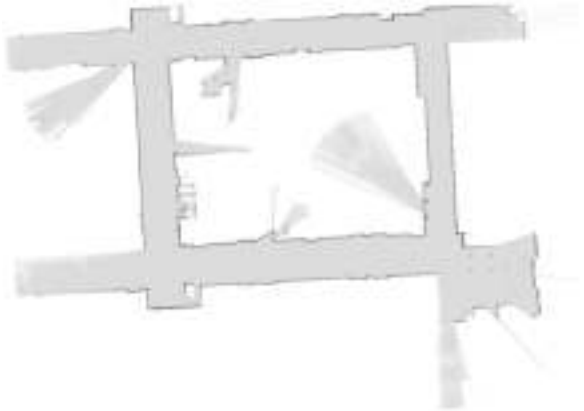
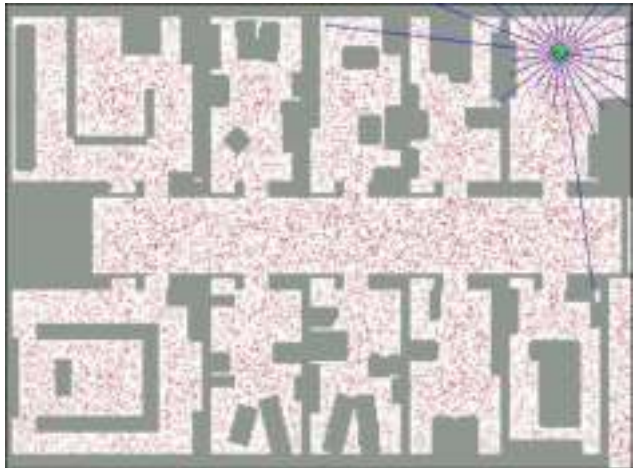
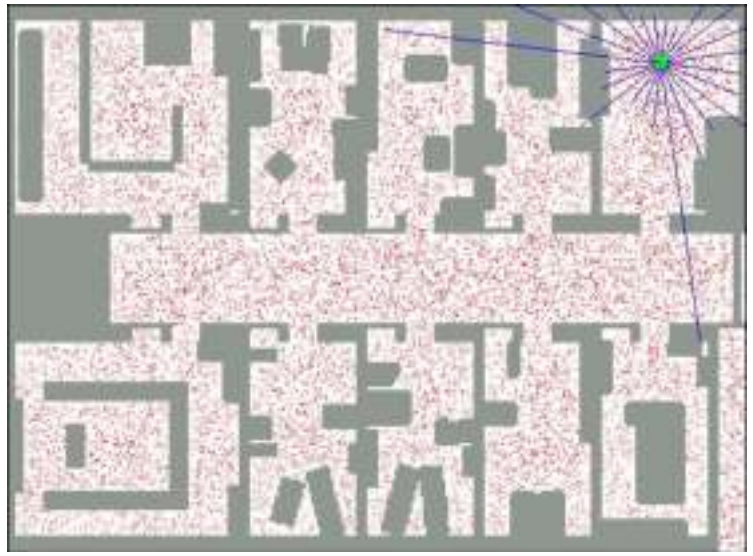
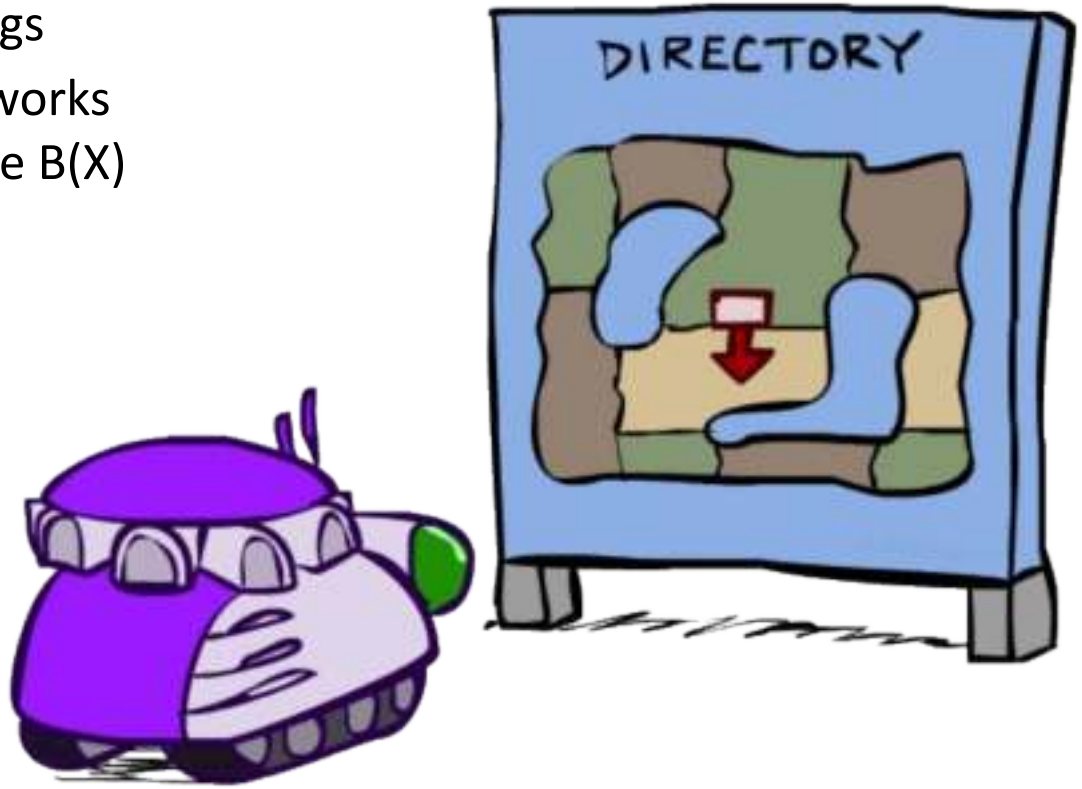




Robot Localization

- In robot localization:
  - We know the map, but not the robot's position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store B(X)
  - Particle filtering is a main technique

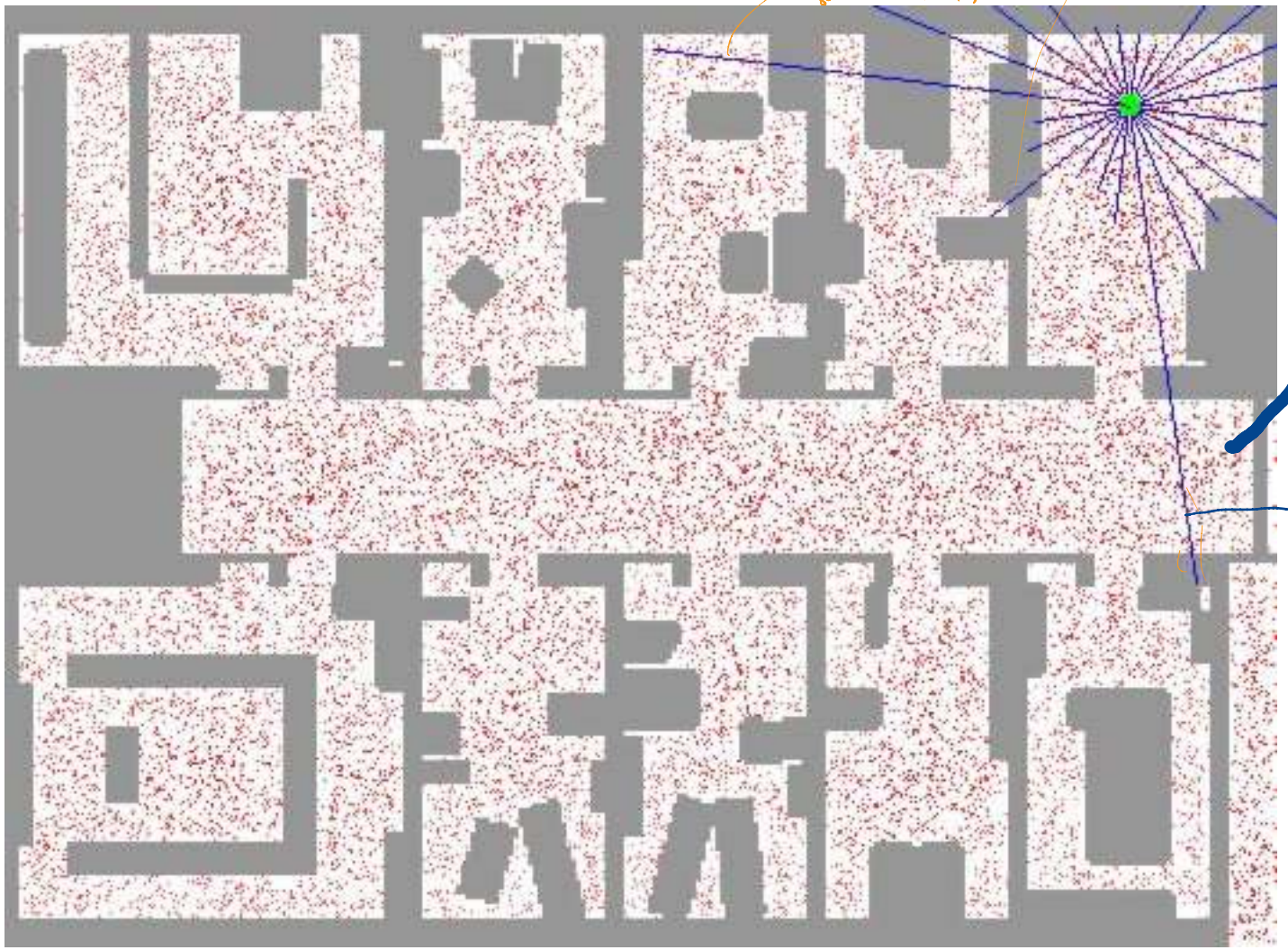




Particle Filter Localization (Sonar)

[Video: global-sonar-uw-annotated.avi]

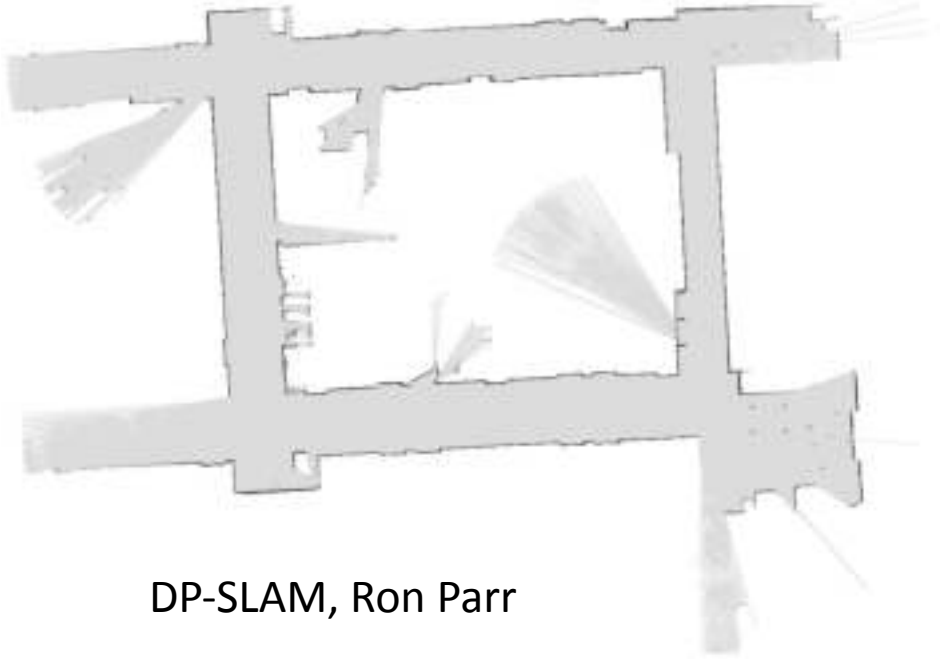
Particle Filter Localization (Laser)



[Video: global-floor.gif]

Robot Mapping

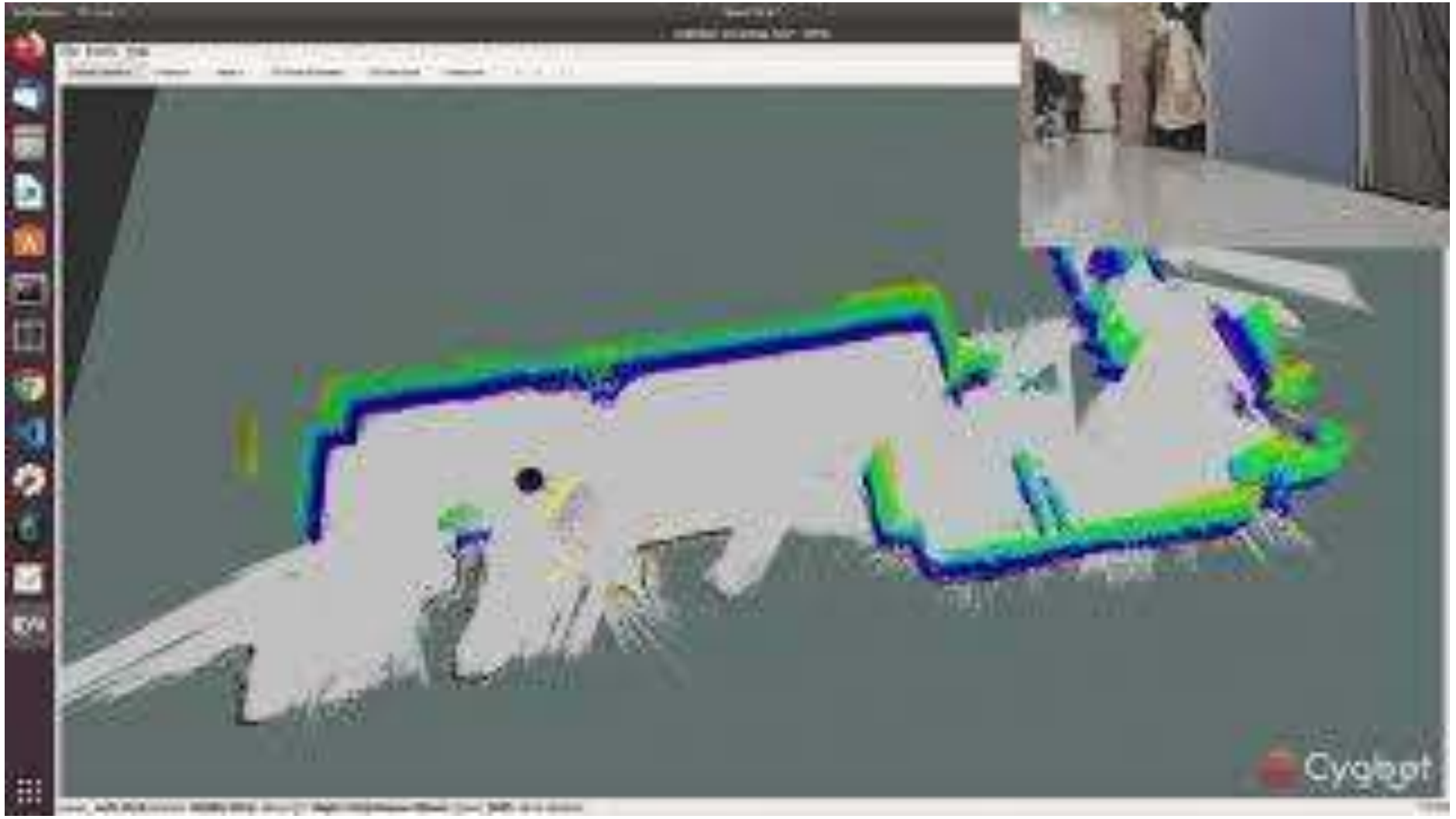
- SLAM: Simultaneous Localization And Mapping
  - We do not know the map or our location
  - State consists of position AND map!
  - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods



DP-SLAM, Ron Parr

[Demo: PARTICLES-SLAM-mapping1-new.avi]

SLAM – Video 1

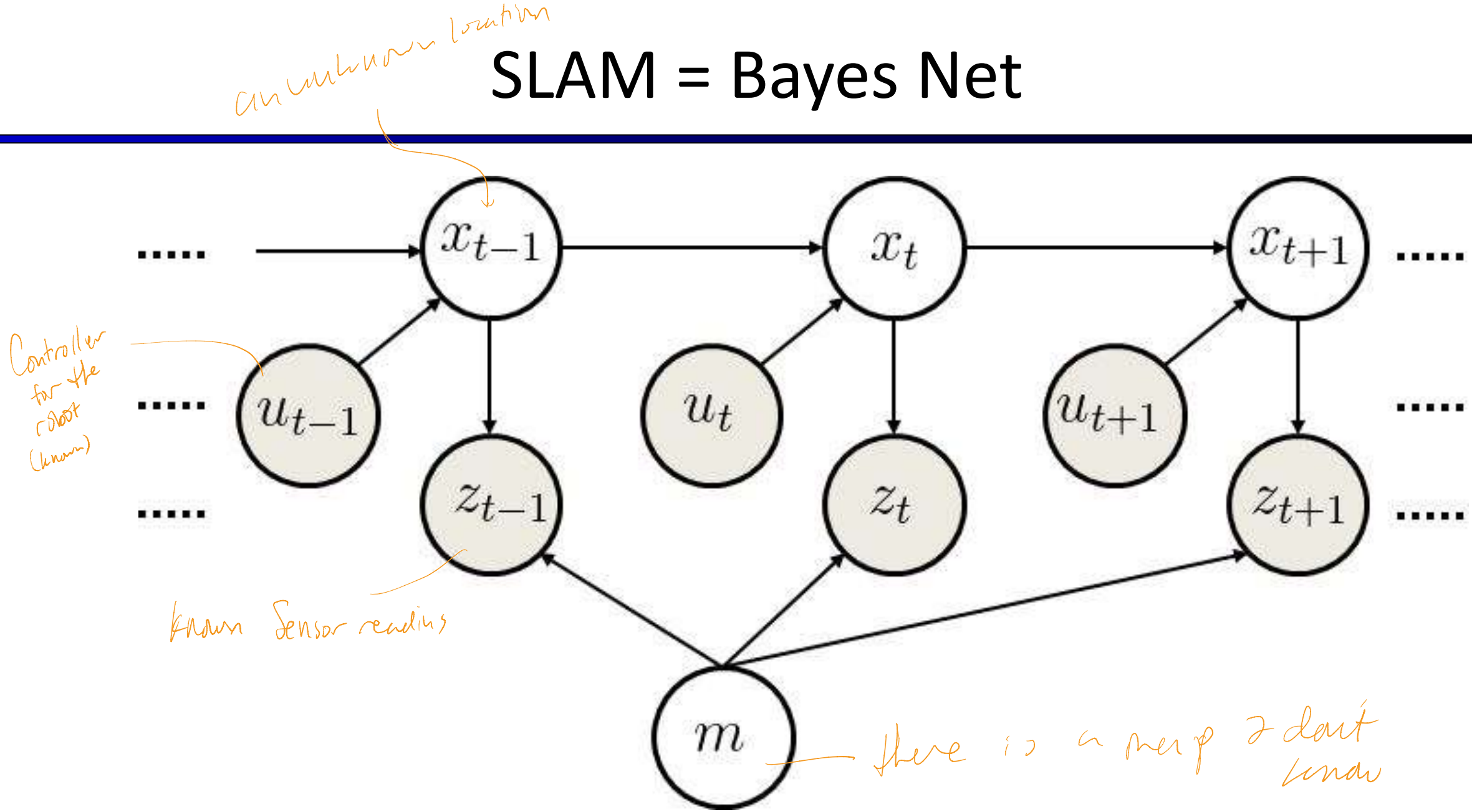


[Demo: PARTICLES-SLAM-mapping1-new.avi]

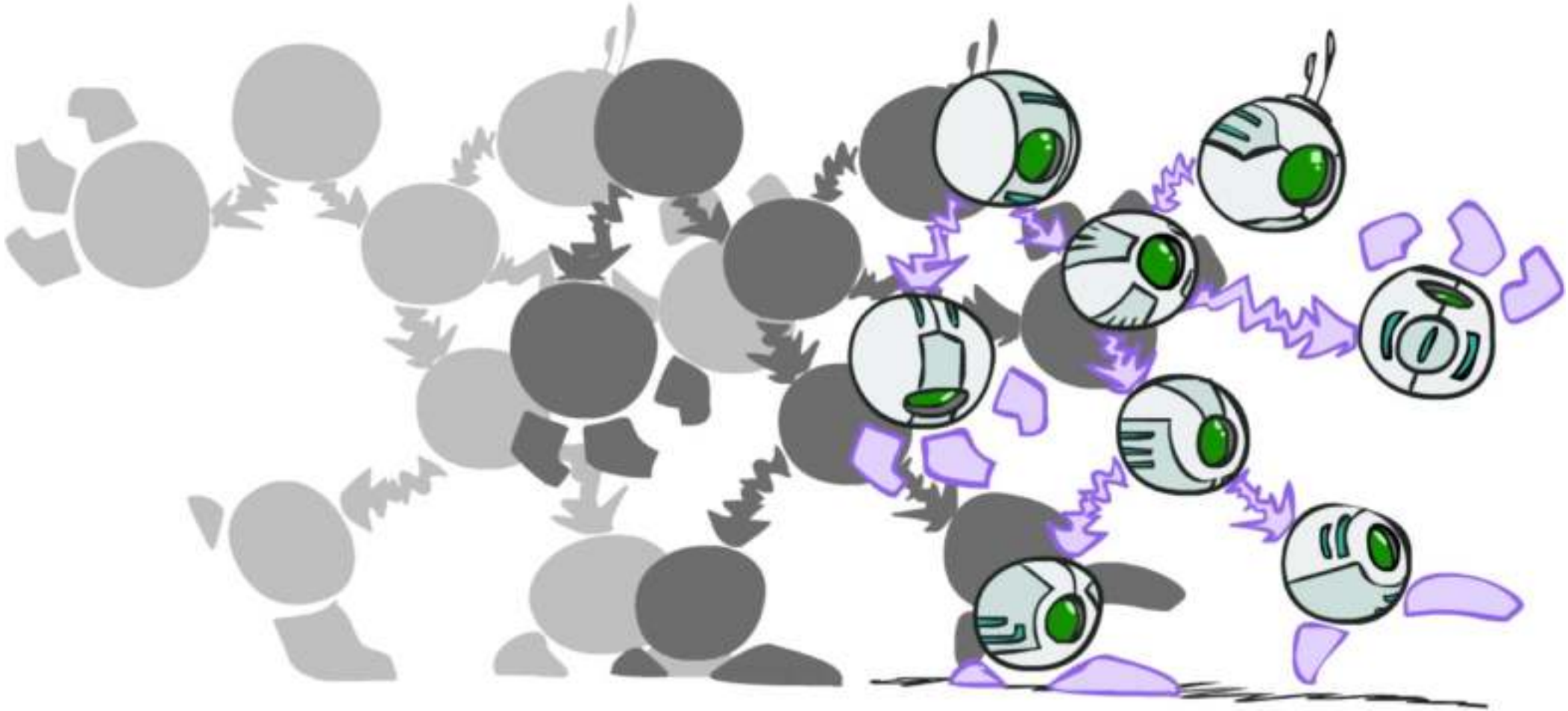
Particle Filter SLAM – Video 2

[Demo: PARTICLES-SLAM-fastslam.avi]

SLAM = Bayes Net

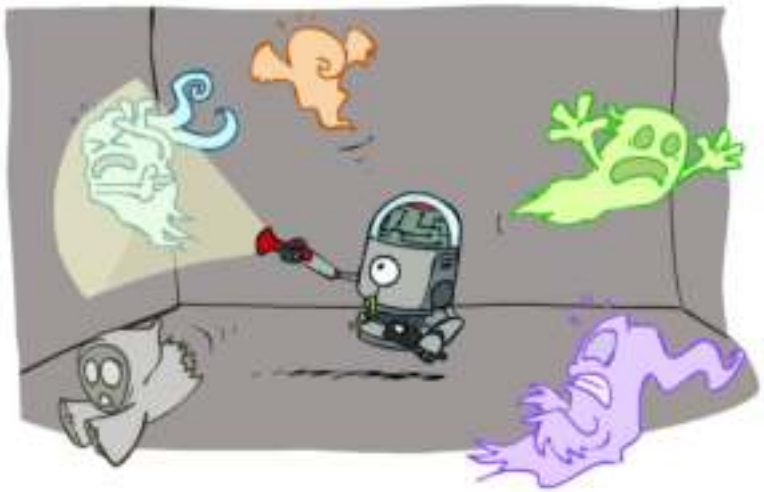


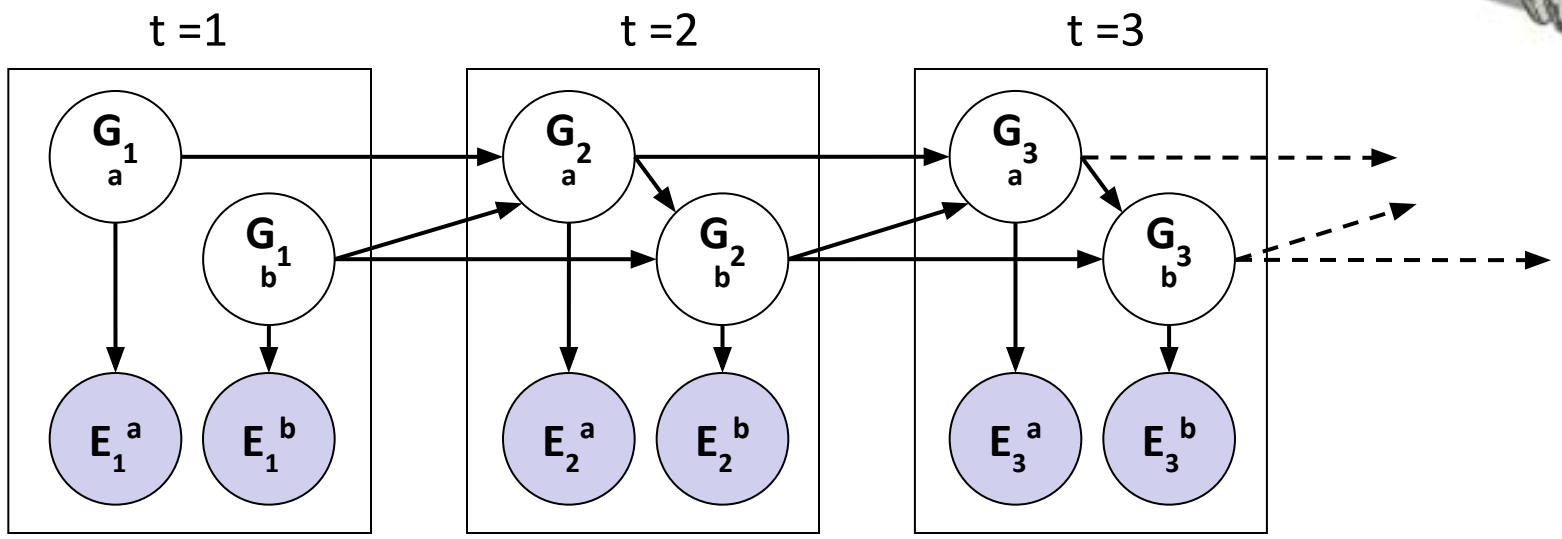
Dynamic Bayes Nets



Dynamic Bayes Nets (DBNs)

- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time *t* can condition on those from *t-1*





- Dynamic Bayes nets are a generalization of HMMs

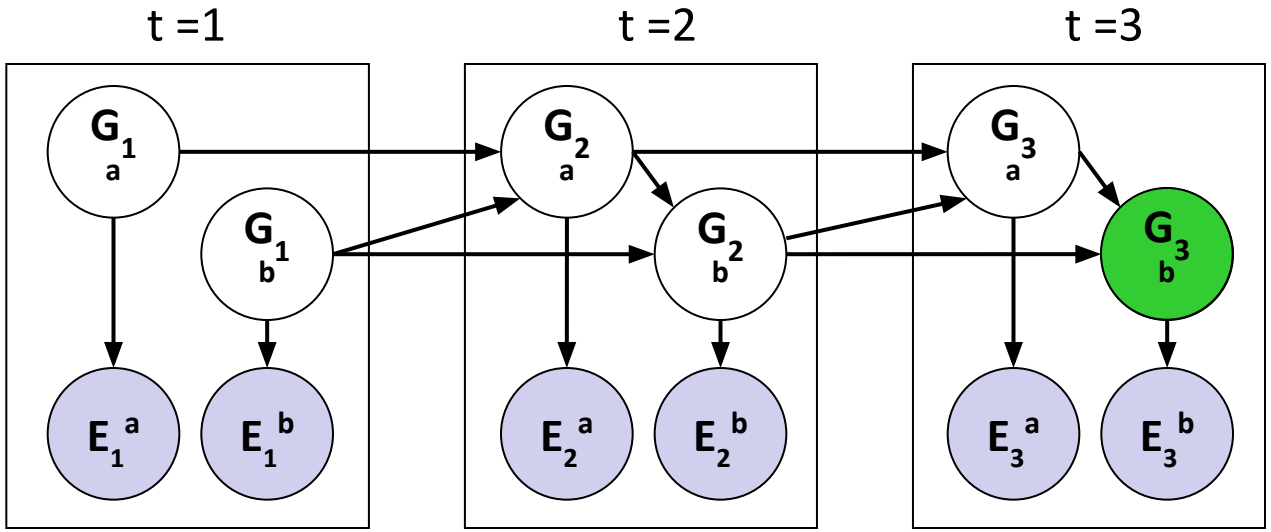
[Demo: pacman sonar ghost DBN model (L15D6)]

Pacman – Sonar (P4)

[Demo: Pacman – Sonar – No Beliefs(L14D1)]

Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Procedure: "unroll" the network for T time steps, then eliminate variables until $P(X_T | e_{1:T})$ is computed



- Online belief updates: Eliminate all variables from the previous time step; store factors for current time only

DBN Particle Filters

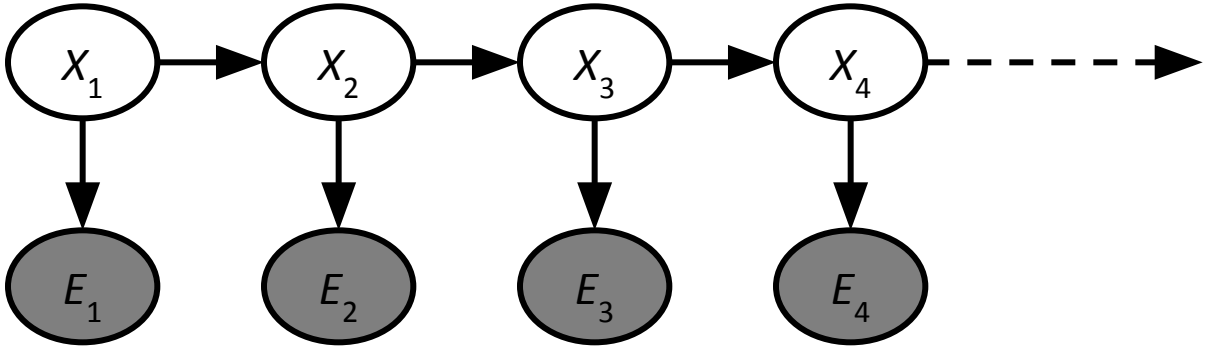
- A particle is a complete sample for a time step
- **Initialize**: Generate prior samples for the t=1 Bayes net
  - Example particle: $\mathbf{G}_1^{\,a} = (3,3)$ $\mathbf{G}_1^{\,b} = (5,3)$
- **Elapse time**: Sample a successor for each particle
  - Example successor: $\mathbf{G}_2^{\,a} = (2,3)$ $\mathbf{G}_2^{\,b} = (6,3)$
- **Observe**: Weight each *entire* sample by the likelihood of the evidence conditioned on the sample
  - Likelihood: $P(\mathbf{E}_1^{\,a} \mid \mathbf{G}_1^{\,a}) * P(\mathbf{E}_1^{\,b} \mid \mathbf{G}_1^{\,b})$
- **Resample:** Select prior samples (tuples of values) in proportion to their likelihood

Most Likely Explanation

Maximum Likelihood Estimate

HMMs: MLE Queries

- HMMs defined by
  - States X
  - Observations E
  - Initial distribution: $P(X_1)$
  - Transitions: $P(X|X_{-1})$
  - Emissions: $P(E|X)$



- New query: most likely explanation:

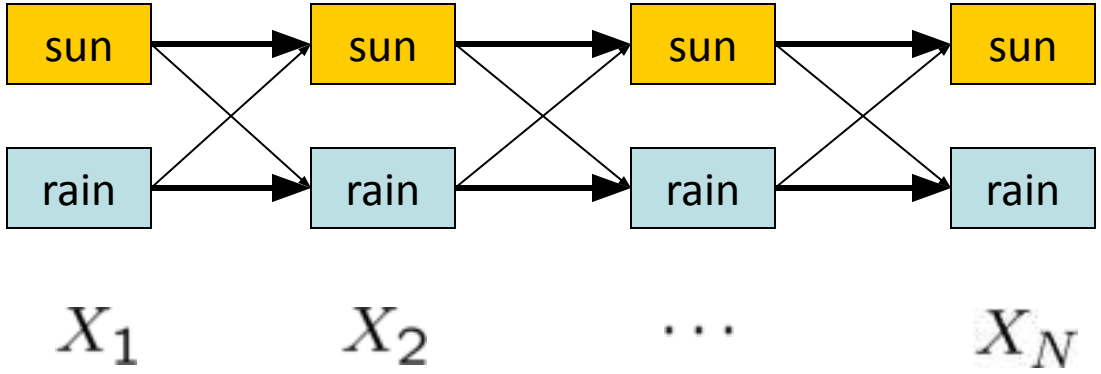
what is the most likely assignment given all hidden state variables & all evidence.

$$\arg\max_{x_{1:t}} P(x_{1:t}|e_{1:t})$$

- New method: the Viterbi algorithm

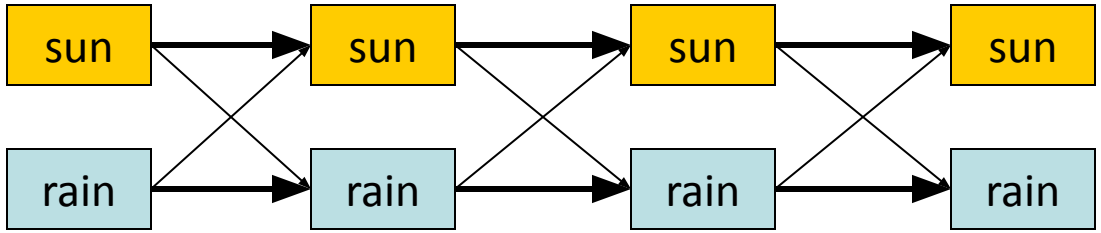
State Trellis

- State trellis: graph of states and transitions over time



- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t|x_{t-1})P(e_t|x_t)$
- Each path is a sequence of states
- The product of weights on a path is that sequence's probability along with the evidence
- Forward algorithm computes sums of paths, Viterbi computes best paths

Forward / Viterbi Algorithms



$X_1 \quad X_2 \quad \cdots \quad X_N$

sums all paths

Forward Algorithm (Sum)

Get most likely path

Viterbi Algorithm (Max)

$$f_t[x_t] = P(x_t, e_{1:t})$$

Forward also has only:
- New evidence
- old ... new
-

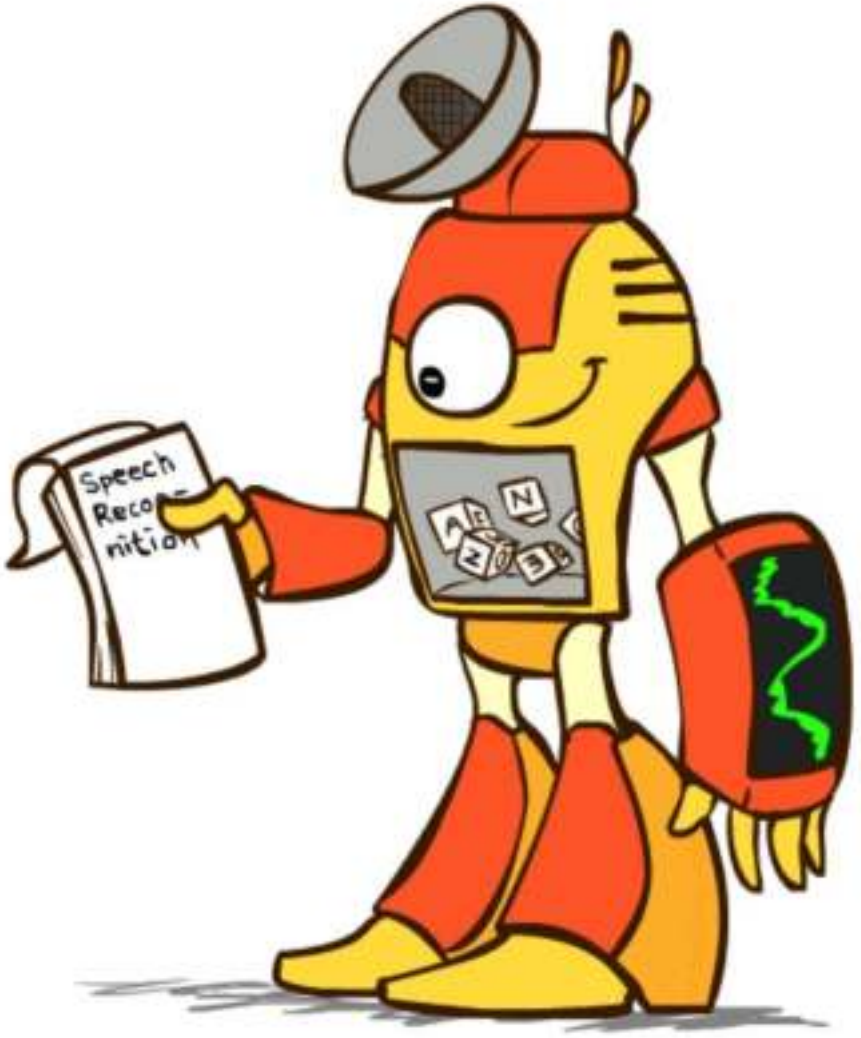
$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}) f_{t-1}[x_{t-1}]$$

beliefs on old state given old evidence

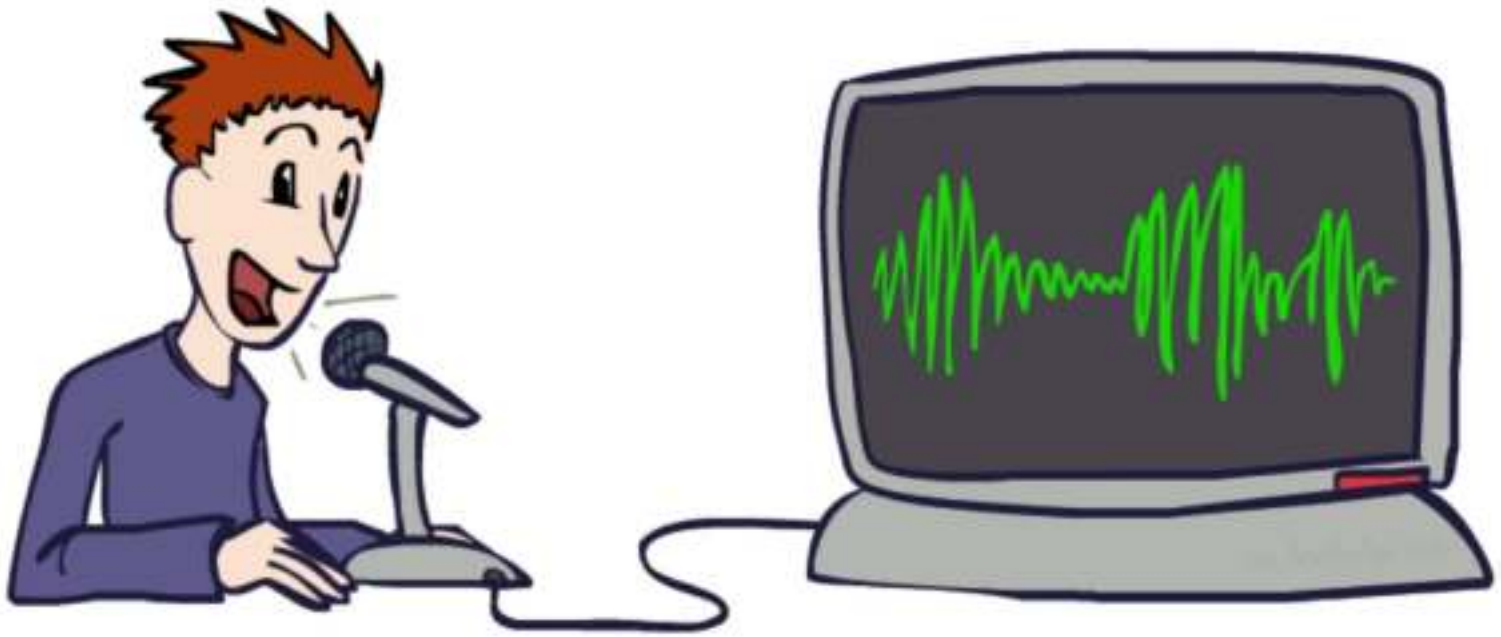
$$m_t[x_t] = \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$$

$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$

Speech Recognition



Digitizing Speech



Speech in an Hour

- Speech input is an acoustic waveform

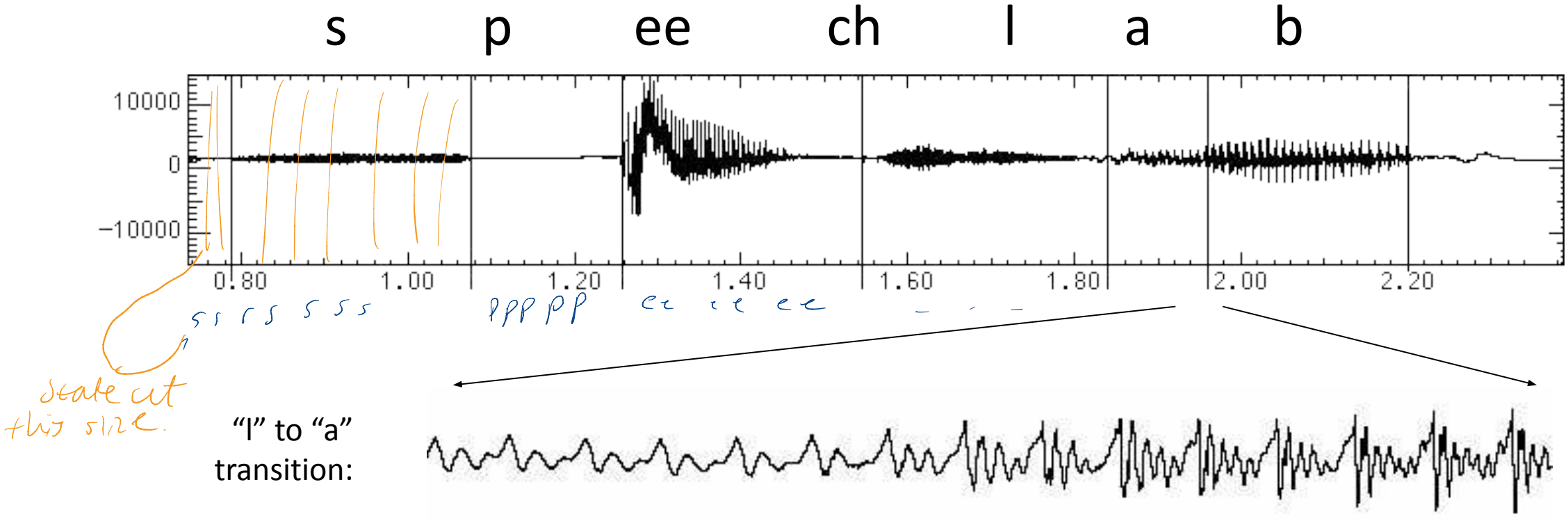
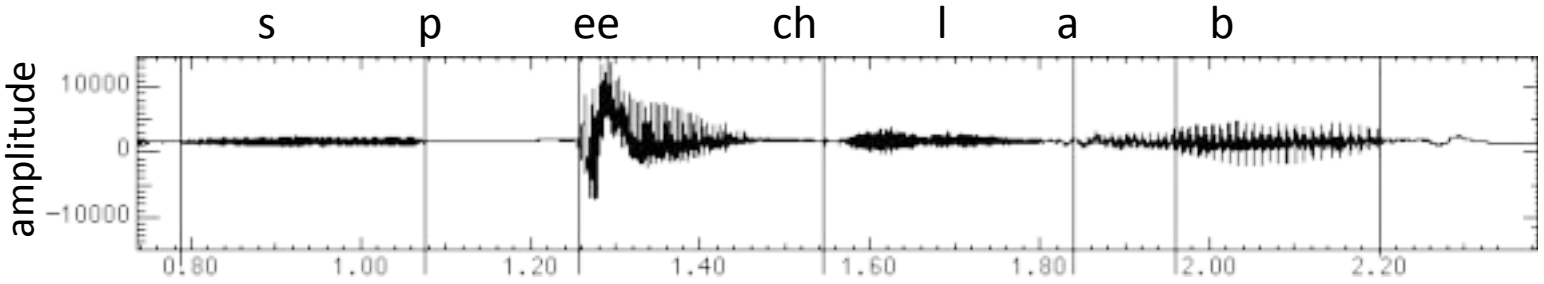
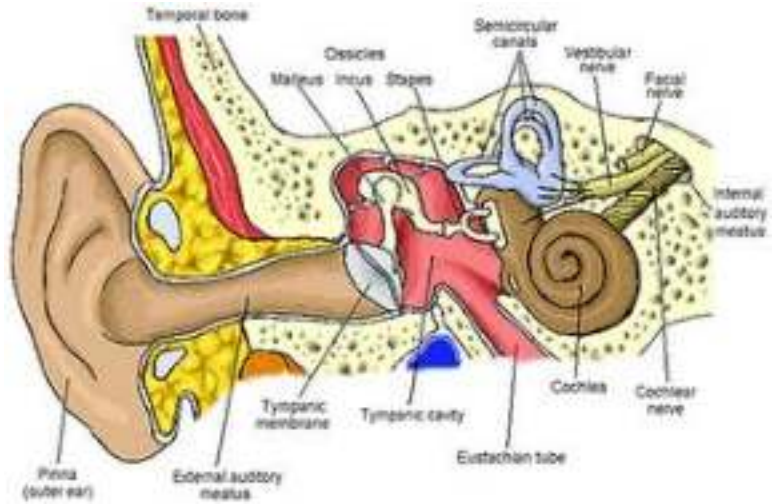


Figure: Simon Arnfield, http://www.psyc.leeds.ac.uk/research/cogn/speech/tutorial/

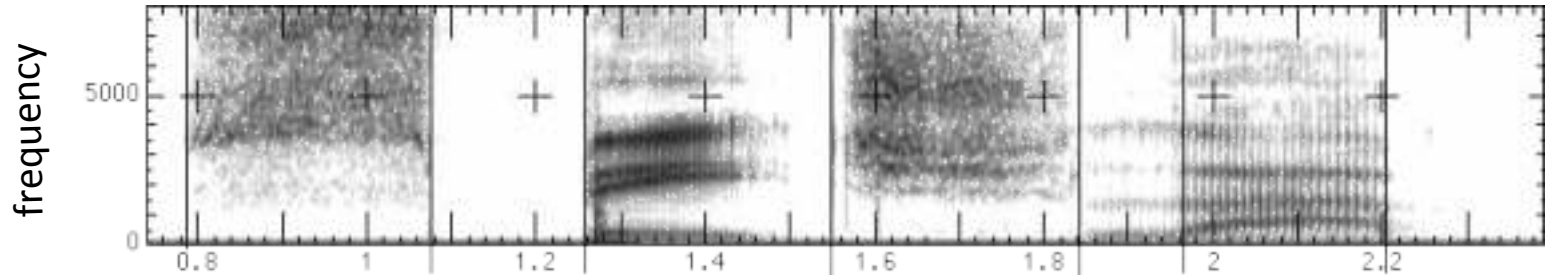
Spectral Analysis

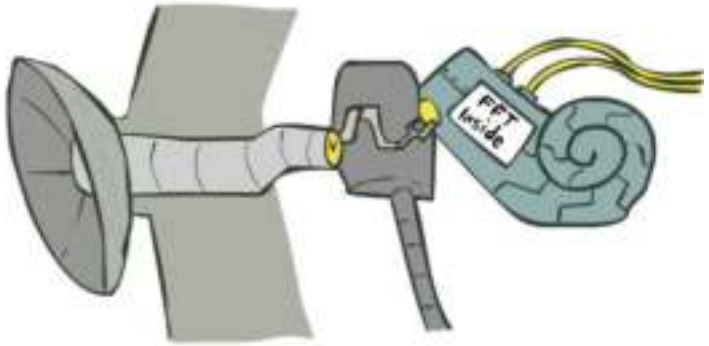
- Frequency gives pitch; amplitude gives volume
  - Sampling at ~8 kHz (phone), ~16 kHz (mic) (kHz=1000 cycles/sec)





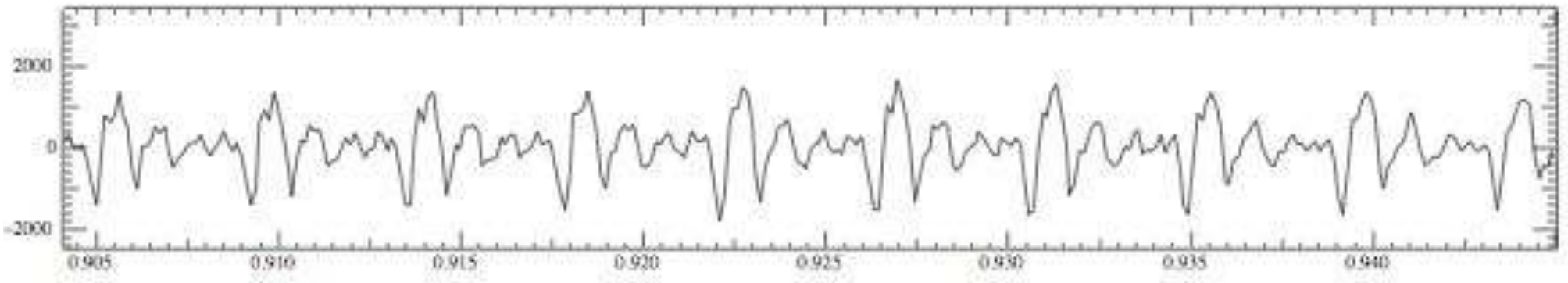
- Fourier transform of wave displayed as a spectrogram
  - Darkness indicates energy at each frequency



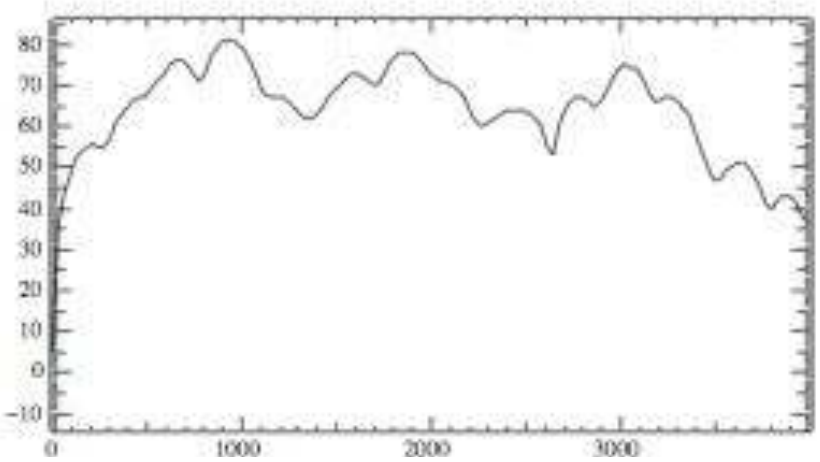


Human ear figure: depion.blogspot.com

Part of [ae] from “lab”

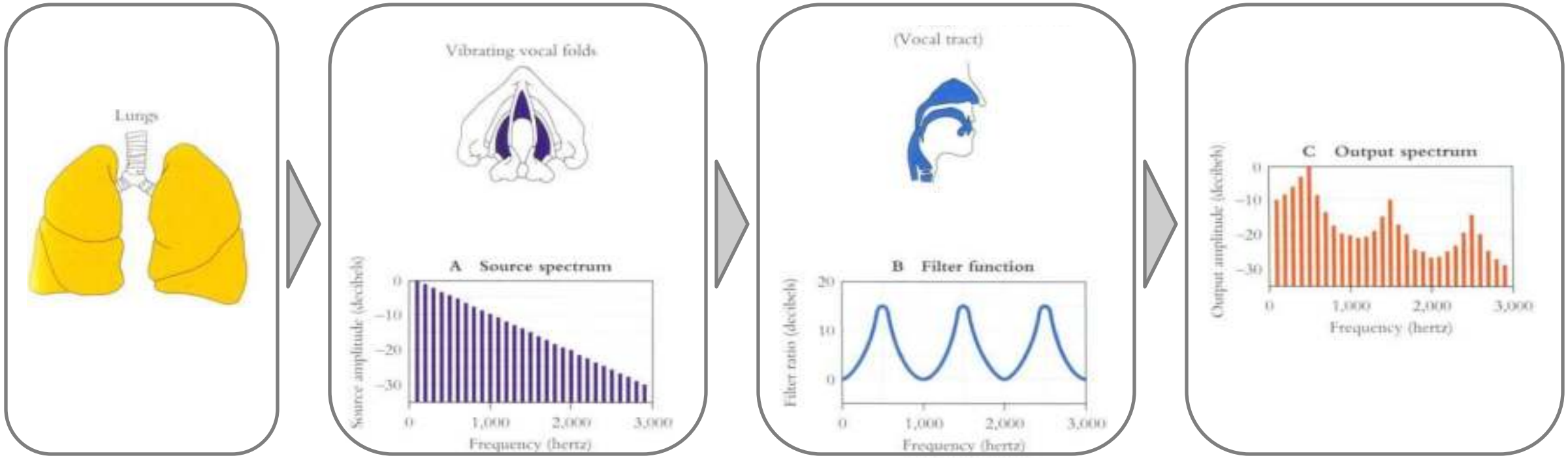


- Complex wave repeating nine times
  - Plus smaller wave that repeats 4x for every large cycle
  - Large wave: freq of 250 Hz (9 times in .036 seconds)
  - Small wave roughly 4 times this, or roughly 1000 Hz



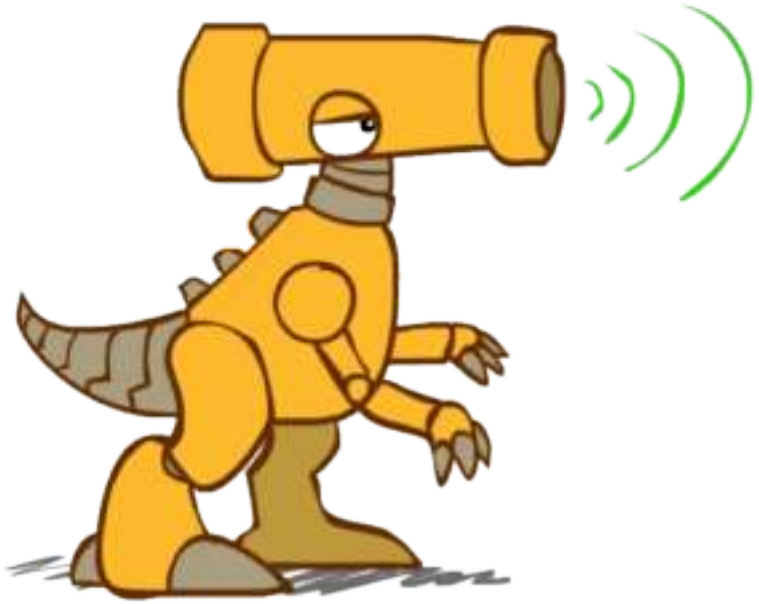
Why These Peaks?

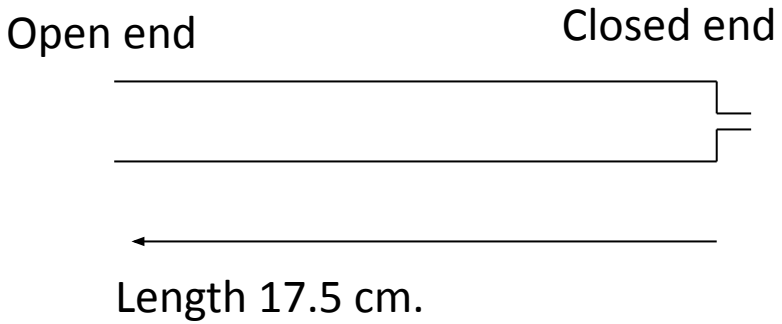
- Articulator process:
  - Vocal cord vibrations create harmonics
  - The mouth is an amplifier
  - Depending on shape of mouth, some harmonics are amplified more than others

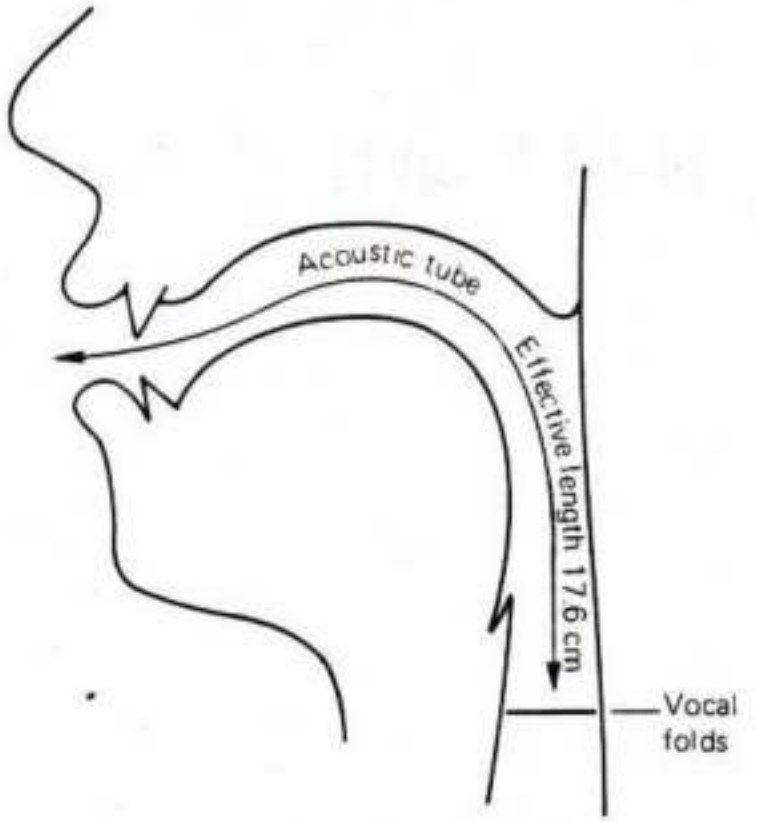


Resonances of the Vocal Tract

- The human vocal tract as an open tube







- Air in a tube of a given length will tend to vibrate at resonance frequency of tube
- Constraint: Pressure differential should be maximal at (closed) glottal end and minimal at (open) lip end

Figure: W. Barry Speech Science slides

Spectrum Shapes

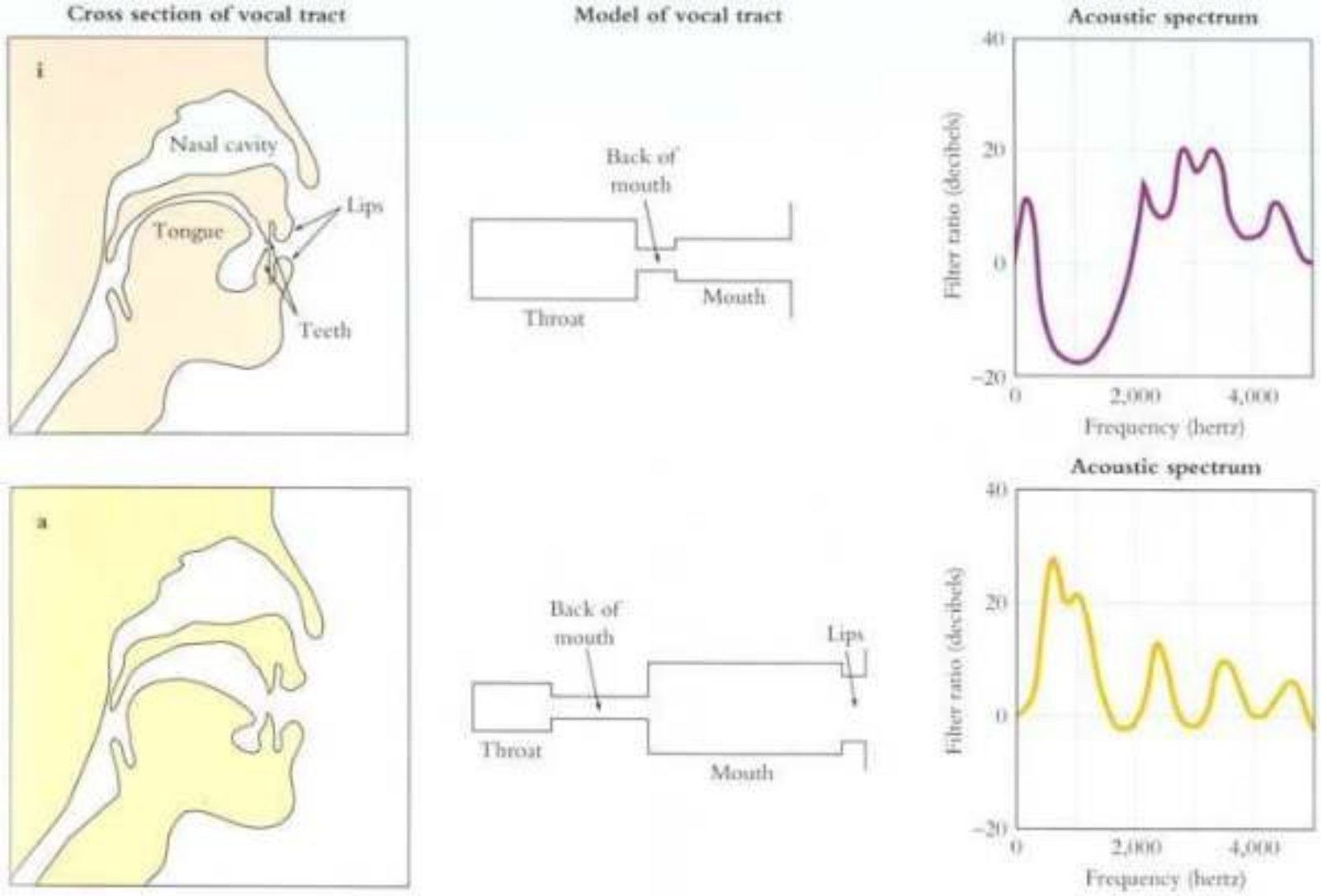
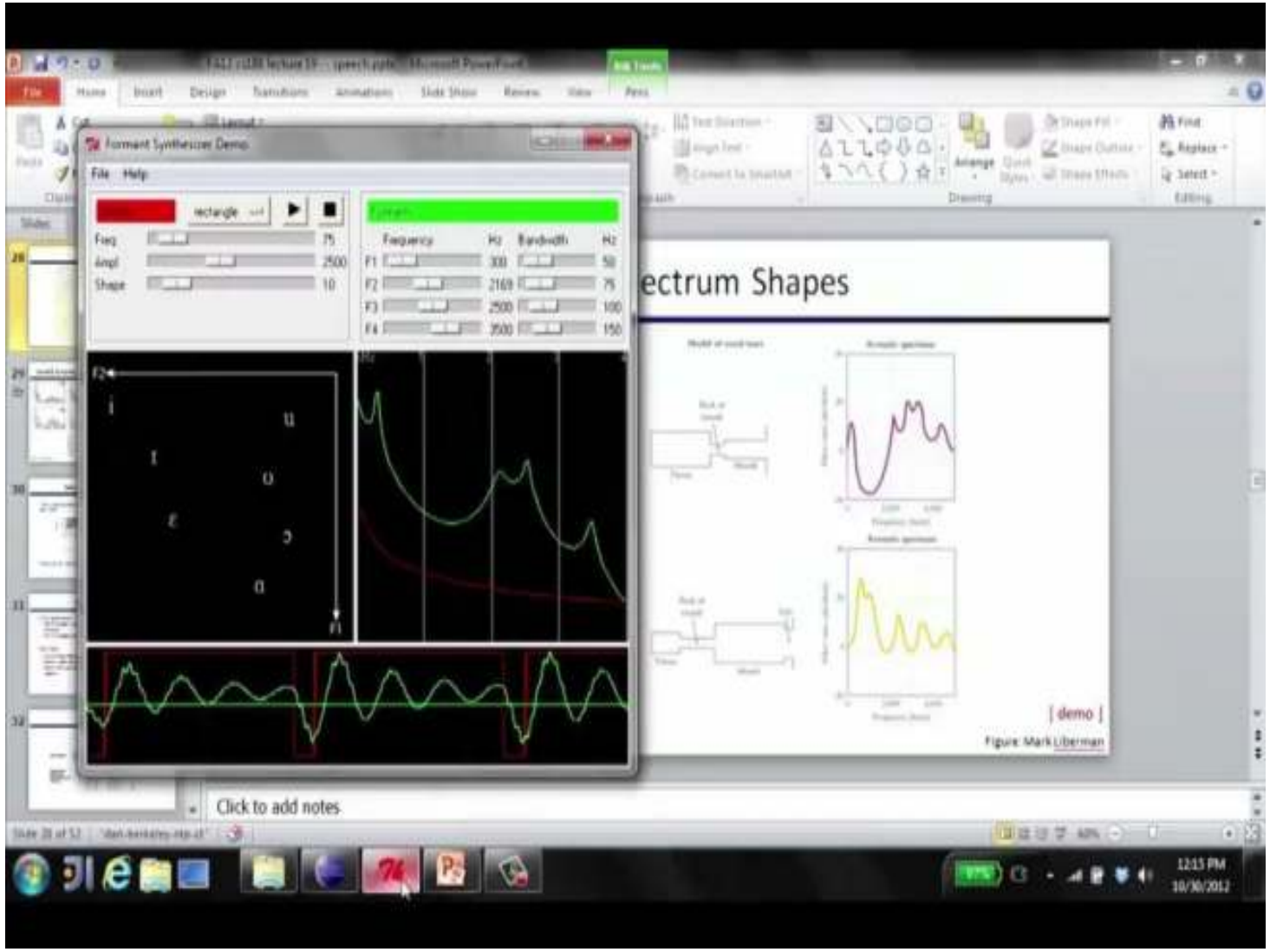


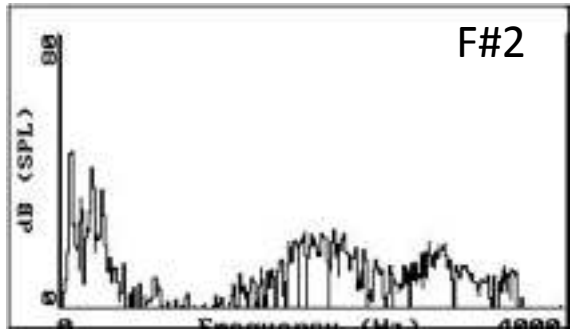
Figure: Mark Liberman

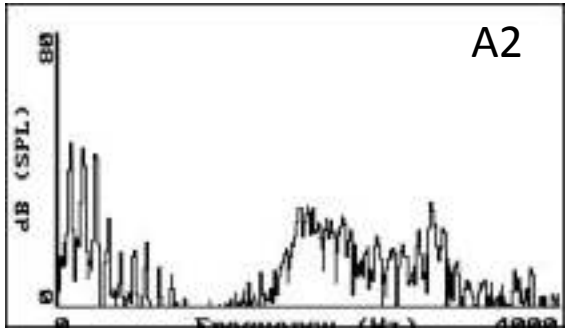
[Demo: speech synthesis]

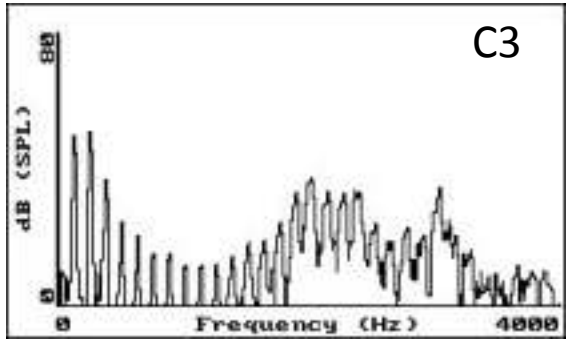
Video of Demo Speech Synthesis

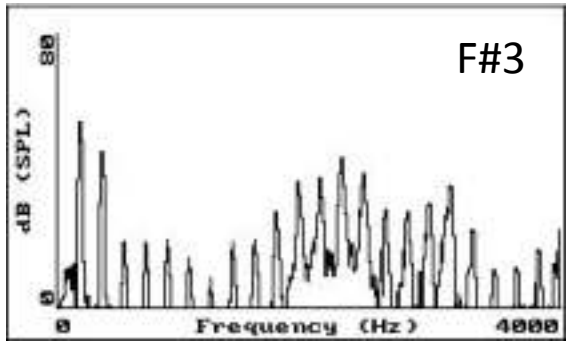


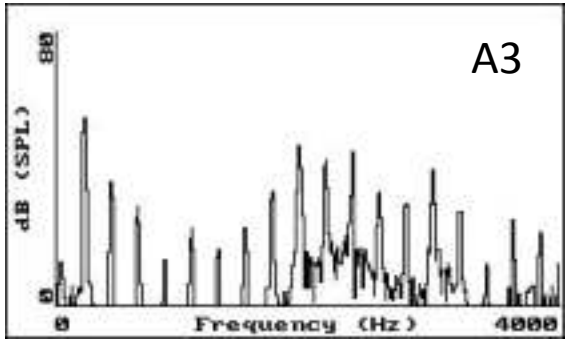
Vowel [i] sung at successively higher pitches

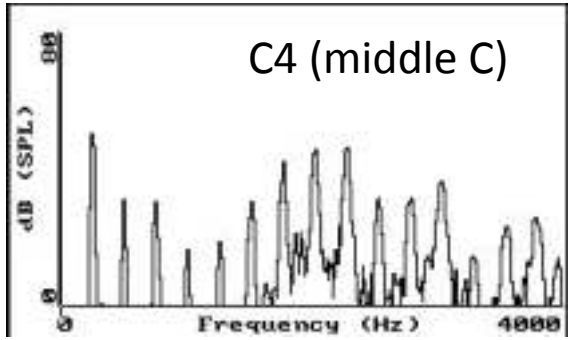


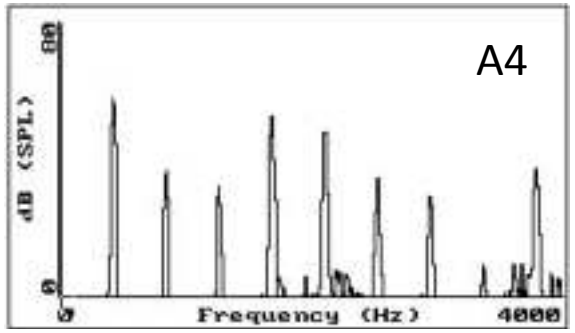


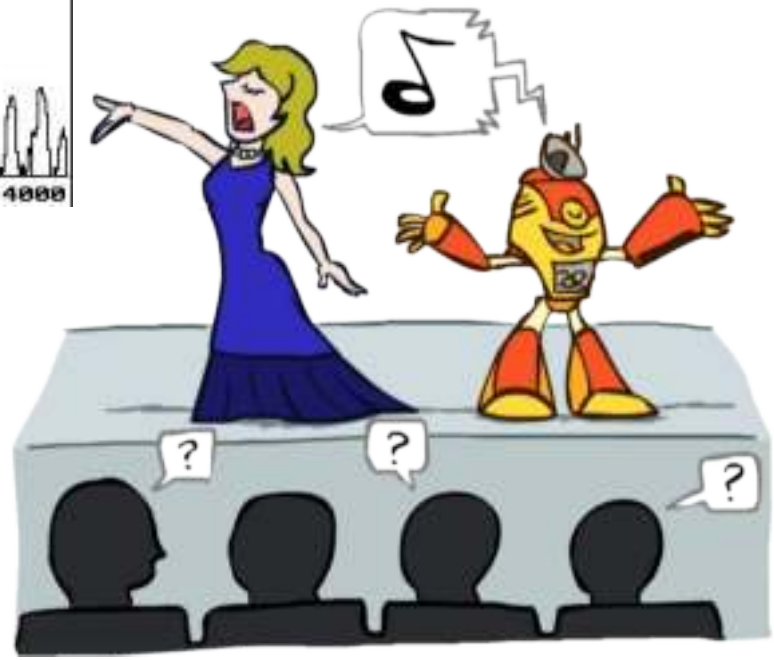








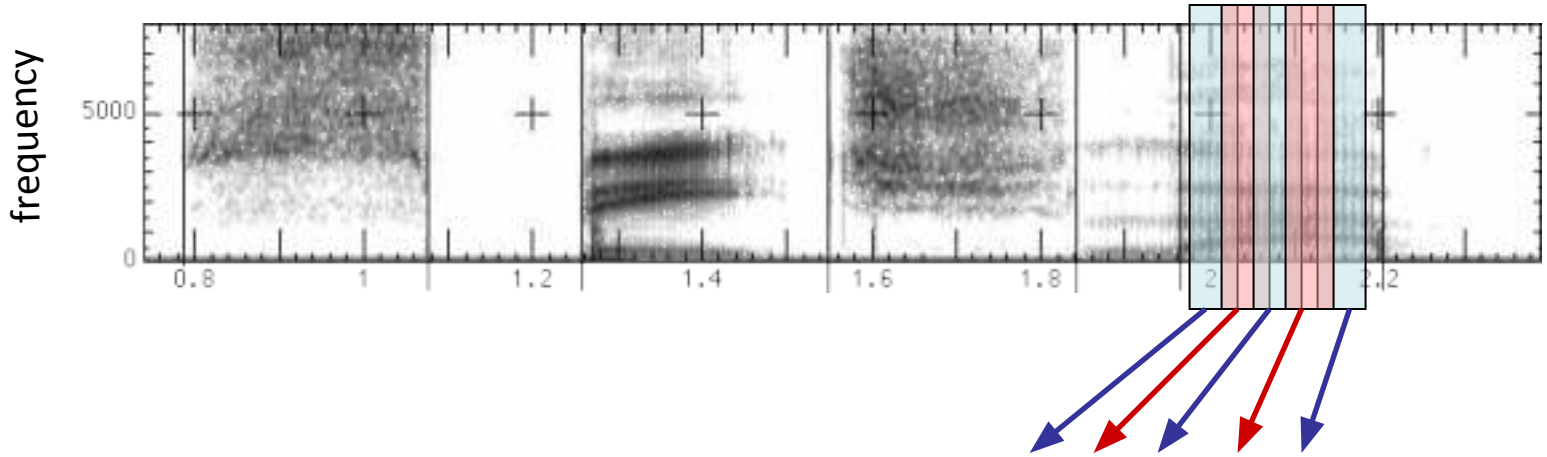




Graphs: Ratree Wayland

Acoustic Feature Sequence

- Time slices are translated into acoustic feature vectors (~39 real numbers per slice)



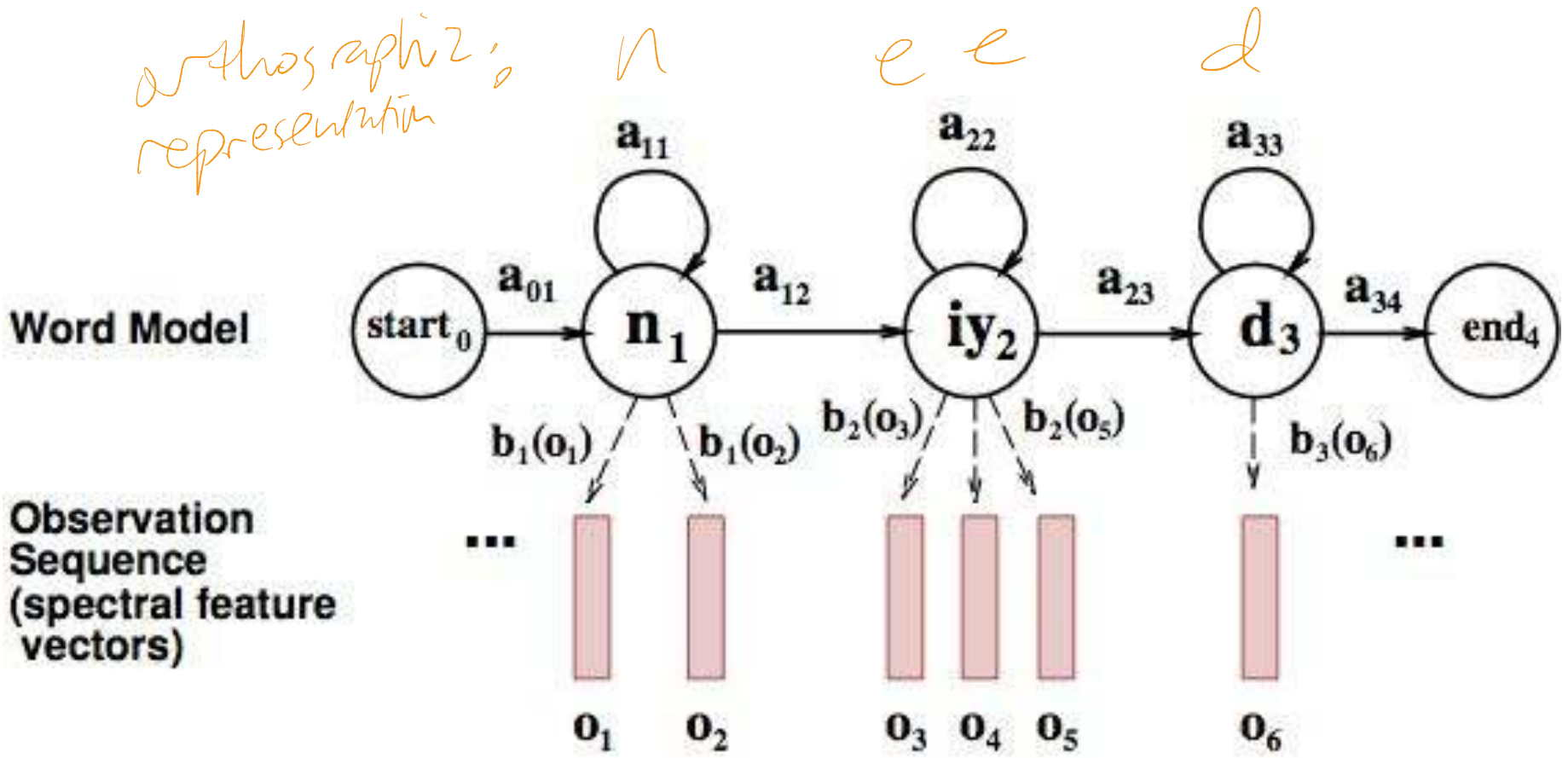
- These are the observations E, now we need the hidden states X

Speech State Space

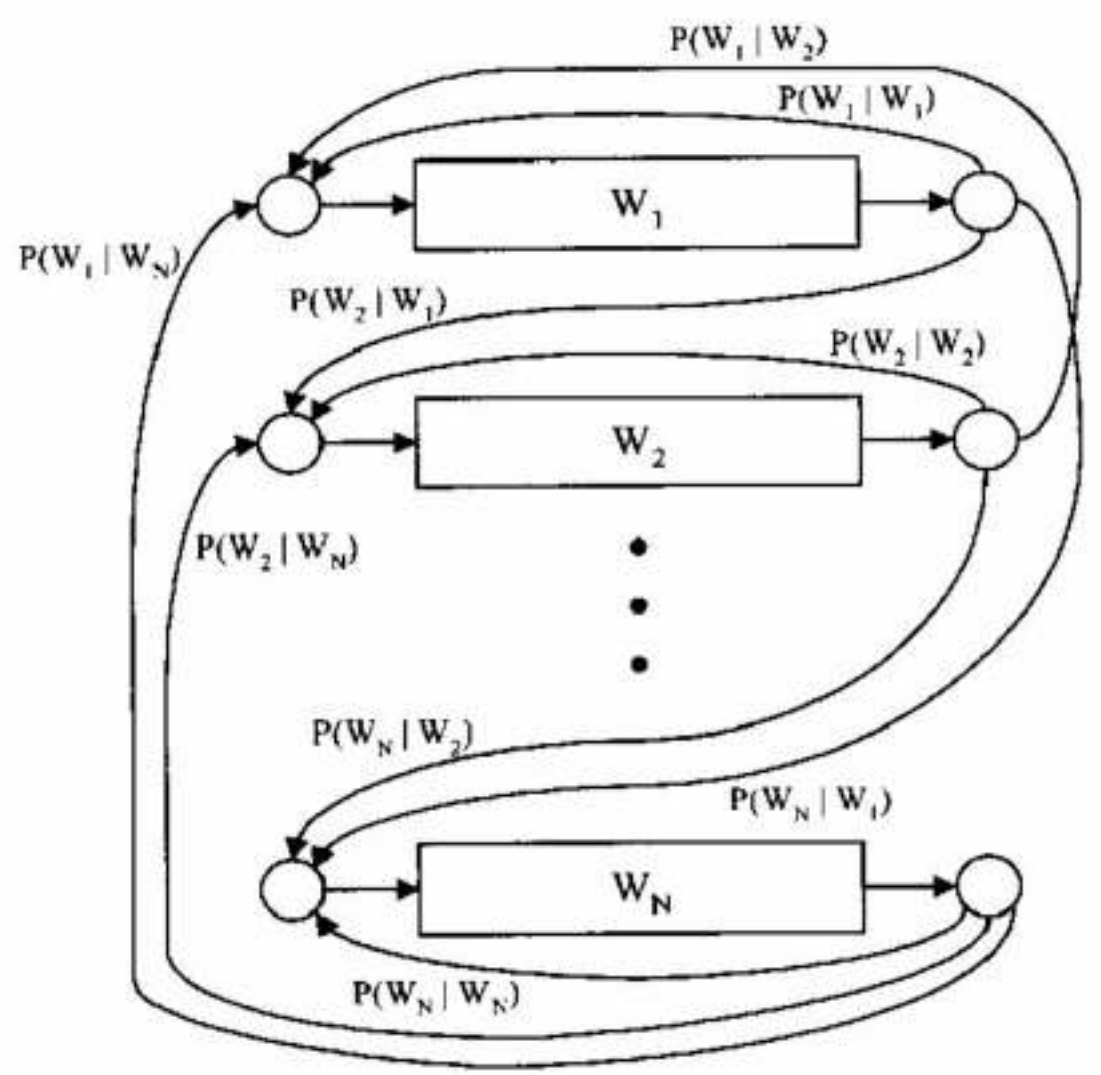
- HMM Specification
  - P(E|X) encodes which acoustic vectors are appropriate for each phoneme (each kind of sound)
  - P(X|X') encodes how sounds can be strung together

- State Space
  - We will have one state for each sound in each word
  - Mostly, states advance sound by sound
  - Build a little state graph for each word and chain them together to form the state space X

States in a Word



Transitions with a Bigram Model



looks at previous word to predict next word.

Training Counts

198015222 the first
194623024 the same
168504105 the following
158562063 the world
…
14112454 the door

23135851162 the *

the is said this # times

$$\hat{P}(\text{door}|\text{the}) = \frac{14112454}{23135851162}$$

$$= 0.0006$$

chance door is next word after the.

Figure: Huang et al, p. 618

Decoding

- Finding the words given the acoustics is an HMM inference problem
- Which state sequence $x_{1:T}$ is most likely given the evidence $e_{1:T}$?

Ex: Find most likely sequence of sounds given evidence

$$x^*_{1:T} = \arg\max_{x_{1:T}} P(x_{1:T}|e_{1:T}) = \arg\max_{x_{1:T}} P(x_{1:T}, e_{1:T})$$

- From the sequence x, we can simply read off the words

Internet Espionage

Challenge

- Setting
  - User we want to spy on use HTTPS to browse the internet
- Measurements
  - IP address
  - Sizes of packets coming in
- Goal
  - Infer browsing sequence of that user

- E.g.: medical, financial, legal, …

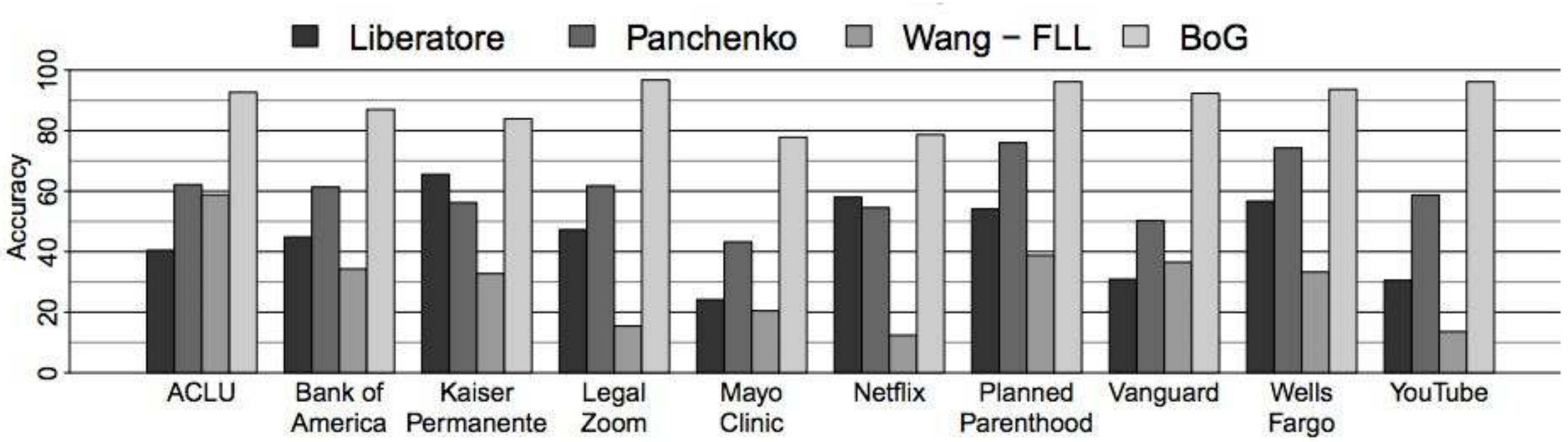
HMM

- Transition model
  - Probability distribution over links on the current page + some probability to navigate to any other page on the site

- Noisy observation model due to traffic variations
  - Caching
  - Dynamically generated content
  - User-specific content, including cookies

  Probability distribution P(packet size | page)

Results



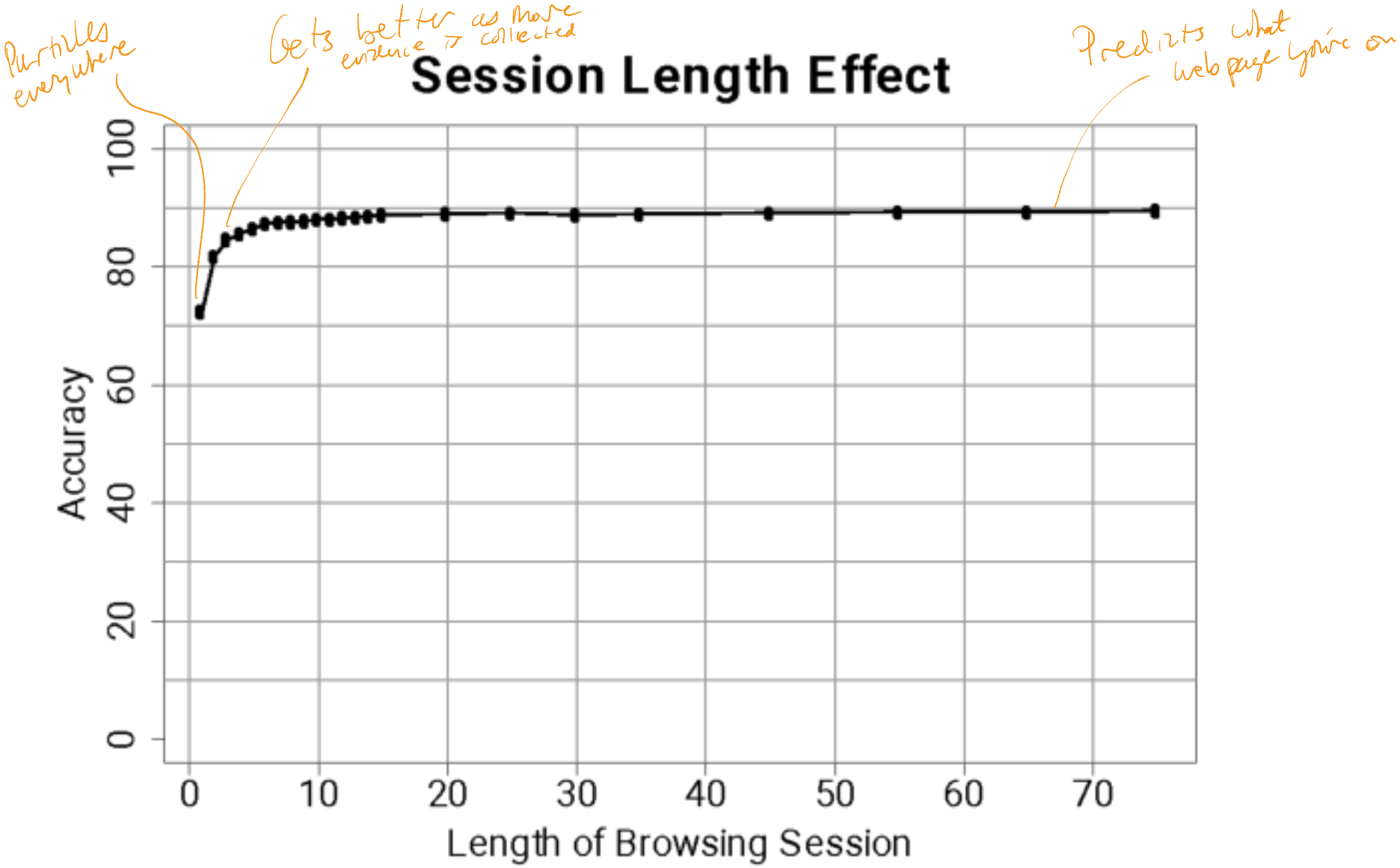
BoG = described approach, others are prior work

Premise:

At the beginning particles everywhere until it collects enough evidence to predict where the ghost is.

HM

Results


Particles everywhere
Gets better as more evidence is collected
Predicts what webpage you're on
Session Length Effect
Accuracy
0
20
40
60
80
100
0
10
20
30
40
50
60
70
Length of Browsing Session

Ethics

Stock answers:

- Maybe some espionage okay
- To defend against attacks, need to know what they are

Another stock answer:

- Maybe it's bad to spy on people

HMS can track your browsing, so you can protect yourself by randomly generating spam that confuses the HM.

Ethics

Volkswagen emissions scandal

Article Talk

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"Dieselgate" and "Emissionsgate" redirect here. For other diesel emissions scandals, see Diesel emissions

The **Volkswagen emissions scandal**, sometimes known as **Dieselgate**[23][24] or **Emissionsgate**,[25][24] began in September 2015, when the United States Environmental Protection Agency (EPA) issued a notice of violation of the Clean Air Act to German automaker Volkswagen Group.[26] The agency had found that Volkswagen had intentionally programmed turbocharged direct injection (TDI) diesel engines to activate their emissions controls only during laboratory emissions testing, which caused the vehicles' NO_x output to meet US standards during regulatory testing. However, the vehicles emitted up to 40 times more NO_x in real-world driving.[27] Volkswagen deployed this software in about 11 million cars worldwide, including 500,000 in the United States, in model years 2009 through 2015.[28][29][30][31]